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Flood regulation using nonlinear model predictive control

Toni Barjas Blanco [a,*], Patrick Willems [b], Po-Kuan Chiang [b], Niels Haverbeke [a], Jean Berlamont [b], Bart De Moor [a]

[a] Department of Electrotechnical Engineering, Katholieke Universiteit Leuven, Kasteelpark Arenberg 10, 3001 Heverlee, Belgium
[b] Department of Civil Engineering, Katholieke Universiteit Leuven, Kasteelpark Arenberg 40, 3001 Heverlee, Belgium

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ABSTRACT

In this paper the flood problem of the river Demer, a river located in Belgium, is discussed. First a simplified model of the Demer basin is derived based on the conceptual reservoir modeling concept. This model was calibrated to simulations results with a more detailed full hydrodynamic model. Afterwards, the focus is shifted to a nonlinear model predictive controller (NMPC) which is based on a new semi-condensed optimization procedure combined with a line search approach. Finally, simulations are performed based on historical data in which the NMPC is compared with the current control strategy used by the local water administration. Uncertainties are added to the rainfall predictions in order to assess the robustness of the NMPC.

1. Introduction

Flooding of rivers are a worldwide cause of great economic losses. This is also the case in the basin of the river Demer in Belgium. In the past the Demer basin experienced several floods. In order to reduce the flood hazard in the area the local water administration installed several flood control reservoirs in order to be able to store the excessive water volume during periods of extreme rainfall. In order to control the flows to and from the reservoirs hydraulic control structures have been put in place. Though these actions have lead to a significant reduction of the flood risk in the basin, during the heavy rainfall periods of 1998 and 2002 the local water administration was not able to prevent flooding along the Demer river. Recent simulations of these past events in a hydrodynamic river model showed that flooding could have been significantly reduced and even avoided if the hydraulic structures would have been controlled in a different way. Therefore, the main interest of this paper is to test a different control strategy than the one adopted currently. Due to the specific nature of the flood problem a nonlinear model predictive controller (NMPC) (Rossiter, 2000) seems the most suitable option. Therefore, in this work such a NMPC has been tested and compared with the current control strategy.

Nonlinear model predictive control is an optimization-based control paradigm that has been used successfully in many control applications due to the fact that it can cope with constraints on the system. Especially in the chemical process industry NMPC has proven its value (Nagy, 2009; Wendt, Li, & Wozny, 2002). With respect to river regulation NMPC has also been used extensively (Malaterre, 1997; Van Overloop, 2006; Wahlin, 2004) but its use has always been limited to fixed setpoint regulation (e.g. irrigation control). Typically, good setpoint regulation can be achieved by assuming a linear model. In this work, however, the main focus will be flood prevention. A simple linear model is no longer sufficient as all the nonlinear dynamics of the river system will be excitated during flood periods. Therefore, in Barjas Blanco et al. (2009), in a first step a nonlinear model has been developed that on the one hand was accurately enough to capture the most important dynamics but on the other hand still fast enough to be used for real-time control purposes. In a second step a nonlinear MPC scheme was proposed based on a trust-region approach. The performance of this NMPC scheme was then compared with that of the current fixed regulation (three-position controller) by simulating the historical rainfall-runoff time series of 1998. First results presented in Barjas Blanco et al. (2009) showed that NMPC outperformed the three-position controller. However, in Barjas Blanco et al. (2009) it was assumed that the rain predictions used by NMPC coincided perfectly with the real rainfall. In practice, this is never the case. Therefore, in order to assess whether NMPC performs better in practice than the three-position controller simulations must be done with a realistic amount of uncertainty added to the

* Corresponding author. Tel.: +32 16 32 8656; fax: +32 16 32 19 70.
E-mail addresses: toni.barjas-blanco@esat.kuleuven.be (T. Barjas Blanco), patrick.willems@bwk.kuleuven.be (P. Willems), pokuan.chiang@student.kuleuven.be (P.-K. Chiang), Niels.Haverbeke@esat.kuleuven.be (N. Haverbeke), jean.berlamont@bwk.kuleuven.be (J. Berlamont), bart.demoor@esat.kuleuven.be (B. De Moor).

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rain predictions. In this work simulations are done based on the rainfall-runoff time series of 1998 but with a realistic amount of uncertainty added to the time series. The amount of uncertainty is estimated by means of the techniques described in Timbe (2007). In order to increase the robustness of the controller the uncertainty added to the rainfall-runoff time series is such that the rainfall-runoff prediction used by NMPC is an overestimation of the real rainfall-runoff. Furthermore, in this work the underlying optimization scheme of the NMPC is improved compared to the optimization scheme used in Barjas Blanco et al. (2009). First, the uncondensed MPC scheme is replaced by a semi-condensed NMPC scheme leading to less memory requirements and less computation time. Second, the trust region approach of Barjas Blanco et al. (2009) is replaced by a line search approach which resulted in better control performance.

2. Background

The study area is the area around the two flood control reservoirs along the Demer river, namely "Webbekom" and "Schulensmeer". A schematic representation of this area is shown in Fig. 1. Because of the complicated shape of the "Schulensmeer" reservoir it is modeled as different reservoirs separated by spills. The spills are indicated by the dark rectangular boxes. There are 12 hydraulic structures that need to be controlled. In Fig. 1 these structures are indicated by white rectangular boxes. All the hydraulic structures are of the gated weir type. Currently these gates are regulated by a three-position controller (OBM, 2003). The rainfall-runoff is indicated by discharges entering the river system at different locations and act as disturbance inputs on the river system. There are eight different locations through which rainfall-runoff (from upstream subbasins) enters the river system. Typically rainfall-runoff predictions go up to 48 h ahead.

