

MIMO Instantaneous Blind Identification Based on Second Order Temporal Structure

Jakob van de Laar, Marc Moonen, *Fellow, IEEE*, and Piet Sommen *Member, IEEE*

Abstract—Blind identification is a crucial subtask in signal processing problems such as Blind Signal Separation (BSS) and Direction Of Arrival (DOA) estimation. This paper presents a procedure for *Multiple-Input Multiple-Output Instantaneous Blind Identification* based on second order temporal statistical variabilities in the data, such as non-whiteness and non-stationarity. The procedure consists of two stages. Firstly, based on a number of assumptions on the statistical structure and diversity of the source signals and mixing system, and using subspace techniques, the problem is reformulated in a particular way such that each column of the unknown mixing matrix satisfies a system of multivariate homogeneous polynomial or ‘polyconjugal’ equations. Then, this nonlinear system is solved by means of a so-called homotopy method. The two-stage blind identification procedure also allows to estimate the mixing matrix for several scenarios with more sources than sensors, something that is often believed to be impossible with second order statistics. Finally, the procedure is applied to the Instantaneous Blind Signal Separation of speech signals.

Index Terms—Blind identification/separation, temporal structure, homogeneous system, homotopy, Second Order Statistics.

I. INTRODUCTION

IN this paper, we consider the so-called *Multiple-Input Multiple-Output (MIMO) Instantaneous Blind Identification (MIBI)* problem. In this problem, a number of mutually statistically independent source signals are mixed by a MIMO instantaneous mixing system and only the mixed signals are available, i.e. both the mixing system and the original source signals are unknown [5]. The goal of MIBI is to recover the instantaneous MIMO mixing system, or its parameters such as in the case of DOA estimation, from the observed mixtures of the source signals only. Fig. 1 shows the MIBI problem setup for S source and D sensor signals. The source, sensor and additive noise signals are denoted by $s_1[n], \dots, s_S[n]$, $x_1[n], \dots, x_D[n]$ and $\nu_1[n], \dots, \nu_D[n]$ respectively. The instantaneous mixing system is modeled by a real- or complex-valued matrix \mathbf{A} of size $D \times S$. A problem closely related to MIBI is *Instantaneous Blind Signal Separation (IBSS)* [4], [5], where the goal is to separate mutually statistically independent source signals from their observed instantaneous mixtures only. Contrary to MIBI, the main interest in IBSS is in the source signals instead of the mixing system. In fact, once MIBI has been performed, the source signals can be recovered (approximately) by applying the (pseudo-)inverse of

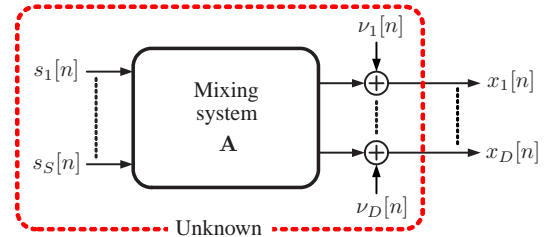


Fig. 1. MIMO Instantaneous Blind Identification setup.

the estimated mixing system to the observed mixtures. In this paper, the main focus is on MIBI, while IBSS is considered as an application.

Many researchers have investigated the use of second order statistics for IBSS [2], [3], [6], [9], [10], [15], [20]. The majority of the available algorithms is based on the (Generalized) Eigenvalue Decomposition or Joint Approximate Diagonalization of two or more sensor correlation matrices of the form $\mathbf{R}^x[n_1, n_2] \triangleq E\{\mathbf{x}[n_1]\mathbf{x}[n_2]^H\}$ with $\mathbf{x}[n]$ the observation vector (see also Section II). For example, see AMUSE [15], SOBI [2], and also [6], [9], [10]. All those algorithms employ sensor correlation matrices containing the sensor correlation values arranged in the same ‘conventional’ manner. In our work we arrange the available sensor correlation values in a particular fashion that allows a different and natural formulation of the problem and the estimation of more columns than sensors. It is widely recognized that many possible applications exist for MIBI and IBSS, see e.g. [4], [5], and the references therein. Examples of (parameterized) MIBI can be found in source localization problems, which are crucial to many sensor array systems, such as radar and sonar. Examples of IBSS can be found in the field of biomedical engineering, where the goal is e.g. to reveal independent sources in biological signals like EEG’s or ECG’s. Other examples can be found in the separation of speech signals, images, etc. Although many practical problems can be described more adequately by more complex MIMO Blind Identification models such as convolutive and/or non-linear models, MIBI can often be used as a good starting point, e.g. for a frequency domain approach in the convolutive case.

This work is a continuation and elaboration of our previous work presented in [16]–[19]. In [16], a practical algorithm based on Second Order Statistics (SOS) and the Generalized Eigenvalue Decomposition (GEVD) was given for the real-valued ‘square MIBI’ case with $D = S$. In [17], we generalized the underlying concepts to real-valued MIBI exploiting the temporal structure in the data of some arbitrary fixed order. Although we briefly touched upon the more general case with

J. van de Laar is with the Digital Signal Processing Group of Philips Research Laboratories, Eindhoven, The Netherlands.

M. Moonen is with the Electrical Engineering Department of the Katholieke Universiteit Leuven, Leuven, Belgium.

P. Sommen is with the Electrical Engineering Department of the Technische Universiteit Eindhoven, Eindhoven, The Netherlands.

complex-valued mixing system and source signals, arbitrary order statistics, and arbitrary conjugation patterns, in [18] the main focus was on the development of the so-called TIME-MUSIC algorithm for Direction Of Arrival estimation which involves a parameterized mixing matrix. In [19] we provided a homotopy method for estimating the mixing matrix from the derived system of equations for the real-valued SOS case. In this paper, we consider real- and/or complex-valued MIBI based on exploiting the Second Order Temporal Structure (SOTS) with arbitrary conjugation pair (see Section III), and again use a homotopy method for estimating the columns of the mixing matrix.

The outline of this paper is as follows. Firstly, the structure and assumptions of the MIBI model are explained in Sections II and III respectively. The derivation of the system of homogeneous polynomial equations is presented in Section IV, and some of the algebraic and geometric properties are highlighted. Then, in Section V we present a homotopy method for solving the system of equations. In Section VI the theory is applied to two MIBI scenarios with three sensors, one with three and the other with four speech source signals. Finally, conclusions are discussed in Section VII.

