Fetal Electrocardiogram Extraction by Blind Source Subspace Separation

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***Abstract*—In this paper, we propose the emerging technique of independent component analysis, also known as blind source separation, as an interesting tool for the extraction of the antepartum fetal electrocardiogram from multilead cutaneous potential recordings. The technique is illustrated by means of a real-life example.**

***Index Terms*—Blind source separation, fetal electrocardiogram, independent component analysis, singular value decomposition.**

I. Introduction

LIKE for adults, it should be possible to visualize the electrical activity of a fetal heart: the *fetal electrocardiogram* (FECG) contains important indications about the health and condition of the fetus. In this respect, analysis of the (instantaneous) *fetal heart rate* (FHR) has become a routine procedure for the evaluation of the well-being of the fetus. The cardiac waveform reveals important diagnostic information as well, e.g., for the diagnosis of arrhytmia.

During delivery accurate recordings can be made by placing an electrode on the fetal scalp. However as long as the membranes protecting the child have not been broken (*antepartum*), one should look for noninvasive techniques. Among the different approaches (measuring of the FHR from a Doppler-shifted ultrasonic heart echo, processing of the fetal magnetocardiogram, phonocardiography, ...), examination of the FECG from ECG-recordings measured on the mother's skin (*cutaneous* recordings) plays an important role.

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The aim of this paper is to show that the emerging technique of *independent component analysis (ICA)*, often called *blind source separation (BSS)*, is a promising tool for the estimation of the FECG from recordings on the mother's skin. We introduced this idea in [9]; the current paper is the first elaborated version of it. Due to lack of space, not all the aspects can be covered in detail. A more elaborated version of this text is available [12]; it contains links to medical applications, places the ECG-approach against other methods for the determination of the FHR, and gives a brief overview of existing signal processing methodologies to examine ECG-recordings.

In Section II, we motivate that cutaneous recordings contain instantaneous linear mixtures of MECG and FECG. The ICA-method itself is further discussed at a conceptual level in Section III, and in its relation to the FECG extraction problem in Section IV. Section V contains application examples.

II. Data Model

Potential measurements on the mother's skin contain contributions from several bioelectric phenomena (maternal and fetal heart activity, potential distributions generated by respiration and stomach activity, ...) and are affected by various kinds of noise (thermal noise, noise from electrode-skin contact, ...). Two aspects have to be discussed here: first, the nature of the occurring signals, and secondly, the characteristics of the propagation from bioelectric source to electrode.

In [18], it is shown that, at a considerable distance from the mother heart, its activity as a bioelectric current source can be represented in first-order approximation by a three-dimensional (3-D) vector signal, that can be imagined as the effect of a rotating current dipole in the chest. The 3-D vector space, described by the discrete-time evolution of the maternal ECG (MECG) after sampling, will be called the *MECG-subspace*. On the other hand [17] states that the observed "dimension" of the fetal heart, i.e., the number of independent signals describing its electrical activity, is not necessarily equal to three, but subject to changes during the period of pregnancy. In this paper, the term *FECG-subspace* will be used. In comparison with the low-voltage range of the FECG, other electrical signals can play an important role too: electromyographic activity (electrical potentials generated by the muscles, the uterus, etc.), 50-Hz net-interference, etc.

The transfer from bioelectric current source to body surface electrode can be assumed linear and resistive [18]. On the other hand the bioelectric source signals are relatively narrow-band, such that the frequency at which the cutaneous potential distribution is sampled (typically 250–500 Hz) can be considered as low, taking into account the high propagation velocity of the

electrical signals. Hence, in first approximation, cutaneous potential measurements can be considered as instantaneous linear mixtures of potential signals generated by underlying bioelectric phenomena; noise can be taken into account as an additive perturbation.

III. ICA

Assume the following basic linear statistical model:

$$Y = \mathbf{M}X + N \tag{1}$$

in which $Y \in \mathbb{R}^I$ is referred to as the *observation vector*, $X \in \mathbb{R}^J$ is called the *source vector* and $N \in \mathbb{R}^I$ represents additive *noise*. $\mathbf{M} \in \mathbb{R}^{I \times J}$ is the *mixing matrix*.

The goal of ICA now consists of the estimation of the transfer matrix $\mathbf{M}$ and/or the corresponding realizations of the source vector X, given only realizations of the output vector Y, under the following assumptions:

- the columns of $\mathbf{M}$ are linearly independent;
- the components of X are mutually statistically independent, as well as statistically independent from the noise components.