3. Hydrodynamic model of the river system

A detailed physically based hydrodynamic model of the river system in the study area was created during earlier studies by the VMM water authority, the local authority responsible for the water management along the Demer river. The model is mainly based on the full hydrodynamic model equations (de St.Venant momentum and continuity equations; see e.g. Chow & Maidment, 1998). These equations are solved based on finite differences (implicit computational scheme). Implementation of that model has been done by means of the InfoWorks-RS river modeling software (Wallingford Software, UK). The model is based on river bed cross-sectional data approximately every 50 m along the modeled rivers, river bed roughness information and geometric data on all hydraulic structures (weirs, culverts, flow and water level control structures) and bridges along the course of all these rivers. Currently, this model is used as a warning system to predict which areas are going to be flooded. However, this model cannot be used for real-time control because of its computational complexity. For this reason in Barjas Blanco et al. (2009) a more simplified model (of the conceptual or grey box type) was calibrated to the detailed hydrodynamic model. The model simplification is reached by lumping the processes in space, and by limiting the study area to the region affected by the flood control. Lumping of the processes in space is done by simulation of the water levels, not every 50 m as the full hydrodynamic model does, but only at the relevant locations. These are the locations up- and downstream of the hydraulic regulation structures, to be controlled by the NMPC-controller, and the locations along the Demer where potential flooding is induced, to be limited by the controller. Depending on these locations, the river is subdivided into reaches, in which water continuity is modeled (in a spatially lumped way per reach) based on reservoir-type of models. A reservoir model simply assumes water continuity (increase in volume v per time step equals inflow q_{in} minus outflow q_{out}): $dv(t)/dt = q_{in}(t) - q_{out}(t)$. The inflow in each reservoir

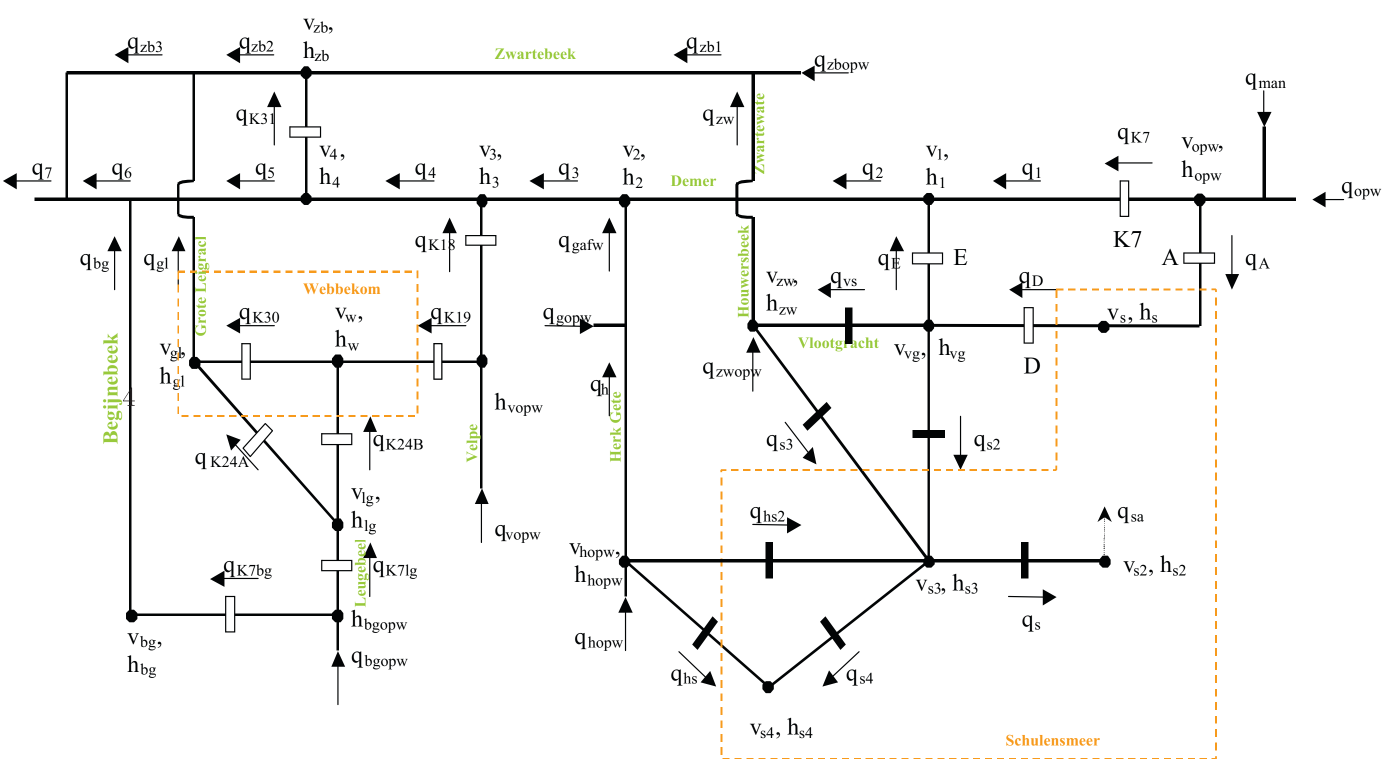

Fig. 1. Schematic overview of the Demer basin.