II. MIBI MODEL STRUCTURE

A block diagram of the MIBI problem setup is shown in Fig. 1. S mutually statistically independent but otherwise unknown source signals are mixed by an unknown MIMO linear instantaneous mixing system, which is represented by the matrix \mathbf{A} , and only D sensor signals $x_1[n], \dots, x_D[n]$ corrupted by D additive noise signals $\nu_1[n], \dots, \nu_D[n]$ are observed. Both the signals and the mixing system may be real- or complex-valued. For convenience and without loss of generality it is assumed that all signals are zero-mean. Mathematically, the MIBI observation model can be written as follows:

$$\mathbf{x}[n] = \sum_{j=1}^S \mathbf{a}^j s_j[n] + \boldsymbol{\nu}[n] = \mathbf{A}\mathbf{s}[n] + \boldsymbol{\nu}[n] \quad \forall n \in \mathbb{Z}, \quad (\text{II.1})$$

where:

$$\mathbf{x}[n] \triangleq \begin{bmatrix} x_1[n] \\ \vdots \\ x_D[n] \end{bmatrix}, \mathbf{s}[n] \triangleq \begin{bmatrix} s_1[n] \\ \vdots \\ s_S[n] \end{bmatrix}, \boldsymbol{\nu}[n] \triangleq \begin{bmatrix} \nu_1[n] \\ \vdots \\ \nu_D[n] \end{bmatrix}, \mathbf{a}^j \triangleq \begin{bmatrix} a_1^j \\ \vdots \\ a_D^j \end{bmatrix}$$

are column vectors of sensor signals, source signals, additive noise signals and mixing elements respectively. Let the symbols \mathbb{C}_M , \mathbb{C}^N , and \mathbb{C}_M^N denote the spaces of complex-valued length- M column vectors, length- N row vectors, and $M \times N$ -matrices respectively. \mathbb{R}_M , \mathbb{R}^N , and \mathbb{R}_M^N are defined similarly for real-valued quantities. The vectors $\mathbf{x}[n]$, $\boldsymbol{\nu}[n]$, and $\mathbf{a}^1, \dots, \mathbf{a}^S$ are elements of \mathbb{R}_D or \mathbb{C}_D , whereas the vector $\mathbf{s}[n]$ is an element of \mathbb{R}_S or \mathbb{C}_S . The unknown mixing matrix \mathbf{A} is an element of \mathbb{R}_D^S or \mathbb{C}_D^S , and can be written in terms of its columns as $\mathbf{A} = [\mathbf{a}^1 \ \dots \ \mathbf{a}^S]$. Sometimes we refer to the mixing matrix as the *array response matrix*, and to its columns as the *array response vectors*. Subscript and superscript indices are used to index the components of a column and row vector

respectively. Furthermore, the symbol n denotes discrete time. According to (II.1), the i -th sensor signal $x_i[n]$ is given by:

$$x_i[n] = \sum_{j=1}^S a_i^j s_j[n] + \nu_i[n] \quad \forall n \in \mathbb{Z}, \quad \forall 1 \leq i \leq D, \quad (\text{II.2})$$

where a_i^j denotes the instantaneous transfer coefficient from the j -th source to the i -th sensor, $s_j[n]$ the j -th source signal at discrete time n , and $\nu_i[n]$ the i -th noise signal at discrete time n . From Eq. (II.1), it follows directly that two indeterminacies are inherent to the MIBI model [4], [5], viz. the norms and order of the mixing matrix columns and the source signals cannot be resolved. This means that the columns and source signals can only be recovered up to a scaling factor and a permutation. Taking into account these indeterminacies, the goal of MIBI is to recover the columns of the mixing system in arbitrary order and with arbitrary nonzero norms. The indeterminacies are by no means problematic because for blind applications the most relevant information is in the ‘directions’ of the columns, or the waveforms of the source signals, rather than in their magnitudes or order.

III. MIBI MODEL ASSUMPTIONS

To be able to exploit the *Second Order Temporal Structure (SOTS)* in the data several assumptions have to be made that mainly serve two purposes. Firstly, they ensure that sufficient temporal structure is present in the source signals, and secondly that the noise signals have a ‘simpler’ temporal structure than the source signals. The assumptions are formulated in terms of correlation functions that we will define soon; see Equations (III.1) and (III.2). A (complex-valued) two-dimensional correlation function can be defined in different ways corresponding to the pattern in which its arguments are conjugated. Such a conjugation pattern is represented by a *conjugation pair*, and is written in the upper-right corner of the considered symbol or function. The most suitable conjugation pair for a particular application depends on the type of signals involved. In the derivations in this paper it is assumed without loss of generality that a specific conjugation pair (c_1, c_2) has been chosen and fixed subsequently, where c_1 and c_2 can either be ‘*’, which means conjugation, or ‘o’, which means no conjugation. For example, let $(v_{i_1}[n_1])^{c_1}$ be the i_1 -th component of a length- G time dependent random vector $\mathbf{v}[n]$ at time index n_1 that is conjugated according to c_1 , and let $(v_{i_2}[n_2])^{c_2}$ be defined similarly. Then, the correlation function of $(v_{i_1}[n_1])^{c_1}$ and $(v_{i_2}[n_2])^{c_2}$ is defined as follows:

$$r_{i_1 i_2}^{v, c_1 c_2}[n_1, n_2] \triangleq E \left\{ (v_{i_1}[n_1])^{c_1} (v_{i_2}[n_2])^{c_2} \right\} \\ \forall 1 \leq i_1, i_2 \leq G, \quad \forall n_1, n_2 \in \mathbb{Z}. \quad (\text{III.1})$$

Likewise, the two-dimensional correlation function of the i -th component of a length- G time dependent random vector $\mathbf{v}[n]$ at time index n_1 that is conjugated according to c_1 , and the j -th component of another length- P random vector $\mathbf{w}[n]$ at time n_2 that is conjugated according to c_2 , is denoted by $r_{ij}^{vw}[n_1, n_2]$ and defined as follows:

$$r_{ij}^{vw}[n_1, n_2] \triangleq E \left\{ (v_i[n_1])^{c_1} (w_j[n_2])^{c_2} \right\} \\ \forall 1 \leq i \leq G, \forall 1 \leq j \leq P, \quad \forall n_1, n_2 \in \mathbb{Z}. \quad (\text{III.2})$$

Let $\Omega_{n_1 n_2}^{s \setminus \nu, c_1 c_2}$ be the so-called Noise-Free Region Of Support (ROS) in the domain of time index pairs (n_1, n_2) on which all correlation functions considered in this paper are defined. Then, the assumptions defining the ROS and underlying the MIBI method presented in the sequel can be formulated as follows:

AS1: The source signals have zero cross-correlation functions on the Noise-Free ROS $\Omega_{n_1 n_2}^{s \setminus \nu, c_1 c_2}$:

$$r_{j_1 j_2}^{s, c_1 c_2}[n_1, n_2] = 0 \quad \forall 1 \leq j_1 \neq j_2 \leq S;$$

AS2: The source auto-correlation functions are linearly independent on the Noise-Free ROS $\Omega_{n_1 n_2}^{s \setminus \nu, c_1 c_2}$:

$$\sum_{j=1}^S \xi^j r_{jj}^{s, c_1 c_2}[n_1, n_2] = 0 \implies \xi^j = 0 \quad \forall 1 \leq j \leq S;$$

AS3: The noise signals have zero auto- and cross-correlation functions on the Noise-Free ROS $\Omega_{n_1 n_2}^{s \setminus \nu, c_1 c_2}$:

$$r_{i_1 i_2}^{\nu, c_1 c_2}[n_1, n_2] = 0 \quad \forall 1 \leq i_1, i_2 \leq D;$$

AS4: The cross-correlation functions between the source and noise signals are zero on the Noise-Free ROS $\Omega_{n_1 n_2}^{s \setminus \nu, c_1 c_2}$:

$$r_{ij}^{\nu, s, c_1 c_2}[n_1, n_2] = r_{ji}^{s, \nu, c_1 c_2}[n_1, n_2] = 0 \\ \forall 1 \leq i \leq D, \quad \forall 1 \leq j \leq S.$$

IV. FORMULATING MIBI AS THE PROBLEM OF SOLVING A SYSTEM OF HOMOGENEOUS POLYCONJUGAL EQUATIONS

Using the assumptions made in the previous section, in this section we will show that the array response vectors $\mathbf{a}^1, \dots, \mathbf{a}^S$ satisfy a well-structured system of D -variate homogeneous so-called *polyconjugal* equations of degree two, thereby ‘projecting’ the MIBI problem onto a mathematical problem, the solution of which yields estimates of the array response vectors. In the course of our derivation we highlight the algebraic structure of the problem formulation.

A. Subspace matrix definition and structure

We start our derivation by expressing the sensor correlation functions in terms of the mixing matrix elements and source auto-correlation functions. Several sensor correlation function values are then arranged in the so-called subspace matrix in a particular fashion. Using AS1-AS4 it follows that the sensor correlation functions can be expressed in terms of the mixing matrix elements and source auto-correlation functions as:

$$r_{i_1 i_2}^{x, c_1 c_2}[n_1, n_2] = \sum_{j=1}^S (a_{i_1}^j)^{c_1} (a_{i_2}^j)^{c_2} r_{jj}^{s, c_1 c_2}[n_1, n_2] \\ \forall (n_1, n_2) \in \Omega_{n_1 n_2}^{s \setminus \nu, c_1 c_2}, \quad 1 \leq i_1, i_2 \leq D. \quad (\text{IV.1})$$

Suppose that we have specified a Noise-Free ROS $\Omega_{n_1 n_2}^{s \setminus \nu, c_1 c_2}$ by a set of N time pairs as follows:

$$\Omega_{n_1 n_2}^{s \setminus \nu, c_1 c_2} = \{(n_1, n_2)^1, \dots, (n_1, n_2)^N\} \\ = \{(n_1^1, n_2^1), \dots, (n_1^N, n_2^N)\}, \quad (\text{IV.2})$$

where $(n_1, n_2)^q = (n_1^q, n_2^q)$ is the q -th time pair of $\Omega_{n_1 n_2}^{s \setminus \nu, c_1 c_2}$, and $N \geq S$. Note that we can choose the time pairs in such a way that the non-whiteness and/or non-stationarity of the

source signals is exploited. Let the $(D)^2 \times N$ subspace matrix $\mathbf{C}_D^{x, c_1 c_2}$ be defined as follows:

$$\mathbf{C}_D^{x, c_1 c_2} \triangleq \begin{bmatrix} \mathbf{r}_D^{x, c_1 c_2}[n_1^1, n_2^1] & \cdots & \mathbf{r}_D^{x, c_1 c_2}[n_1^N, n_2^N] \\ r_{11}^{x, c_1 c_2}[n_1^1, n_2^1] & \cdots & r_{11}^{x, c_1 c_2}[n_1^N, n_2^N] \\ \vdots & \ddots & \vdots \\ r_{DD}^{x, c_1 c_2}[n_1^1, n_2^1] & \cdots & r_{DD}^{x, c_1 c_2}[n_1^N, n_2^N] \end{bmatrix}, \quad (\text{IV.3})$$

where the q -th column is given by:

$$\mathbf{r}_D^{x, c_1 c_2}[n_1^q, n_2^q] \triangleq E \left\{ (\mathbf{x}[n_1^q])^{c_1} \otimes (\mathbf{x}[n_2^q])^{c_2} \right\} \quad (\text{IV.4})$$

with \otimes denoting the Kronecker product. Hence, the elements of $\mathbf{r}_D^{x, c_1 c_2}[n_1^q, n_2^q]$ and the rows of $\mathbf{C}_D^{x, c_1 c_2}$ are stacked in ‘Kronecker order’. The linear space spanned by the rows of the subspace matrix $\mathbf{C}_D^{x, c_1 c_2}$ is called the *signal subspace*. Thus, the dimension $d_D^{x, c_1 c_2}$ of the signal subspace equals the rank of the subspace matrix. Similarly to Eq. (IV.3), the $S \times N$ source auto-correlation matrix $\mathbf{C}_S^{s, c_1 c_2}$ is defined as follows:

$$\mathbf{C}_S^{s, c_1 c_2} \triangleq \begin{bmatrix} \mathbf{r}_S^{s, c_1 c_2}[n_1^1, n_2^1] & \cdots & \mathbf{r}_S^{s, c_1 c_2}[n_1^N, n_2^N] \\ r_1^{s, c_1 c_2}[n_1^1, n_2^1] & \cdots & r_1^{s, c_1 c_2}[n_1^N, n_2^N] \\ \vdots & \ddots & \vdots \\ r_S^{s, c_1 c_2}[n_1^1, n_2^1] & \cdots & r_S^{s, c_1 c_2}[n_1^N, n_2^N] \end{bmatrix}, \quad (\text{IV.5})$$

where the q -th column is given by:

$$\mathbf{r}_S^{s, c_1 c_2}[n_1^q, n_2^q] \triangleq E \left\{ (\mathbf{s}[n_1^q])^{c_1} \odot (\mathbf{s}[n_2^q])^{c_2} \right\}, \quad (\text{IV.6})$$

with \odot denoting the element-wise product. The linear space spanned by the rows of the source auto-correlation matrix is called the *source subspace*. Thus, the dimension $d_S^{s, c_1 c_2}$ of the source subspace equals the rank of the source auto-correlation matrix. From Equations (IV.1), (IV.3) and (IV.5) it follows that:

$$\mathbf{C}_D^{x, c_1 c_2} = \mathbf{A}_{D, \diamond}^{c_1 c_2} \mathbf{C}_S^{s, c_1 c_2}, \quad (\text{IV.7})$$

where $\mathbf{A}_{D, \diamond}^{c_1 c_2}$ is the so-called second order *Khatri-Rao product* of \mathbf{A} with conjugation pair $\mathbf{c}_2 = (c_1, c_2)$, which is defined as:

$$\mathbf{A}_{D, \diamond}^{c_1 c_2} \triangleq \left[(\mathbf{a}^1)^{c_1} \otimes (\mathbf{a}^1)^{c_2} \quad \cdots \quad (\mathbf{a}^S)^{c_1} \otimes (\mathbf{a}^S)^{c_2} \right]. \quad (\text{IV.8})$$

Hence, the j -th column of this matrix equals the Kronecker product of the j -th column of \mathbf{A} conjugated according to c_1 with the j -th column of \mathbf{A} conjugated according to c_2 .

B. Dimensions of signal and source subspaces

We now show how the signal subspace dimension $d_D^{x, c_1 c_2}$ depends on the mixing matrix and the source signal properties. First, from assumption AS2 it follows that the source auto-correlation matrix in Eq. (IV.5) has full rank. Hence, the source subspace dimension equals the number of sources:

$$d_S^{s, c_1 c_2} \triangleq \text{rank}(\mathbf{C}_S^{s, c_1 c_2}) = S. \quad (\text{IV.9})$$

Using the fact that the rank of a matrix is unchanged upon either left or right multiplication by a nonsingular matrix [11], and that $\mathbf{C}_S^{s, c_1 c_2}$ has full rank, it follows from Eq. (IV.7) that:

$$d_D^{x, c_1 c_2} \triangleq \text{rank}(\mathbf{C}_D^{x, c_1 c_2}) = \text{rank}(\mathbf{A}_{D, \diamond}^{c_1 c_2}). \quad (\text{IV.10})$$

Hence, the signal subspace dimension d_D^{x,c_1c_2} equals the rank of the Khatri-Rao product matrix $\mathbf{A}_{D,\diamond}^{c_1c_2}$. Essentially our subspace approach to MIBI is based on the fact that we can compute or estimate the (properties of the) various subspaces of the unknown matrix $\mathbf{A}_{D,\diamond}^{c_1c_2}$ from the known matrix \mathbf{C}_D^{x,c_1c_2} . The rank of the Khatri-Rao product matrix has been studied in several works, e.g. [7], [13], [21].

C. Deriving the system of polyconjugal equations

If the number of rows of the subspace matrix \mathbf{C}_D^{x,c_1c_2} is larger than the dimension d_D^{x,c_1c_2} of the signal subspace spanned by its rows, then \mathbf{C}_D^{x,c_1c_2} has a non-zero left null space $\mathcal{N}_1(\mathbf{C}_D^{x,c_1c_2})$. Defining a matrix Φ such that its rows form a basis for the complex conjugate $(\mathcal{N}_1(\mathbf{C}_D^{x,c_1c_2}))^*$ of the left null space $\mathcal{N}_1(\mathbf{C}_D^{x,c_1c_2})$ of \mathbf{C}_D^{x,c_1c_2} , it follows that:

$$\Phi \mathbf{C}_D^{x,c_1c_2} = \mathbf{0}. \quad (\text{IV.11})$$

The matrix Φ can be determined directly from the Singular Value Decomposition (SVD) of \mathbf{C}_D^{x,c_1c_2} by choosing and conjugate transposing the left singular vectors that correspond to (near-)zero singular values. The maximum number of linearly independent rows of Φ equals the dimension $Q_{D,S}^{c_1c_2}$ of $\mathcal{N}_1(\mathbf{C}_D^{x,c_1c_2})$ and is given by the difference between the number of rows of \mathbf{C}_D^{x,c_1c_2} and its rank [11]:

$$Q_{D,S}^{c_1c_2} \triangleq \dim(\mathcal{N}_1(\mathbf{C}_D^{x,c_1c_2})) = (D)^2 - d_D^{x,c_1c_2}. \quad (\text{IV.12})$$

Substituting Eq. (IV.7) into Eq. (IV.11) yields:

$$\Phi \mathbf{C}_D^{x,c_1c_2} = \Phi \mathbf{A}_{D,\diamond}^{c_1c_2} \mathbf{C}_S^{s,c_1c_2} = \mathbf{0}. \quad (\text{IV.13})$$

Because \mathbf{C}_S^{s,c_1c_2} has full rank due to AS2, it follows that:

$$\Phi \mathbf{A}_{D,\diamond}^{c_1c_2} = \mathbf{0}. \quad (\text{IV.14})$$

This system describes the relation between the *unknown* coefficients of the mixing matrix \mathbf{A} and the *known* coefficients of the matrix Φ . Let $\tilde{\varphi}_q \in \mathbb{C}^{(D)^2} (\mathbb{R}^{(D)^2})$ be the q -th row of Φ , let $\mathbf{z} \in \mathbb{C}_D$ be a vector of variables with the same size as a column of \mathbf{A} , and define the functions $f_{D,1}^{c_1c_2}(\mathbf{z}), \dots, f_{D,Q_{D,S}^{c_1c_2}}^{c_1c_2}(\mathbf{z})$ as follows:

$$f_{D,q}^{c_1c_2}(\mathbf{z}) \triangleq \tilde{\varphi}_q \cdot [(\mathbf{z})^{c_1} \otimes (\mathbf{z})^{c_2}] \quad \forall 1 \leq q \leq Q_{D,S}^{c_1c_2}. \quad (\text{IV.15})$$

Then, (IV.14) states that all columns $\mathbf{a}^1, \dots, \mathbf{a}^S$ of \mathbf{A} satisfy:

$$f_{D,q}^{c_1c_2}(\mathbf{a}^j) = 0 \quad \forall 1 \leq q \leq Q_{D,S}^{c_1c_2}, \quad 1 \leq j \leq S. \quad (\text{IV.16})$$

Hence, at this point *the MIBI problem has been 'projected' onto the problem of solving the following system of equations for the columns of the mixing matrix \mathbf{A} :*

$$\{f_{D,q}^{c_1c_2}(\mathbf{z}) = 0\}_{1 \leq q \leq Q_{D,S}^{c_1c_2}}. \quad (\text{IV.17})$$

By 'projected' we mean that the system of equations follows from our MIBI problem definition and formulation, but not necessarily the other way around. We refer to Alg. 1 for a summary of the results that we have developed so far, and Section VI for examples.