Most of the current ICA-algorithms rely on the first assumption for identifiability. The second assumption is the actual key ingredient for ICA. It is a very strong hypothesis, but also quite natural in lots of applications.

It is impossible to determine the norm of columns of $\mathbf{M}$ in (1), since a rescaling of these vectors can be compensated by the inverse scaling of the source signal values. Similarly the ordering of the source signals, having no physical meaning, cannot be identified. For non-Gaussian sources, these indeterminacies are the only way in which an ICA-solution is not unique [8], [20].

The ICA-assumptions do not allow to distinguish between the signal and the noise term in (1). Hence, the source signals will be estimated as $\hat{X}$, by a simple matrix multiplication

$$\hat{X} = \mathbf{W}^T Y. \tag{2}$$

As an example, $\mathbf{W}^T$ can take the form of the pseudoinverse $\hat{\mathbf{M}}^\dagger$, with $\hat{\mathbf{M}}$ an estimate of the mixing matrix. More generally, various beamforming strategies [22] can be applied.

Exploitation of the fact that the source signals are uncorrelated leads to a classical *principal component analysis* (PCA), which only allows to estimate the sources as well as the mixing matrix up to an orthogonal transformation. To illustrate this, let us assume that the sources have unit variance. Then we have (we omit the noise term at this point, for clarity)

$$\mathbf{C}_Y = \mathbf{M}\mathbf{M}^T \tag{3}$$

in which $\mathbf{C}_Y$ is the covariance matrix of Y. Substitution of the singular value decomposition (SVD) of the mixing matrix $\mathbf{M} = \mathbf{U}\mathbf{S}\mathbf{V}^T$ shows that the eigenvalue decomposition (EVD) of the observed covariance allows to estimate the column space of $\mathbf{M}$ while the factor $\mathbf{V}$ remains unknown

$$\mathbf{C}_Y = \mathbf{U}\mathbf{S}^2\mathbf{U}^T = (\mathbf{U}\mathbf{S})(\mathbf{U}\mathbf{S})^T. \tag{4}$$

As is well known, $\mathbf{U}$ and $\mathbf{S}$ might be found directly, in a numerically more reliable way, from the SVD of the observed dataset [13].

The solution to the ICA-problem lies in the fact that the assumption of *statistical independence* is stronger than the notion of *uncorrelated* signals. Statistical independence is not only a claim on the second-order statistics of the signals, but also on their higher order statistics (HOS) [16]. More precisely, it is not sufficient that the source covariance $\mathbf{C}_X$ is a diagonal matrix—in addition, the higher order cumulants of the source vector should be diagonal higher order tensors. (A higher order tensor can intuitively be imagined as a multi-way matrix, of which the entries are characterized by more than two indexes; its diagonal is defined as the entries for which all the indexes are equal.)

If we focus at the fourth-order level (third-order cumulants vanish for even probability density functions), then we have the following. The fourth-order cumulant $\mathcal{C}_X^{(4)}$ of a real zero-mean stochastic vector X is defined by

$$\begin{aligned}(\mathcal{C}_X^{(4)})_{i_1 i_2 i_3 i_4} \stackrel{\text{def}}{=} {} & \mathrm{E}\{X_{i_1}X_{i_2}X_{i_3}X_{i_4}\} - \mathrm{E}\{X_{i_1}X_{i_2}\}\mathrm{E}\{X_{i_3}X_{i_4}\} \\ & - \mathrm{E}\{X_{i_1}X_{i_3}\}\mathrm{E}\{X_{i_2}X_{i_4}\} \\ & - \mathrm{E}\{X_{i_1}X_{i_4}\}\mathrm{E}\{X_{i_2}X_{i_3}\}\end{aligned} \tag{5}$$

for all index values; E denotes the expectation. For every component X_i of X that has a nonzero mean, X_i has to be replaced by $X_i - \mathrm{E}\{X_i\}$. It can be proven that the link between the cumulant of the observations and the cumulant of the sources is a straight generalization of its second-order counterpart, (3)