(submodel representing a river reach) is the discharge from the more upstream river reach (result of the more upstream submodel). The outflow depends on the water storage in the reach ($q_{out}=f(v)$) or is assumed equal to the sum of the upstream discharge and the other inflows along the reach (e.g. from tributary rivers). Water level differences ($h_{upstr}-h_{downstr}$) along the reach are modeled proportional to the ratio of the squared discharge (q) in the reach and the squared water depth downstream along the reach ($h_{downstr}$ is the water level downstream of the reach and $h_{downstr,0}$ the level of river bed):

$$h_{upstr}(t)=h_{downstr}(t)+\frac{q(t)^2}{(h_{downstr}(t)-h_{downstr,0}(t))^2} \tag{1}$$

The relation between the water level difference and (1) is expected for most river reaches after the equation of Manning, well known in river hydraulic sciences and engineering (e.g. Chow & Maidment, 1998). The precise relation is calibrated based on the simulation results for a few historical flow events (including flood events) with the detailed InfoWorks-RS (IWRS) model. This type of calibration is done for all conceptual submodels. Fig. 2 shows an example of a calibration result for the water level difference along one of the river reaches in the model. The conceptual model has 75 states consisting of 35 discharges, 20 water levels and 20 node volumes. Table 1 displays the flood levels for all the water levels in the river system. All water levels in this paper are in

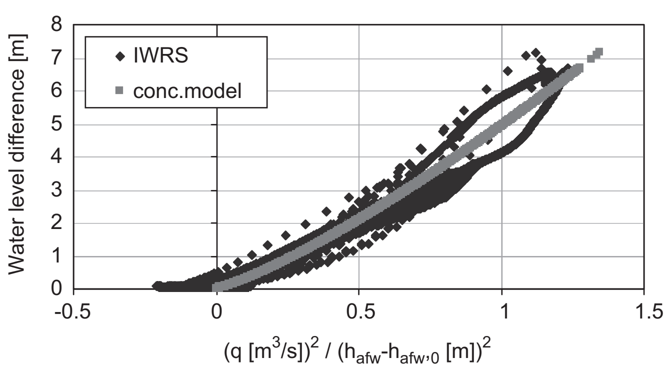

Fig. 2. Calibration result of the conceptual model (conc.model) for one of the river reaches (h_{gl}) in the full hydrodynamic (IWRS) model.

Table 1
The water levels in the river system with their corresponding flood level.

Water level	Flood level (m)
h_{opw}	23.2
h_v	24
h_{opw}	24.8
h_h	23.6
h_s	23.2
h_w	22.4
h_1	24.3
h_2	22.7
h_3	22.9
h_4	22.5
h_{zw}	21.6
h_{vg}	22.5
h_{zb}	21.5
h_{bg}	24.2
h_{lg}	23
h_{gl}	22
h_{afw}	20.5
hs_2,hs_3,hs_4	22.75

meter above the "TAW" level, which is the topographical reference level for Belgium. More detailed information about the hydraulic model can be found in Barjas Blanco et al. (2009).

4. Controller design

Several control strategies for river systems have been proposed in the literature, see Malaterre (1997) and Clemmens, Ruiz Carmona, and Schuurmans (1998) for recent reviews. Currently, in the Demer basin the hydraulic structures are controlled by a three-position controller (OBM, 2003; Rogers & Goussard, 1998). A three-position controller consists of a basic control mode that responds to a deviation from the setpoint water level by moving the control gate at a predetermined movement speed. The standard three controller states are:

1. Off—no corrective action.
2. On, above setpoint—move the gate to decrease water level.
3. On, below setpoint—move the gate to increase water level.

This type of controller has only one goal: to steer the corresponding water level to the desired reference level. The three-position controller used by the local water administration to control the river system in the Demer basin is more advanced and complex than the standard three-position controller. The controller is more advanced in the sense that it consists of more logical rules. These logical rules are based on expert knowledge, where the main concern is to avoid flooding rather than optimal setpoint regulation. This more advanced controller suffers from a very important drawback, namely that the controller determines its control action based on the current state of the system only, namely the up- and downstream water levels and discharges. The controller does not use future water level predictions to determine an appropriate control action. A better alternative when trying to avoid or decrease flooding in a river basin is a nonlinear model predictive controller (NMPC) (Camacho & Bordons, 2005; Diehl, Ferreau, & Haverbeke, 2008; Rossiter, 2000). The main characteristics of NMPC that justify its use for flooding regulation are the following:

- NMPC can cope with all the constraints that are present in a river system like physical upper and lower bounds of the gates and maximal gate movement. Also upper constraints on the water levels can be taken into account which is necessary for flood prevention.
- By combining the rainfall predictions with the mathematical model of the river system NMPC can make predictions of the future water levels and use this information for making better decisions with respect to flooding avoidance.
- During a flood event all the nonlinear dynamics of the river system are excitated. So in order to make accurate predictions of the future water level it is necessary to have a nonlinear model of the river system. In the literature (Camacho & Bordons, 2005; Diehl et al., 2008) there exist efficient nonlinear NMPC schemes that can cope with nonlinear models and at the same time are fast enough for on-line implementation.
- River systems are typically highly interactive multi-input–multi-output systems (MIMO). It is known that traditional control design techniques based on transfer function models are very difficult to use for such kind of systems because they make use of relatively little information about the system. NMPC, however, can effectively deal with MIMO systems.
- NMPC solves at each sampling time an optimization problem. NMPC can only be applied if this optimization problem can be

solved within the sampling time. Because river systems have relatively slow dynamics the necessary sampling time is typically in the range of 15–60 min, which is large enough to solve the on-line optimization problem. Note, however, that this is true if a simplified model is used and the horizon is not too long. If a very detailed prediction model is used with a long horizon solving the optimization can be very time demanding (days).

In the next section the principles of NMPC will be explained as well as the implementation for the case study.