Algorithm 1 $D \times S$ MIBI exploiting Second Order Temporal Structure with arbitrary conjugation pair (c_1, c_2) .

- 1: Estimate sensor correlation functions for time index tuples (n_1, n_2) in Noise-Free ROS $\Omega_{n_1 n_2}^{s \setminus \nu, c_1 c_2}$;
- 2: Arrange these values in subspace matrix \mathbf{C}_D^{x,c_1c_2} ;
- 3: Compute SVD of \mathbf{C}_D^{x,c_1c_2} and split result into signal and noise subspace parts as follows:

$$\mathbf{C}_D^{x,c_1c_2} = \mathbf{U}\Sigma(\mathbf{V})^* = \mathbf{U}^s \Sigma_s^s (\mathbf{V}_s)^* + \mathbf{U}^\nu \Sigma_\nu^\nu (\mathbf{V}_\nu)^* ;$$
- 4: Let Φ be a matrix whose rows span complex conjugate left null space $(\mathcal{N}_1(\mathbf{C}_D^{x,c_1c_2}))^*$ of \mathbf{C}_D^{x,c_1c_2} :

$$\Phi \triangleq (\mathbf{U}^\nu)^H ;$$
- 5: With each row $\tilde{\varphi}_q$ of Φ for $1 \leq q \leq Q_{D,S}^{c_1c_2}$ associate a D -variate homogeneous polyconjugal of degree 2:

$$f_{D,q}^{c_1c_2}(\mathbf{z}) \triangleq \tilde{\varphi}_q \cdot [(\mathbf{z})^{c_1} \otimes (\mathbf{z})^{c_2}] ;$$
- 6: The following system remains to be solved for the columns of the mixing matrix (see Alg. 2 on the following page):

$$\{f_{D,q}^{c_1c_2}(\mathbf{z}) = 0\}_{1 \leq q \leq Q_{D,S}^{c_1c_2}}.$$

All functions in system (IV.17) have the same specific form. Firstly, from the definition in (IV.15) it is clear that each function $f_{D,q}^{c_1c_2}(\mathbf{z}) = f_{D,q}^{c_1c_2}(z_1, \dots, z_D)$ is a D -variate 'polynomial-like' function containing product terms of degree two. Strictly speaking, $f_{D,q}^{c_1c_2}(\mathbf{z})$ is only a polynomial in z_1, \dots, z_D if $(c_1, c_2) = (0, 0)$ because for $(c_1, c_2) \in \{(0, *), (*, 0), (*, *)\}$ the product terms (also) contain conjugates of the variables. Therefore, in general we refer to a function of the type defined by $f_{D,q}^{c_1c_2}(\mathbf{z})$ as a 'polyconjugal'. From (IV.15) it follows directly that each function $f_{D,q}^{c_1c_2}(\mathbf{z})$ in (IV.17) is a D -variate polyconjugal of degree two, which is *homogeneous of degree two with conjugation pair (c_1, c_2)* , meaning that:

$$f_{D,q}^{c_1c_2}(\eta \mathbf{z}) = (\eta)^{c_1} (\eta)^{c_2} f_{D,q}^{c_1c_2}(\mathbf{z}) \quad \forall \eta \in \mathbb{C}, \quad \forall \mathbf{z} \in \mathbb{C}_D. \quad (\text{IV.18})$$

This property implies the following:

$$f_{D,q}^{c_1c_2}(\mathbf{v}) = 0 \quad \implies \quad f_{D,q}^{c_1c_2}(\eta \mathbf{v}) = 0 \quad \forall \eta \in \mathbb{C}. \quad (\text{IV.19})$$

Hence, if \mathbf{v} is a solution of (IV.17), then so is $\eta \mathbf{v}$ for all $\eta \in \mathbb{C}$. This is compatible with the scaling indeterminacy inherent to MIBI, see also Section II. It also implies that the norms of the solutions of system (IV.17) can be chosen arbitrarily. Algebraically, the zero contour level of each function $f_{D,q}^{c_1c_2}(z_1, \dots, z_D)$ defines a *cone* in the D -dimensional Euclidian space. Hence, geometrically, solving (IV.17) is equivalent to finding the one-dimensional intersections between $Q_{D,S}^{c_1c_2}$ cones in a D -dimensional Euclidian space, where ideally each intersection is a one-dimensional linear subspace that corresponds to a column of the mixing matrix. We refer to Section VI for examples.

V. SOLVING THE SYSTEM OF POLYCONJUGAL EQUATIONS BY HOMOTOPY

In this section we summarize the main ideas behind so-called homotopy methods. The resulting algorithm will be employed in the examples presented in the next section for solving systems of the form (IV.17); see also [19]. Homotopy methods provide a deterministic means for solving a system of nonlinear equations. They are based on the so-called path following or continuation techniques. Excellent discussions can be found in several articles and books, e.g. [1], [8], [12], [14]. The rationale behind homotopy methods is to smoothly deform the known solutions of a known (and possibly simple) start system into the desired solutions of the target system, see also Alg. 2. The start and target systems are embedded in a family of systems, called the homotopy, and then all members in the family are solved in a sequential and iterative manner. Since each new system is close to the previous system, under some mild uniqueness and smoothness conditions its solutions deviate only slightly from those of the previous system and each path converges to a geometrically isolated solution [1], [12]. Here, we denote a general target system by $\mathbf{p}(\mathbf{z}) = \mathbf{0}$ and assume that $\mathbf{p} : \mathbb{C}_P \rightarrow \mathbb{C}_P$. Furthermore, its solution set is denoted by \mathcal{P} , i.e. $\mathcal{P} \triangleq \{\mathbf{z}_p \in \mathbb{C}_P \mid \mathbf{p}(\mathbf{z}_p) = \mathbf{0}\}$. Similarly, we denote the start system by $\mathbf{g}(\mathbf{z}) = \mathbf{0}$ with $\mathbf{g} : \mathbb{C}_P \rightarrow \mathbb{C}_P$, and its solution set is denoted by \mathcal{G} , i.e. $\mathcal{G} \triangleq \{\mathbf{z}_g \in \mathbb{C}_P \mid \mathbf{g}(\mathbf{z}_g) = \mathbf{0}\}$. This system is constructed in such a way that it has the same structure and number of solutions as the target system $\mathbf{p}(\mathbf{z}) = \mathbf{0}$. In Alg. 2 we summarize the homotopy method that we will use in the next section. The path \mathcal{C} is a simple curve in the complex plane that has to be predefined by the user. The continuation parameter $\lambda \in \mathbb{C}$ follows \mathcal{C} from λ_0 to λ_e with $\lambda_0 \neq \lambda_e$. Here, we use the arc $\mathcal{C} \triangleq \{\cos(\theta) + j\frac{3}{10}\sin(\theta) \mid 0 \leq \theta \leq \pi\}$ with starting point $\lambda_0 = 1$ and end point $\lambda_e = -1$. The constants $\gamma_g \in \mathbb{C}$ and $\gamma_p \in \mathbb{C}$ are randomly chosen fixed constants that serve to avoid singularities and crossings along the different paths. The parameter *MaxNnwIt* defines the maximum number of Newton corrector steps.