$$\begin{aligned}(\mathcal{C}_Y^{(4)})_{i_1 i_2 i_3 i_4} = \sum_{j_1 j_2 j_3 j_4} & (\mathbf{M})_{i_1 j_1}(\mathbf{M})_{i_2 j_2} \\ & \cdot(\mathbf{M})_{i_3 j_3}(\mathbf{M})_{i_4 j_4}(\mathcal{C}_X^{(4)})_{j_1 j_2 j_3 j_4}\end{aligned} \tag{6}$$

for all index values, in which $\mathcal{C}_X^{(4)}$ is diagonal. A nice property is that higher order cumulants are insensitive to additive Gaussian noise. Equation (6) means that the unknown mixing matrix $\mathbf{M}$ is not only a diagonalizer of the covariance matrix $\mathbf{C}_Y$, but also of the cumulant tensor $\mathcal{C}_Y^{(4)}$, which leads to a sufficient amount of constraints to solve the problem. From an algebraic point of view, this means that the ICA-solution can be obtained by means of multilinear generalizations of the EVD (see e.g., [6], [8], and [10]). Actually, since the first paper on the subject [14], ICA has become a hot topic in the signal processing world. Apart from multilinear algebra, solutions have been based on principles of neural networks, information theory, etc. Instead of discussing one particular algorithm, we refer the reader to [7], [15] and the references therein.

Although generally PCA does not allow to identify the mixing matrix nor the source signals, there are some cases in which it does lead to a reasonably good source separation. A straightforward example consists of the situation in which the mixing matrix has mutually orthogonal columns (having mutually distinct norms, if we assume that the sources have unit variance), as is clear from (4). A second example is the situation in which the source variances are very different (assuming that the norms of the corresponding columns of $\mathbf{M}$ have a comparable magnitude). Next, consider a setup with, e.g., two sources, of which

the variances are given by σ_1^2 and σ_2^2, with $\sigma_1^2 \gg \sigma_2^2$. Reference [21] proved that in this case PCA yields, for both source estimates, an interference-to-signal ratio of the order of σ_2^2/σ_1^2. This corresponds to the fact that the dominant eigenvector of $\mathbf{C}_Y$ turns out to be an accurate estimate of the first column of $\mathbf{M}$ in this scenario; the second eigenvector however, is not necessarily a good estimate of the second column of $\mathbf{M}$ but it is approximately orthogonal to the first one. In the context of research on ICA, similar results have independently been obtained in [11] and [19].

IV. Extraction of the FECG by Means of BSSS

As explained in Section II, the propagation of q bioelectric sources to an array of p body surface electrodes ($p \geqslant q$), can be formulated as

$$Y(t) = \mathbf{M}X(t) + N(t) \tag{7}$$

where $Y(t) = (y_1(t) \cdots, y_p(t))^T$ contains the potential recordings, $X(t) = (x_1(t) \cdots, x_q(t))^T$ contains the signal values of the bioelectric sources, and the noise on each channel is represented by $N(t) = (n_1(t) \cdots n_p(t))^T$. The matrix $\mathbf{M}$ describes the propagation from source to electrode, i.e., its entry with row number i and column number j gives the gain of the jth bioelectric source signal with respect to the ith channel data ($1 \leqslant i \leqslant p$; $1 \leqslant j \leqslant q$). It is natural to assume that the different bioelectric sources—since they originate at different locations, correspond to different mechanisms, etc.—can be approximately modeled as statistically independent. The noise components $n_i(t)$ ($1 \leqslant i \leqslant p$) are assumed to be Gaussian, with variance σ_N^2, mutually independent as well as independent from the source signals.

As a conclusion, the derivation of the antepartum FECG from multilead cutaneous recordings can be considered as an example of BSS, as discussed in Section III, in which however the sources are of a multidimensional nature; we will use the term *blind source subspace separation* (BSSS). The fact that only the different source subspaces have to be separated, instead of all the source components allows to reduce the computational cost, in comparison to conventional ICA, without loss of medical information. For example, in the Jacobi-type algebraic algorithms of [6], [8], [10] the multidimensional character of the sources limits the number of Jacobi-rotation angles that have to be identified, since rotations of the basis vectors within one and the same source subspace are irrelevant.

Since there is a large gap between the amplitudes of the MECG and the FECG, a good separation can already be expected from merely PCA, as explained in Section III. This is the philosophy behind the important class of SVD-techniques for the extraction of the FECG [3]–[5]. To enhance the performance, one often tries to choose the electrode positions in a way that is more or less likely to correspond to an orthogonal transfer (see also Section III), but this is still a matter of heuristic rules and trial-and-error.