5. Nonlinear model predictive control

NMPC is a control strategy that uses a model of the system in order to make future predictions on which an optimal input sequence is determined in order to minimize an objective function taking constraints into account. The basic components of NMPC for water systems are the following:

1. A process model combined with the future rain predictions is used in order to predict the future outputs within a predetermined window with length N.
2. An objective function is minimized taking constraints on the inputs and outputs into consideration. The objective function is typically a quadratic function trying to minimize the deviation of the water level with the reference level on the one hand, and the gate movement on the other hand.
3. After the minimization of the objective function a sequence of future inputs is obtained of which only the first one is actually applied to the system.
4. In the next sampling time the new state of the system is measured or estimated, new predictions of the rainfall are obtained and the complete process is repeated. Because of the repetition of this process and the re-estimation of the state and the rainfall predictions the NMPC strategy has a certain robustness against model uncertainties and rainfall prediction errors.

In the remainder of this section the different components of the developed NMPC algorithm are described. First the standard uncondensed NMPC scheme is discussed. This scheme is frequently used in practice but it has the drawback that it uses a lot of memory, especially in high dimensional applications with large horizons. One way to solve this problem is to use a condensed NMPC scheme in which the states are removed from the optimization problem leading to a small optimization problem. However, this scheme was not applicable to the river control problem due to bad conditioning of the optimization problem. Therefore in a next step a novel scheme is proposed that offers the benefits of the condensed scheme and applicable to the problem of river control. This scheme is referred at as the semi-condensed NMPC scheme. Note that this semi-condensed scheme is the main theoretical contribution in this paper. It is a novel scheme that according to the authors has not previous appeared in the literature. Further it is discussed in detail how to properly handle the constraints on the system. During periods of heavy rainfall it is unavoidable to allow water levels to violate their flood limit and therefore a constraint strategy is applied to deal with this. Finally, the uncontrollability of the gates is explained in detail. The origin of the uncontrollability problem is explained as well as a method to effectively deal with it.

5.1. Uncondensed NMPC

The process model used in this work is a nonlinear state space model described as

$$\mathbf{x}_{k+1} = f(\mathbf{x}_k, \mathbf{u}_k, \mathbf{d}_k) \tag{2}$$

$$\mathbf{y}_k = \mathbf{C}\mathbf{x}_k \tag{3}$$

with $\mathbf{x}_k$ the state of the system at time step k containing the water levels, discharges and water volumes in the water system, $\mathbf{u}_k$ the input of the system at time step k containing all the gated weirs in the water system, $\mathbf{d}_k$ the disturbance input at time step k representing the rainfall-runoff at time step k and $\mathbf{y}_k$ the output of the system at time step k which in this case coincides with the water levels in the Demer basin.

For a prediction horizon N NMPC solves the following optimization program at each sampling time:

Optimization Problem 1.

$$\min_{\substack{\mathbf{u}_k,\ldots,\mathbf{u}_{k+N-1}\\ \mathbf{x}_k,\ldots,\mathbf{x}_{k+N}\\ \mathbf{y}_{k+1},\ldots,\mathbf{y}_{k+N}}} \sum_{i=1}^{N} (\boldsymbol{y}_{k+i}-\boldsymbol{y}_r)^T \boldsymbol{Q} (\boldsymbol{y}_{k+i}-\boldsymbol{y}_r) + (\boldsymbol{u}_{k+i-1}-\boldsymbol{u}_r)^T \boldsymbol{R} (\boldsymbol{u}_{k+i-1}-\boldsymbol{u}_r) \tag{4}$$

subject to the following constraints for each $j=1,\ldots,N$:

$$\boldsymbol{x}_k = \hat{\boldsymbol{x}}_k \tag{5}$$

$$\boldsymbol{x}_{k+j} = f(\boldsymbol{x}_{k+j-1}, \boldsymbol{u}_{k+j-1}, \boldsymbol{d}_{k+j-1}) \tag{6}$$

$$\boldsymbol{y}_{k+j} = \boldsymbol{C}\boldsymbol{x}_{k+j} \tag{7}$$

$$\boldsymbol{u}_{min} \le \boldsymbol{u}_{k+j-1} \le \boldsymbol{u}_{max} \tag{8}$$

$$|\boldsymbol{u}_{k+j}-\boldsymbol{u}_{k+j-1}| \le \Delta\max, \quad j<N \tag{9}$$

$$\boldsymbol{L}\boldsymbol{y}_{k+j} \le \boldsymbol{L}\boldsymbol{y}_{max} \tag{10}$$

with $\mathbf{Q}$ and $\mathbf{R}$ positive-definite diagonal matrices and $\hat{\mathbf{x}}_k$ the measured or estimated state at time step k. Constraint (8) ensures the gates do not exceed their lower and upper limits, constraint (9) ensures the maximal possible gate movement is not exceeded and (10) represents constraints bounding the different water levels in order to avoid flooding. Note that the matrix $\mathbf{L}$ is added in order to be able to deal with the constraint as a soft constraint. Ideally the matrix $\mathbf{L}$ is the unitary matrix. However, during periods of heavy rainfall the upper limits cannot always be satisfied and in that case the matrix $\mathbf{L}$ needs to be adjusted in order to remove the constraints that cannot be satisfied. This is achieved by replacing the values on the diagonal corresponding to the violated water levels by a 0. The optimization problem 1 is a constrained nonlinear programming problem due to the presence of the nonlinear equality constraint (6) representing the dynamical model of the river system. Nonlinear programming problems are typically solved in an iterative way (Nocedal & Wright, 1999). Suppose a point is described as $\mathbf{p} = [\mathbf{u}_k^T \ \ldots \ \mathbf{u}_{k+N-1}^T \ \mathbf{x}_{k+1}^T \ \ldots \ \mathbf{x}_{k+N}^T]^T$, then first a starting point $\mathbf{p}_0$ is chosen which in this work is typically a relatively good estimate of the (local) optimal solution of the nonlinear programming problem (see further). Then a sequence of iterates $\{\mathbf{p}_k\}_{k=0}^{\infty}$ with decreasing value for the cost function (4) are generated that terminate when no more progress can be made, when it seems a solution point has been approximated with sufficient accuracy or when the available time (sampling time) has expired.