VI. EXAMPLES WITH THREE SENSORS

In this section the steps of Alg. 1 and 2 are performed for two MIBI examples with $D = 3$ sensors, a real-valued mixing system, and real-valued source signals, so that visualization of the procedure and results is possible. In the first example three and in the second four speech source signals are mixed according to Model (II.1) with mixing matrices given by (VI.2) and (VI.4) respectively. For convenience, and without loss of generality, the columns of \mathbf{A} have unit Euclidian norm. Since the mixing system and source signals are real-valued the conjugation pair (c_1, c_2) is irrelevant and will be omitted from the notation. All speech source signals are sampled at 8 kHz, and the noise signals $\nu_1[n]$, $\nu_2[n]$ and $\nu_3[n]$ are mutually statistically independent white Gaussian noise sequences with variance 0.1. The number of time samples in all signals involved is 10000, which equals 1250 milliseconds. In each example we exploit both the non-stationarity and non-whiteness by using sensor correlation values for different times and

Algorithm 2 Overview of homotopy continuation method.

```

1: Define path  $\mathcal{C}$  and initialize  $\gamma_g, \gamma_p, MaxNnwIt$ , etc.;
2: Construct start system  $\mathbf{g}(\mathbf{z}) = \mathbf{0}$ ;
3: Embed  $\mathbf{g}(\mathbf{z})$  and  $\mathbf{p}(\mathbf{z})$  in convex homotopy:
    $\mathbf{h}(\mathbf{z}, \lambda) = \gamma_g(\lambda - \lambda_e) \mathbf{g}(\mathbf{z}) + \gamma_p(\lambda - \lambda_0) \mathbf{p}(\mathbf{z}) \quad \forall \lambda \in \mathcal{C}$ ;
4: Compute solutions of start system  $\mathbf{g}(\mathbf{z}) = \mathbf{0}$  and store them in set  $\mathcal{G}$ ;
5: for all  $\mathbf{z}_g \in \mathcal{G}$  do
    $\mathbf{z} := \mathbf{z}_g$ ;
   for  $\lambda = \lambda_0 \rightarrow \lambda_e$  along  $\mathcal{C}$  do
     % Euler predictor step:
      $\frac{d\mathbf{z}}{d\lambda} = -[\nabla_{\mathbf{z}} \mathbf{h}]^{-1} \cdot \partial_{\lambda} \mathbf{h}$ ;
      $\mathbf{z} := \mathbf{z} + \Delta\lambda \cdot \frac{d\mathbf{z}}{d\lambda}$ ;

     % Newton corrector steps:
      $\lambda := \text{next } \lambda \text{ from } \mathcal{C}$ ;

     for  $m = 1 \rightarrow MaxNnwIt$  do
        $\Delta\mathbf{z} := -[\nabla_{\mathbf{z}} \mathbf{h}]^{-1} \cdot \mathbf{h}$ ;
        $\mathbf{z} := \mathbf{z} + \Delta\mathbf{z}$ ;

     end for
   end for
   Store solution  $\mathbf{z}_p = \mathbf{z}$  in set  $\mathcal{P}$ ;
end for
Return  $\mathcal{P}$ ;

```

different lags. To do so, the signal sequences are partitioned into disjoint blocks consisting of 2000 samples, and for each block the one-dimensional sensor correlation functions are computed for lags 1, 2 and 3. Lag zero is omitted because the corresponding correlation values are noise-contaminated. Because the number of available samples is 10000, the number of blocks equals 5. Hence, in total for each sensor correlation function 15 values are estimated and employed. Calling the block index b with $1 \leq b \leq 5$, and the lag index k , the employed Noise-Free ROS in the domain of block-lag pairs $(b; k)$ used here is:

$$\Omega_{b;k}^{s \setminus \nu} = \{(1; 1), (1; 2), (1; 3), \dots, (5; 1), (5; 2), (5; 3)\}.$$

For each block the sensor correlation values are estimated from the sensor signals by averaging products of the form $x_{i_1}[n] x_{i_2}[n - k]$ with $k = 1, 2, 3$ over the block length.

Note that the Kronecker product $\mathbf{z} \otimes \mathbf{z}$ in the definition of $f_{D,q}^{c_1 c_2}(\mathbf{z})$ in Eq. (IV.15) is a vector of length $(D)^2 = 9$ that contains only $\frac{1}{2}D(D + 1) = 6$ different products, viz. $z_1 z_1, z_1 z_2, z_1 z_3, z_2 z_2, z_2 z_3$, and $z_3 z_3$. Combining the coefficients of $\tilde{\varphi}_q$ corresponding to equal products, and also combining the corresponding rows of the subspace matrix yields a system of $\frac{1}{2}D(D + 1) - d_D^x$ equations of the following form:

$$f_p(z_1, z_2, z_3) \triangleq \sum_{i_1=1}^D \sum_{i_2=i_1}^D \alpha_p^{i_1 i_2} z_{i_1} z_{i_2}, \quad (VI.1)$$