Conceptually, the higher order processing step in ICA may add the following advantages to the second-order approach.

- It is possible to enhance the quality of separation: whereas the PCA-error only decreases proportionally to the ratio of the power of the weak source vs the power of the strong source, ICA directly aims at a correct reconstruction of the mixing matrix. Section V contains an illustration. In case the higher order ICA-step would fail, one can still resort to the results of the PCA, which forms the first step in many ICA-algorithms.
- The propagation of the electrical signals can be characterized in an essentially unique way. We mention three important implications:

  The transfer vectors indicate how strongly the different electrodes capture each source signal; from this information, better measurement positions might be deduced. We mention that the positioning of the electrodes is still the most crucial factor for the success of the PCA-method [5].

  An important aspect in the evaluation of the fetal well-being is the quantification of fetal movements [4]. At this moment the required information can only be obtained by echography or, simply, by asking the mother. The number of significant changes in the FECG-subspace, which could be obtained from an on-line adaptive ICA-implementation, could be very useful information here.

  The properties of the human body as a conducting medium are, in their own, subject of medical research [18]. The study of the propagation of the fetal heart signal to the mother's skin is an important subaspect [17]. The transfer matrix can provide more understanding with respect to the propagation of electrical signals through the body.
- The physician can resort to a more intuitive interpretation of the results: the separation of the measured signals into statistically independent source signals with a physical meaning, is easier to interpret than a decomposition in time-orthogonal principal components.

We stress the fact that the FECG-extraction is formulated as a *blind identification* problem, since it is less meaningful in practice to resort to a more parametric approach.

- The transfer coefficients are subject to a large uncertainty: the development of propagation models is still in its infancy. Moreover, it is clear that length, weight, contour, etc. are significantly different from patient to patient.
- The geometrical and resistivity parameters of the body of a single patient are not constant in time. Fetal growth, a different position of the fetus in the uterus, the variation in the characteristics of the amniotic fluid and the placenta during pregnancy, the changing geometry, ... imply important changes of the transfer matrix.
- For the application in medical diagnosis and treatment it is crucial that *un*expected ECG-patterns can be detected and examined. For example, the parametric formulation of the quasiperiodicity of a regular heart rate pattern would hamper the detection of extrasystoles (extra heartbeats between the regular beat-to-beat pattern).
- Potentially interesting is also the application of BSSS to cardiac electrical imaging, a recent generalization of the

ECG, in which more information is acquired by using a larger array of (e.g., 200) electrodes to record a sequence of "electrical images" of the body [2]. This technique can be seen as an emerging modality for medical imaging, complementary to, e.g., computed tomography and magnetic resonance imaging; it is worth mentioning that in Japan the technique is already common practice.

We may conclude that conceptually BSSS is a very promising technique to tackle the problem of FECG-extraction. Section V contains a real-life example. At this moment however, our database is too limited to assess to which extent the assumptions, underlying the ICA-model, are valid in medical practice. With this respect, hard conclusions on the merits and drawbacks of the method can only be drawn after intensive medical testing.

V. Examples

Fig. 1 shows the first 5 seconds of a set of potential signals measured in a 1 min eight-channel experiment. The horizontal axis displays the time in seconds; with respect to the vertical axes only the relative values are important. The sampling frequency was 500 Hz. For details about the data acquisition we refer to [5]. Channels 1–5 show abdominal signals; for channels 6–8 the electrodes have been placed further away from the fetus, e.g., on the thorax. Channels 1 and 3 clearly contain weak fetal contributions. Due to the large amplitudes of the MECG in the thoracic signals, the FECG is less visible.

The source estimates after PCA are displayed in Fig. 2. Two MECG-free FECG-components were obtained as, respectively, the sixth and the seventh right singular vector of the data-matrix. The signals 1 and 2 partially describe the MECG-subspace; the MECG also appears in signals 3 and 5. Channels 4 and 8 mainly show noise contributions.