In order to obtain a sequence of improving iterates $\{\mathbf{p}_k\}_{k=0}^{\infty}$ a NMPC scheme is used (Camacho & Bordons, 2005). Basically, the NMPC scheme approximates the nonlinear behavior of the system by a linear time variant system. A linear time variant system has the following form:

$$\mathbf{x}_{k+1} = \mathbf{A}_k \mathbf{x}_k + \mathbf{B}_k \mathbf{u}_k + \mathbf{d}_k \tag{11}$$

$$\mathbf{y}_k = \mathbf{C}\mathbf{x}_k \tag{12}$$

with $\mathbf{A}_k$, $\mathbf{B}_k$ and $\mathbf{C}_k$ the system matrices of the system at time step k. Remark that the system matrices are time-dependent which is why the system is called time variant. Also note that the disturbance input $\mathbf{d}_k$ is time variant and makes it possible to take future rain predictions into account.

The nonlinear scheme starts by doing a simulation with the future optimal inputs obtained at the previous time step. This leads to a trajectory of the future states. The future optimal inputs computed at time step $k-1$ are defined as

$$[\mathbf{u}_{k-1|k-1} \quad \cdots \quad \mathbf{u}_{k+N-2|k-1}] \tag{13}$$

with $\mathbf{u}_{k+i|k}$ denoting the input at time step $k+i$ predicted at time step k. At time step k a simulation of the river model is done using the following inputs:

$$[\mathbf{u}_{k|k-1} \quad \cdots \quad \mathbf{u}_{k+N-2|k-1} \quad \mathbf{u}_{k+N-2|k-1}] \tag{14}$$

This simulation gives raise to a sequence of future states

$$[\mathbf{x}^0_{k+1} \quad \cdots \quad \mathbf{x}^0_{k+N}] \tag{15}$$

Combining (14) and (15) and assuming for convenience that

$$[\mathbf{u}_{k|k-1} \quad \cdots \quad \mathbf{u}_{k+N-2|k-1} \quad \mathbf{u}_{k+N-2|k-1}] = [\mathbf{u}^0_k \quad \cdots \quad \mathbf{u}^0_{k+N-1}]^T \tag{16}$$

an initial starting point

$$\mathbf{p}_0 = [\mathbf{u}^0_k \quad \cdots \quad \mathbf{u}^0_{k+N-1} \quad \mathbf{x}^0_{k+1} \quad \cdots \quad \mathbf{x}^0_{k+N}]^T \tag{17}$$

can be obtained that is a relatively good estimate of the (local) solution of the nonlinear optimization problem 1. The nonlinear equality constraint (6) representing the nonlinear behavior of the system can now be linearized around the point $\mathbf{p}_0$ leading to the following linear time-varying approximation of the behavior of the nonlinear model around the point $\mathbf{p}_0$:

$$\mathbf{x}_{k+j+1} = \mathbf{A}_{k+j}(\mathbf{x}_{k+j} - \mathbf{x}^0_{k+j}) + \mathbf{B}_{k+j}(\mathbf{u}_{k+j} - \mathbf{u}^0_{k+j}) + \mathbf{x}^0_{k+j+1} \tag{18}$$

with $\mathbf{x}^0_{k+j}$ and $\mathbf{u}^0_{k+j}$ the simulated state and input at time step $k+j$ and $\mathbf{x}^0_{k+j+1} = f(\mathbf{x}^0_{k+j}, \mathbf{u}^0_{k+j})$. The matrices $\mathbf{A}_{k+j}$ and $\mathbf{B}_{k+j}$ are obtained by the following forward finite difference scheme:

$$\mathbf{A}_{k+j}(:,s) = \frac{f(\mathbf{x}^0_{k+j} + \Delta\mathbf{x}(s), \mathbf{u}^0_{k+j}) - f(\mathbf{x}^0_{k+j}, \mathbf{u}^0_{k+j})}{\Delta\mathbf{x}(s)} \tag{19}$$

$$\mathbf{B}_{k+j}(:,s) = \frac{f(\mathbf{x}^0_{k+j}, \mathbf{u}^0_{k+j} + \Delta\mathbf{u}(s)) - f(\mathbf{x}^0_{k+j}, \mathbf{u}^0_{k+j})}{\Delta u(s)} \tag{20}$$

with $\mathbf{A}_{k+j}(:,s)$, $\mathbf{B}_{k+j}(:,s)$ denoting the s-th column of $\mathbf{A}_{k+j}, \mathbf{B}_{k+j}$, $\Delta\mathbf{x}(s)$ and $\Delta\mathbf{u}(s)$ two column vectors containing only zeros with exception from the s-th element which is equal to a small perturbation δ. The linearized system (18) can be re-written into the form of (11) by noting that $\mathbf{d}_k = \mathbf{x}^0_{k+1} - \mathbf{A}_k\mathbf{x}^0_k - \mathbf{B}_k\mathbf{u}^0_k$. Also note that the rain predictions are taken into account implicitly by the simulated states $\mathbf{x}^0_{k+j} j = 1,\ldots,N$. Replacing the nonlinear system equation (6) by the time-varying system (18), optimization problem 1 converts into a quadratic programming problem (QP) :

Optimization Problem 2.

