3. Discussion by T. Van Gestel, B. De Moor, B.D.O. Anderson and P. Van Overschee

Frequency weighted balancing [1] is an important technique with interpretations and applications in system identification and controller design. Unfortunately, the result of this paper may seem pessimistic as properties, that made internally balanced truncation popular, are formmally disproven for the frequency weighted balanced case. By means of a constructive algorithm [4], counterexamples can be generated which allow to refute the upper error bound $E_\infty \leq 2(1+\alpha)\sum_{k=r+1}^{n}\sigma_k$ for any first order system $g(s)$ (that is being reduced) and for any value of $\alpha \in \mathbb{R}^+$. Simpler counterexamples are obtained when also the system $g(s)$ can be chosen, as is seen from the discussion above. Although this result may be intuitively clear, it is still somewhat surprising that the frequency weighted balanced singular values do not contain the necessary information for an upper error bound. Moreover, Example 2 shows that the relative value of the frequency weighted singular values of the same system give no information on which state truncation yields the best approximation.

A closer look at the frequency weighted balancing formulation reveals that the nice properties of internally balancing are lost because of the cross-terms in the expressions for the frequency weighted balancing case. However, these cross-terms are essential to have frequency weightings. Alternative frequency weighted balanced truncation methods have been proposed [3, 5], where our method yields a $B_\perp = 0$ when B has full row rank. These methods basically aim at reformulating the problem in such a way that results from internally balanced truncation [1, 2] can be used.

As endorsed by Dr. Hurak, we have pointed out problems in Enns' frequency weighted balancing formulation and refuted the conjecture on the upper error bound. As he does, we hope that this result may contribute to the development of alternative frequency weighted balanced truncation techniques that have more interesting properties. We agree that the Sreeram and Wang example now provides a simpler way to obtain a counter example than as illustrated in our paper. We endorse their other comments.

References

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