The result after BSSS is shown in Fig. 3 (we used the algorithm proposed in [8], which is an approximate maximum-likelihood solver; e.g., the methods reported in [6], and [10] yield comparable results). The result is an excellent source separation. We remark that, just like in the PCA-approach [3]–[5], the statistics of the nonstationary signals have been estimated "roughly" by simple time-averaging. Whereas the PCA-method obtained only two clear MECG-components (the third signal is heavily perturbated by noise and the fifth principal component contains important FECG-contributions), BSSS accurately reconstructed the full 3-D MECG-subspace (signals 1–3 in Fig. 3). As far as the FECG is concerned, the quality of the seventh principal component and the eighth BSSS-signal are comparable, but in the sixth BSSS-signal the signal-to-noise ratio is somewhat better than in the sixth PCA-estimate. The off-set in the sixth PCA-signal is found back as an extra source signal (the seventh signal in Fig. 3; this sequence continues as a low-periodic signal and deserves further medical interpretation—it might, e.g., be due to respiration). The fifth BSSS-signal mainly shows noise contributions.

Figs. 4 and 5 visualize some information extracted from the sixth ICA-component. Fig. 4 plots the evolution of the instantaneous beat-to-beat FHR. Fig. 5 shows the average FECG waveform. In short, we first determined the position of the fetal heartbeats by developing a high-precision robust fetal QRS-complex detector (the QRS-complex is the central part of the cardiac waveform, with high potential values); both an expert-system and a pattern classification approach were followed. In a second step, the instantaneous FHR and the average waveform were calculated as accurately as possible by maximizing the correlation

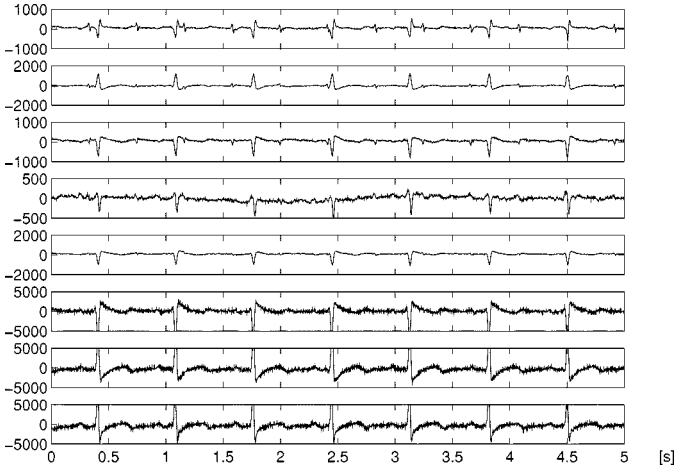

Fig. 1. Eight-channel set of cutaneous data recordings.

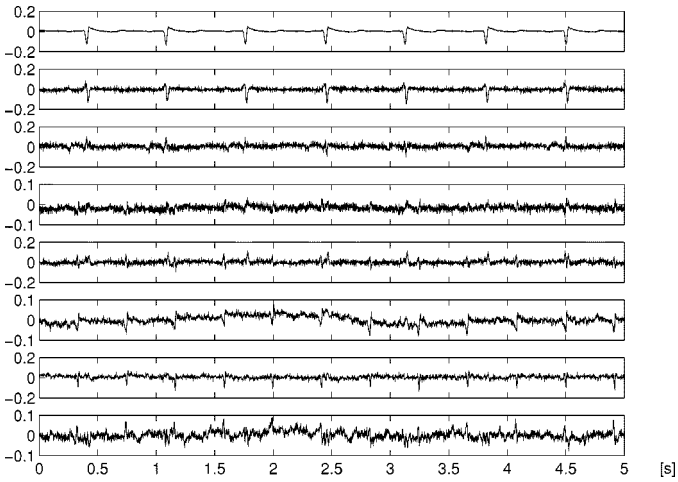

Fig. 2. Source estimates obtained by means of PCA.

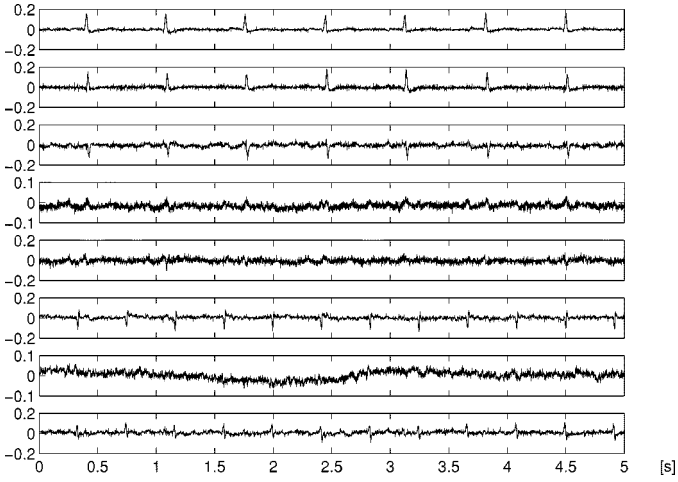

Fig. 3. Source estimates obtained by means of BSSS.