$$\min_{\substack{\mathbf{u}_k,\ldots,\mathbf{u}_{k+N-1} \\ \mathbf{x}_{k+1},\ldots,\mathbf{x}_{k+N} \\ \mathbf{y}_{k+1},\ldots,\mathbf{y}_{k+N}}} \sum_{i=1}^{N} (\boldsymbol{y}_{k+i} - \boldsymbol{y}_r)^T \mathbf{Q} (\boldsymbol{y}_{k+i} - \boldsymbol{y}_r) + (\boldsymbol{u}_{k+i-1} - \boldsymbol{u}_r)^T \boldsymbol{R} (\boldsymbol{u}_{k+i-1} - \boldsymbol{u}_r) \tag{21}$$

subject to the following constraints for each $j = 1,\ldots,N$:

$$\boldsymbol{x}_k = \hat{\boldsymbol{x}}_k \tag{22}$$

$$\boldsymbol{x}_{k+j} = \boldsymbol{A}_{k+j-1}\boldsymbol{x}_{k+j-1} + \boldsymbol{B}_{k+j-1}\boldsymbol{u}_{k+j-1} + \boldsymbol{d}_{k+j-1} \tag{23}$$

$$\boldsymbol{y}_{k+j} = \boldsymbol{C}\boldsymbol{x}_{k+j} \tag{24}$$

$$\boldsymbol{u}_{min} \le \boldsymbol{u}_{k+j-1} \le \boldsymbol{u}_{max} \tag{25}$$

$$|\boldsymbol{u}_{k+j} - \boldsymbol{u}_{k+j-1}| \le \Delta\max, j < N \tag{26}$$

$$\boldsymbol{L}\boldsymbol{y}_{k+j} \le \boldsymbol{L}\boldsymbol{y}_{max} \tag{27}$$

Note that in optimization problem 2 the future input trajectories as well as the future state trajectories are considered as optimization variables which is the reason why this NMPC scheme is called uncondensed NMPC. This scheme was used in the results obtained in Barjas Blanco et al. (2009).

5.1.1. *Line search*

Note that the linear time-varying system (11) is only an approximation for the nonlinear system (2) around the point of linearization. Therefore taking a full Newton step in the direction found by solving optimization problem 2 does not necessarily yield an improved point. In order to enforce progress in every optimization step either a trust region constraint can be added to the QP or a line search can be performed. In this work is opted for the line search approach. The reason for this choice is the added uncertainty on the rainfall-runoff predictions. At each time step k an initial solution is created based on the solution obtained at time step $k-1$. However, due to the uncertainty of the rainfall-runoff predictions this initial solution can be quite different from the local optimum of the NLP to be solved at time step k. It is known that in such cases line search methods perform better than trust region methods (Nocedal & Wright, 1999). In the following the line search approach used in the simulations is discussed in more detail.

Assume the point $\mathbf{p}^*$ to be the solution of the QP, then a line search method will approximately search for the best point lying on the line that connects the points $\mathbf{p}_0$ and $\mathbf{p}^*$. Because each point on this line can be written as $\mathbf{p} = \mathbf{p}_0 + \alpha(\mathbf{p}^* - \mathbf{p}_0)$ for $0 \le \alpha \le 1$ a line search basically tries to find that value for α that leads to the best point $\mathbf{p}$. In order to evaluate the cost corresponding to a point on the line, the sequence of future control moves corresponding to the point is applied to the nonlinear model (6) and a simulation is done with $\hat{\mathbf{x}}$ as initial state. This gives raise to a sequence of future states. The best point on the line is the point for which this simulation yields the best value for the cost function (4) and satisfies the constraints (5)–(10). Because the point $\mathbf{p}^*$ is a solution of the QP described in optimization problem 2 and the evaluation of the point has been done by a simulation of the nonlinear model starting with $\hat{\mathbf{x}}$ as initial state of the simulation, constraints (5)–(9) are automatically satisfied. For constraints (5)–(7) this is trivial. Because the line search involves a search on a line with two feasible points as endpoints and because the constraint defined by (8) is convex, constraint (8) is satisfied for all points on the defined line. In order to see that constraint (9) is also satisfied for each point on the defined line if the constraint is satisfied for the two endpoints, let $\mathbf{u}^0_{k+i}, \mathbf{u}^0_{k+i+1}$ denote two successive inputs of the vector $\mathbf{p}_0$ and $\mathbf{u}^*_{k+i}, \mathbf{u}^*_{k+i+1}$ two successive inputs of the vector $\mathbf{p}$. Because $\mathbf{p}$ and $\mathbf{p}_0$ satisfy constraint (9) the following conditions are valid:

$$|\mathbf{u}^0_{k+i} - \mathbf{u}^0_{k+i+1}| \le \Delta\max \tag{28}$$

$$|\mathbf{u}^*_{k+i} - \mathbf{u}^*_{k+i+1}| \le \Delta\max \tag{29}$$

It is straightforward to show that for each point on the line the value the corresponding inputs on time steps $k+i$ and $k+i+1$ can be written as

$$\mathbf{u}_{k+i} = (1-\alpha)\mathbf{u}^0_{k+i} + \alpha\mathbf{u}^*_{k+i} \tag{30}$$

$$\mathbf{u}_{k+i+1} = (1-\alpha)\mathbf{u}^0_{k+i+1} + \alpha\mathbf{u}^*_{k+i+1} \tag{31}$$