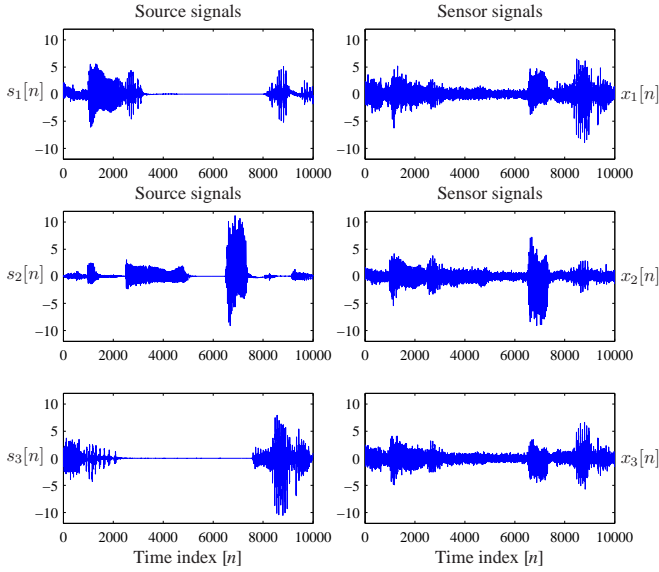


Fig. 2. Speech sources (left) and their noise-contaminated mixtures (right) for 3×3 example.

where the coefficients $\alpha_p^{11}, \alpha_p^{12}, \alpha_p^{13}, \alpha_p^{22}, \alpha_p^{23}, \alpha_p^{33}$ for $p = 1, \dots, \frac{1}{2}D(D+1) - d_D^x$ follow from the SVD of the corresponding ‘reduced subspace matrix’.

A. Three mixtures of three speech signals

The sensor signals are obtained from (II.1) with:

$$\mathbf{A} = \begin{bmatrix} 0.5774 & 0.4082 & 0.8083 \\ 0.5774 & -0.8165 & 0.1155 \\ 0.5774 & -0.4082 & -0.5774 \end{bmatrix}. \quad (\text{VI.2})$$

Fig. 2 shows the source signals $s_1[n]$, $s_2[n]$ and $s_3[n]$ at the left side, and the three noise-contaminated mixtures $x_1[n]$, $x_2[n]$ and $x_3[n]$ at the right side. Using only the sensor data the functions in the system to be solved can be obtained by means of Alg. 1. Eq. (IV.10) implies that $d_D^x = 3$. Hence, there are $\frac{1}{2}D(D+1) - d_D^x = 3$ functions of the form (VI.1) in the system, which becomes $\{f_1(z_1, z_2, z_3) = 0, f_2(z_1, z_2, z_3) = 0, f_3(z_1, z_2, z_3) = 0\}$. Because all quantities involved are real-valued, the surfaces describing the zero contour levels are conventional quadric cones in three-dimensional space. In Fig. 3 we have depicted the intersections of each cone with the unit sphere in different colors/grey shades. Those intersections are a kind of ‘ellipses on the sphere’ that we will call ‘spherical ellipses’. Note that each intersection consists of two parts that are point-symmetric with respect to the origin. The black arrows represent the ideal columns of \mathbf{A} , whereas the grey arrows are the opposites of the black ones. The figure shows that the columns of \mathbf{A} point to the points denoted by the black and grey dots where three spherical ellipses intersect each other. Hence, the array response vectors are determined uniquely by the intersections induced by the system of equations. The magenta arrows and dots indicate the columns of \mathbf{A} as estimated by Alg. 2. The start system that we use is exactly similar to the target system with the unit-norm constraint added. The coefficients of the homogeneous

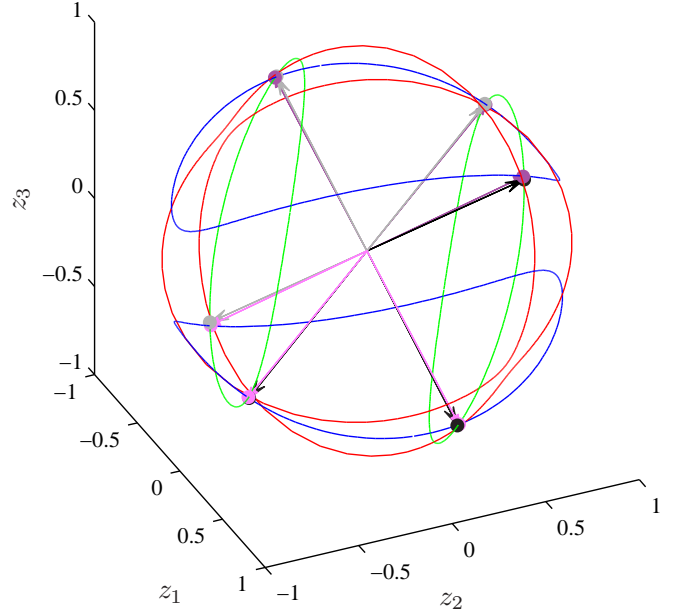


Fig. 3. Zero contour spherical ellipses of $f_1(z_1, z_2, z_3)$, $f_2(z_1, z_2, z_3)$ and $f_3(z_1, z_2, z_3)$ for 3×3 example.

part of the start system can be obtained by computing the left null space of the second order Khatri-Rao product (IV.8) of a known matrix (here we have chosen the identity matrix). Alg. 2 yields the following estimate of \mathbf{A} :

$$\hat{\mathbf{A}} = \begin{bmatrix} 0.5666 & -0.4089 & -0.8116 \\ 0.5773 & 0.8195 & -0.1219 \\ 0.5878 & 0.4015 & 0.5713 \end{bmatrix}.$$

Because our purpose is to demonstrate MIBI and IBSS, and not noise reduction, we apply the inverse of $\hat{\mathbf{A}}$ to the noise-free sensor signals to recover the source signals (recall however, that $\hat{\mathbf{A}}$ has been estimated from the noisy sensor data):

$$\mathbf{y}[n] = \hat{\mathbf{A}}^{-1} (\mathbf{x}[n] - \boldsymbol{\nu}[n]). \quad (\text{VI.3})$$

Fig. 4 shows the source signals at the left side (see also Fig. 2), and the noise-free estimated source signals at the right side. It is evident that the signals are well-separated. As a measure of performance, we compute the total transfer matrix from the source to the output signals:

$$\mathbf{T} \triangleq \hat{\mathbf{A}}^{-1} \mathbf{A} = \begin{bmatrix} 0.9971 & -0.0082 & -0.0082 \\ -0.0002 & -0.9919 & -0.0022 \\ -0.0153 & -0.0090 & -1.0006 \end{bmatrix}.$$

It can readily be seen that this total transfer matrix is approximately equal to the identity matrix with two ‘flipped’ signs. This is also clear from Fig. 4. Listening to the separated signals confirmed that the separation was successful.