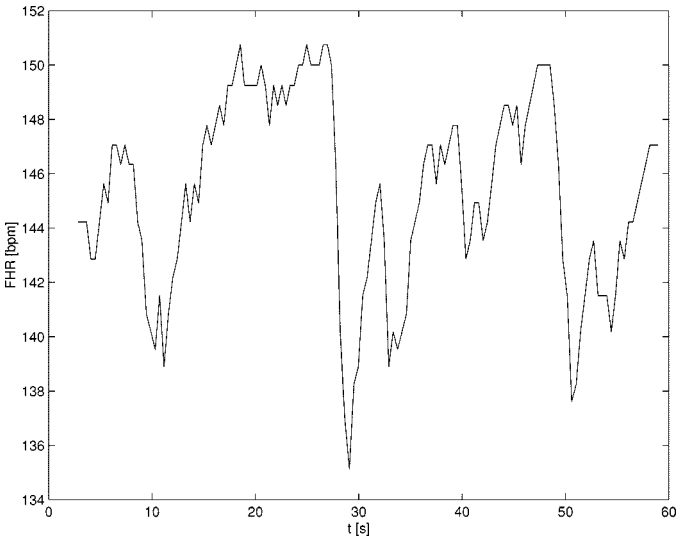

Fig. 4. Evolution of the instantaneous FHR.

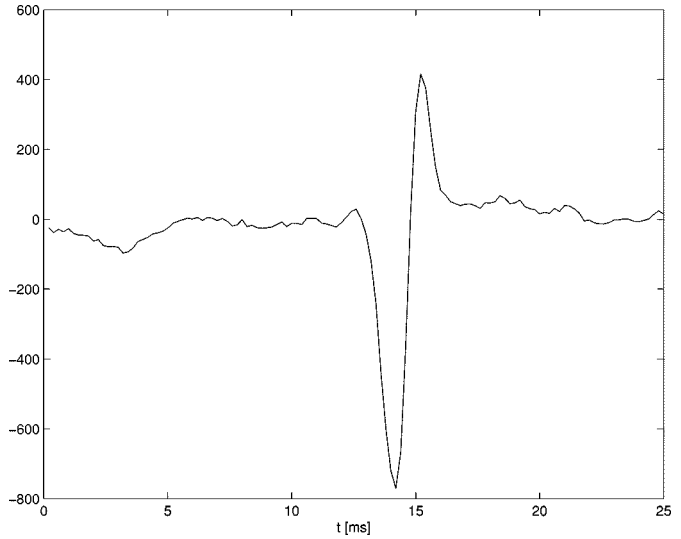

Fig. 5. Average waveform of the fetal heartbeat in the sixth ICA component (Fig. 3).

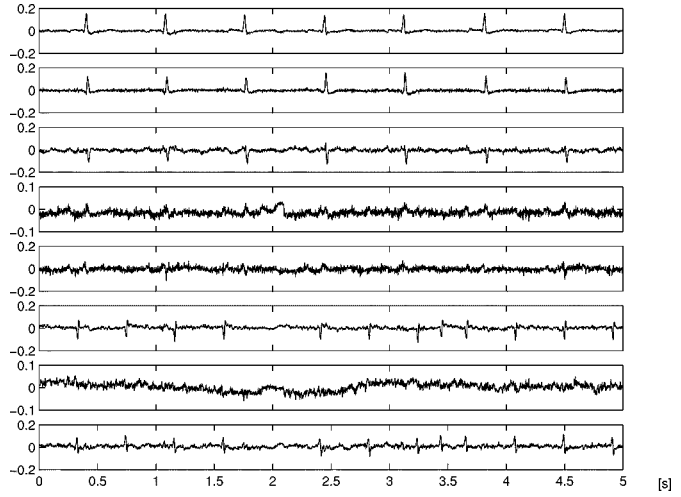

Fig. 6. Source estimates obtained by means of BSSS from data, containing an extrasystole around $t = 3.5$ s and missing a fetal heartbeat around $t = 2$ s.