Therefore the difference between these two inputs can be written as

$$\mathbf{u}_{k+i+1} - \mathbf{u}_{k+i} = (1-\alpha)(\mathbf{u}^0_{k+i+1} - \mathbf{u}^0_{k+i}) + \alpha(\mathbf{u}^*_{k+i+1} - \mathbf{u}^*_{k+i}) \tag{32}$$

which for $0 \le \alpha \le 1$ is a convex combination of two differences with an absolute value lower than Δmax and therefore it follows that $|\mathbf{u}_{k+i+1} - \mathbf{u}_{k+i}| \le \Delta$max for each value of α, so constraint (9) is satisfied for each point on the line. The only constraint that needs to be verified during the line search is constraint (10). The best point found by the line search is the next point in the sequence $\{\mathbf{p}_k\}_{k=0}^{\infty}$. Again a linearization around this new point is done and the QP of optimization problem 2 is solved and after a line search the next point in the sequence is obtained. These steps are repeated until convergence. Convergence occurs when the difference $\|\mathbf{p}_i - \mathbf{p}_{i-1}\|_\infty \le \varepsilon$, with $\mathbf{p}_i$ and $\mathbf{p}_{i-1}$ the optimal points calculated at iteration i and $i-1$, respectively, and ε a small value. Note that there exist many sophisticated line search algorithms in the literature (Nocedal & Wright, 1999). In this work a simple line search algorithm is implemented in which the point $\mathbf{p}$ is evaluated for several values of α going from $\alpha = 0$ to 1 in steps of 1/25.

The nonlinear optimization procedure can be summarized as follows:

- At iteration n a simulation of the future state trajectory is performed using the optimal inputs $\{\mathbf{u}_k^{n-1}, \ldots, \mathbf{u}_{k+N-1}^{n-1}\}$ from the previous iteration $n-1$ leading to the linear time variant system (11).
- The QP of optimization problem 2 is solved leading to an optimal input sequence $\{\mathbf{u}_k^*, \ldots, \mathbf{u}_{k+N-1}^*\}$.
- A line search is performed in which the inputs at iteration n are determined as

  $$\{\mathbf{u}_k^n, \ldots, \mathbf{u}_{k+N-1}^n\} = (1-\alpha)\{\mathbf{u}_k^{n-1}, \ldots, \mathbf{u}_{k+N-1}^{n-1}\} + \alpha\{\mathbf{u}_k^*, \ldots, \mathbf{u}_{k+N-1}^*\} \tag{33}$$

  with α obtained as the value from the set $\{0, \frac{1}{25}, 2\frac{1}{25}, \ldots, 1\}$ that gives the maximum value for the cost function (4) and at the same time satisfies constraint (10).
- A convergence check is performed as follows:

  $$\|[(\mathbf{u}_k^n)^T \ \ldots \ (\mathbf{u}_{k+N-1}^n)^T]^T - [(\mathbf{u}_k^{n-1})^T \ \ldots \ (\mathbf{u}_{k+N-1}^{n-1})^T]^T\|_\infty \le \varepsilon \tag{34}$$

- If the convergence condition (34) is not satisfied n is increased by 1 and the algorithm jumps to the first step. If the convergence condition is satisfied, convergence is obtained and the optimal input u_k^n corresponding to the current time step is applied to the real system.

5.2. Semi-condensed NMPC

As stated before, the NMPC scheme of optimization problem 2 is an uncondensed NMPC scheme. The drawback of this scheme is the amount of memory required, especially for large horizons. A way to solve this problem is to use a condensed NMPC scheme. In a condensed NMPC scheme the states are eliminated from the optimization variables by writing them as a function of the initial state and the input trajectory. As a result of this the number of optimization variables reduces significantly resulting to a smaller QP to be solved *on-line*. In Barjas Blanco, Willems, De Moor, and Berlamont (2008) this scheme was used in order to control the upstream part of the model. However, the drawback of this approach is that the resulting QP turns out to be ill-conditioned for large horizons. This is the reason why in Barjas Blanco et al. (2009) an uncondensed NMPC scheme was used. In this paper a new NMPC scheme is proposed that combines the advantages of both approaches and that will be referred to as semi-condensed NMPC. In the following this scheme will be outlined in more detail.

Define a set φ of positive integers as follows:

$$\varphi = \{n_1, n_2, \ldots, n_r\} \tag{35}$$

$$n_1 > 0 \tag{36}$$

$$n_r \le N \tag{37}$$

$$n_{i+1} > n_i, 0 < i < r \tag{38}$$

This set defines the set of states $\chi = \{\mathbf{x}_{k+n_1}, \mathbf{x}_{k+n_2}, \ldots, \mathbf{x}_{k+n_r}\}$ which are taken explicitly into account as optimization variables of the semi-condensed NMPC scheme. Similar to the condensed NMPC scheme the states in between two successive states from χ can always be written in function of one of the states in χ and a subset of the unknown input variables. Assume the states $\mathbf{x}_{k+n_i}, \mathbf{x}_{k+n_{i+1}} \in \chi$ for $i \in \{1, \ldots, r-1\}$ it then follows that the states in between $\mathbf{x}_{k+n_i}, \mathbf{x}_{k+n_{i+1}}$ can be written as

$$\begin{bmatrix} \mathbf{x}_{k+n_i+1} \\ \mathbf{x}_{k+n_i+2} \\ \vdots \\ \mathbf{x}_{k+n_{i+1}-1} \end{bmatrix} = \mathbf{F}(n_i)\mathbf{x}_{k+n_i} + \mathbf{G}(n_i) \begin{bmatrix} \mathbf{u}_{k+n_i} \\ \mathbf{u}_{k+n_i+1} \\ \vdots \\ \mathbf{u}_{k+n_{i+1}-2} \end{bmatrix} \tag{39}$$

with

$$\mathbf{F}(n_i) = \begin{bmatrix} \mathbf{A}_{k+n_i} \\ \mathbf{A}_{k+n_i+1}\mathbf{A}_{k+n_i} \\ \vdots \\ \mathbf{A}_{k+n_{i+1}-2} \ldots \mathbf{A}_{k+n_i+1}\mathbf{A}_{k+n_i} \end{bmatrix} \tag{40}$$

and

$$\mathbf{G}(n_i) = \begin{bmatrix} \mathbf{B}_{k+n_i} & \mathbf{0} & \ldots & \ldots \\ \mathbf{A}_{k+n_i+1}\mathbf{B}_{k+n_i} & \mathbf{B}_{k+n_i+1} & \mathbf{0} & \ldots \\ \mathbf{A}_{k+n_i+2}\mathbf{A}_{k+n_i+1}\mathbf{B}_{k+n_i} & \mathbf{A}_{k+n_i+2}\mathbf{B}_{k+n_i+1} & \mathbf{B}_{k+n_i+2} & \ldots \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix} \tag{41}$$