B. Three mixtures of four speech signals

Appending one column to the mixing matrix in (VI.2) gives:

$$\mathbf{A} = \begin{bmatrix} 0.5774 & 0.4082 & 0.8083 & -0.1690 \\ 0.5774 & -0.8165 & 0.1155 & -0.5071 \\ 0.5774 & -0.4082 & -0.5774 & 0.8452 \end{bmatrix}. \quad (\text{VI.4})$$

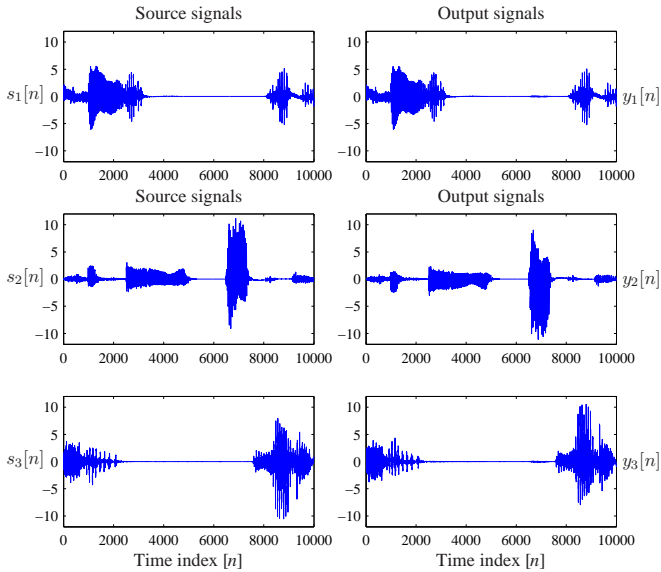


Fig. 4. Speech sources (left) and noise-free estimated source signals (right) for 3×3 example.

Using one additional speech signal we again compute the three sensor signals by means of (II.1). Now $d_D^x = 4$ and there are only $\frac{1}{2}D(D+1) - d_D^x = 2$ functions of the form (VI.1) in the system. Similarly to Fig. 3, in Fig. 5 we have depicted the intersections of each zero contour level (cone) with the unit sphere in different colors/grey shades. Using Alg. 2 yields the following estimate of \mathbf{A} :

$$\hat{\mathbf{A}} = \begin{bmatrix} 0.8200 & -0.4079 & -0.1740 & -0.5762 \\ 0.1331 & 0.8156 & -0.5061 & -0.5595 \\ -0.5567 & 0.4103 & 0.8447 & -0.5958 \end{bmatrix}.$$

Again we see that the estimated columns, which are indicated in Fig. 5 by the magenta arrows and dots, approximately equal the ideal ones and are determined uniquely by the two intersecting cones.

C. Number of identifiable columns for real-valued scenario

As we have seen in the examples above, geometrically, solving a system consisting of homogeneous polynomial equations of degree two in three variables is equivalent to finding the intersections between the corresponding two-dimensional quadric hypersurfaces that are embedded in a three-dimensional Euclidian space. Ideally, each intersection is a one-dimensional subspace that corresponds to a column of the mixing matrix. In a three-dimensional Euclidian space at least two two-dimensional surfaces are required for uniquely defining one-dimensional intersections. Algebraically, this means that the dimension of the solution set of a system of two homogeneous polynomial equations in three variables generally is one-dimensional. Since for the 3×3 example above the system contains three equations, the one-dimensional solutions sets corresponding to the three columns of the mixing matrix are overdetermined. In other words, one degree of freedom is left unused, which, as we have seen for the 3×4 example, can be exploited to determine one

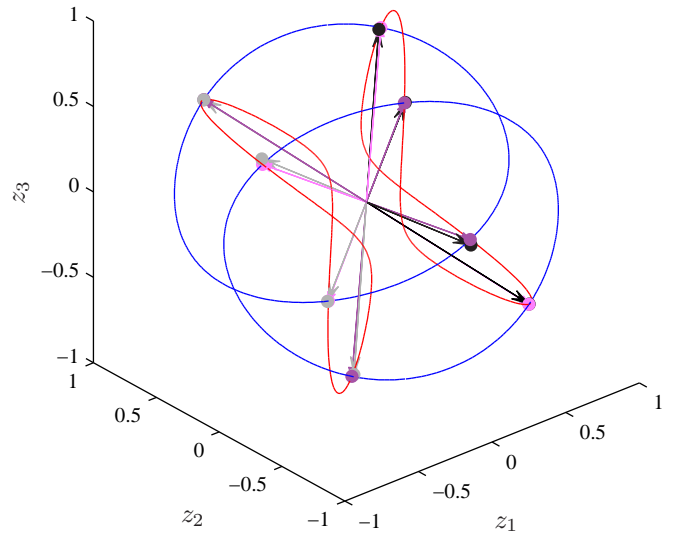


Fig. 5. Zero contour spherical ellipses of $f_1(z_1, z_2, z_3)$ and $f_2(z_1, z_2, z_3)$ for 3×4 example.

more column. Because at least two cones are required for determining one-dimensional solutions sets, no more than four columns can be identified with three sensors and second order statistics. Using the same reasoning we can deduce that for real-valued MIBI the maximum number of columns that can be identified with D sensors and SOS equals:

$$S_{\max} = \frac{1}{2}D(D+1) - (D-1). \quad (\text{VI.5})$$

VII. CONCLUSIONS

We have presented a procedure for exploiting Second Order Temporal Structure such as non-whiteness and/or non-stationarity in the source signals for performing MIMO Instantaneous Blind Identification (MIBI). Based on a number of assumptions and using subspace techniques, the MIBI problem has been reformulated in a particular way such that each column of the unknown mixing matrix satisfies a system of multivariate polyconjugal equations. The assumptions underlying the problem formulation have been defined on a Noise-Free Region Of Support and mainly serve to ensure that the source signals are mutually unrelated, that sufficient temporal structure is present in the source signals, that the source and noise signals are mutually unrelated, and that the noise signals have a simpler temporal structure than the source signals. This latter property has been employed for eliminating the effect of additive sensor noise on the mixing matrix estimation. The resulting nonlinear system of equations was solved by means of a homotopy method. The two-stage identification procedure can also handle several scenarios with more sources than sensors. For complex-valued systems and signals, the procedure can deal with arbitrary conjugation patterns chosen in accordance with the characteristics of the signals involved. Finally, we have demonstrated the principles by means of examples.

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