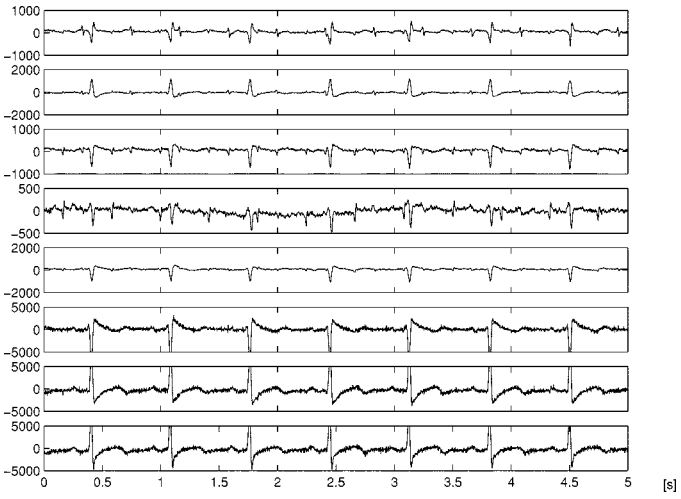

Fig. 7. Eight-channel set of observations containing heartbeats of fetal twins.

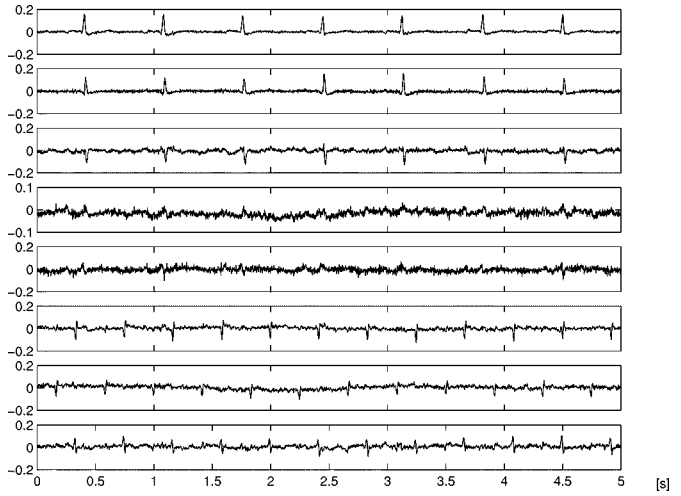

Fig. 8. Source estimates obtained from the data in Fig. 7 by means of BSSS.

between consecutive pulses. For details about the procedure we refer to [1].

Figs. 7 and 8 show an artificially constructed situation of fetal twins. The data of Fig. 7 were obtained as follows. First, the two fetal ICA-components of Fig. 3 were shifted over approximately $t = -0.25$ s to artificially generate an independent heartbeat, to be attributed to a second fetus. These signals were added to the original dataset after multiplication by mixing vectors, obtained by independent random permutations of the abdominal and the thoracic entries of the original mixing vectors; the permutations are meant to ensure that the dimensionality of the intersection of both FECG-subspaces is zero. Fig. 8 shows that eight-channel data were sufficient for the extraction of a two-dimensional FECG-subspace (channels 6 and 8; first fetus) and an additional FECG signal (channel 7; second fetus).

Fig. 6 illustrates what happens in the case of an atypical FHR and shows the importance of a blind approach, as already motivated in Section IV. The input for the ICA algorithm was constructed as follows. A small piece of data around $t = 0.75$ s in Fig. 1 was copied to $t = 3.5$ s, to simulate an extrasystolic fetal heartbeat. In addition, the fetal heartbeat around $t = 2$ s was

skipped by setting the five abdominal signals to zero. Nevertheless, Fig. 6 still shows an excellent BSSS.

VI. Conclusion

In this paper, we have proposed BSSS as an innovating way to solve a classical problem in biomedical engineering, namely the extraction of the FECG from multilead potential recordings on the mother's skin. In comparison to the important class of SVD-based methods, proposed earlier, the higher order ICA-step additionally requires the estimation and the (partial) diagonalization of the fourth-order cumulant tensor of the data. From a conceptual point of view, ICA is a very ambitious approach: it aims at the direct reconstruction of the different statistically independent bioelectric source signals, as well as the characteristics of their propagation to the electrodes, each revealing important medical information. It is nonparametric and is not based on pattern averaging, which could hamper the detection and analysis of atypical fetal heartbeats.

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