Note that (39) can be re-written as

$$\begin{bmatrix} \mathbf{x}_{k+n_i+1} \\ \mathbf{x}_{k+n_i+2} \\ \vdots \\ \mathbf{x}_{k+n_{i+1}} \end{bmatrix} = [\mathbf{F}(n_i) \ \ \mathbf{G}(n_i)]\mathbf{z}(n_i) \tag{42}$$

with

$$\mathbf{z}(n_i) = [\mathbf{x}^T_{k+n_i} \ \ \mathbf{u}^T_{k+n_i} \ \ \mathbf{u}^T_{k+n_i+1} \ \ \ldots \ \ \mathbf{u}^T_{k+n_{i+1}+1}]^T \tag{43}$$

Now define $J(n_i, n_{i+1})$ as the cost related to the states in between the states $\mathbf{x}_{k+n_i}, \mathbf{x}_{k+n_{i+1}}$, from (42) it can be shown that this cost can be expressed as a quadratic function in $\mathbf{z}(n_i)$

as follows:

$$J(n_i,n_{i+1}) = z(n_i)^T \begin{bmatrix} \mathbf{F}^T\hat{\mathbf{Q}}\mathbf{F} & \mathbf{F}^T\hat{\mathbf{Q}}\mathbf{G} \\ \mathbf{G}^T\hat{\mathbf{Q}}\mathbf{F} & \mathbf{G}^T\hat{\mathbf{Q}}\mathbf{G} \end{bmatrix} \mathbf{z}(n_i) - 2\mathbf{p}_r^T\hat{\mathbf{Q}}[\mathbf{F}\ \ \mathbf{G}]\mathbf{z}(n_i) \tag{44}$$

with

$$\hat{\mathbf{Q}} = \begin{bmatrix} \mathbf{C}^T\mathbf{QC} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{C}^T\mathbf{QC} & \dots & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{C}^T\mathbf{QC} \end{bmatrix} \tag{45}$$

and

$$\mathbf{p}_r = [\mathbf{x}_r^T\ \ \mathbf{x}_r^T\ \ \dots\ \ \mathbf{x}_r^T]^T \tag{46}$$

Taking this into account the cost function (21) can be re-written as

$$\min_{\substack{\mathbf{u}_k,\dots,\mathbf{u}_{k+N-1} \\ \mathbf{x}_{k+n_1},\dots,\mathbf{x}_{k+n_r} \\ \mathbf{y}_{k+n_1},\dots,\mathbf{y}_{k+n_r}}} \sum_{i=1}^{r-1} (\mathbf{y}_{k+n_i} - \mathbf{y}_r)^T \mathbf{Q} (\mathbf{y}_{k+n_i} - \mathbf{y}_r) + J(n_i,n_{i+1}) + J(0,n_1) + (\mathbf{y}_{k+n_r} - \mathbf{y}_r)^T \mathbf{Q} (\mathbf{y}_{k+n_r} - \mathbf{y}_r) + (\mathbf{u}_{k+i-1} - \mathbf{u}_r)^T \mathbf{R} (\mathbf{u}_{k+i-1} - \mathbf{u}_r) \tag{47}$$

which is again a quadratic function in the unknown variables. Changing the constraints of optimization problem 2 accordingly is straightforward and combining this with cost function (47) leads to a semi-condensed QP formulation. Note that the number of optimization variables has reduced from N state vectors and N input vectors to n_r state vectors and N input vectors. In this application this is a significant reduction as the number of state variables is large, more specific there are 75 state variables. Also note that a line search is still needed to ensure reduction of the cost function.

5.3. Constraint strategy

The constraints (8) and (9) in optimization problem 1 are hard constraints that should always be satisfied because they represent physical limitations of the hydraulic structures. Constraint (10) on the other hand represents the flood levels of the different water levels in the river system. During heavy rainfall it is not always possible to satisfy these constraints. Therefore, this constraint should be implemented as a soft constraint which means that under normal operation this constraint should be satisfied but during heavy rainfall constraint violations are tolerated. Since this constraint is a vector inequality, a priorization strategy is necessary.

In order to achieve this a number of constraint sets with different priorities are defined. The set with the highest priority is called S_1 and represents all the upper limits on all the water levels. If the QP of optimization problem 2 returns no feasible solution for the constraint set S_1, the constraints related with the less important water levels are omitted leading to a new constraint set $S_2 \supseteq S_1$. The weights in the cost function of optimization problem 1 corresponding to the water levels of the omitted constraints are increased by which the QP tries to minimize the violation of the omitted constraints and tries to make the set S_1 feasible in the next QP iteration or in the future time steps. This procedure of omitting less important constraints is repeated until the QP returns a feasible solution. If a feasible solution is obtained, a line search will be performed and in the next iteration this procedure will be repeated starting from the constraint set S_1. Note that this procedure always leads to a feasible solution because in the *worst-case* all soft constraints will be omitted and therefore a feasible solution is ensured. Also note that omitting constraints can be achieved by setting the corresponding values on the diagonal of $\mathbf{L}$ equal to 0.

Another aspect that must be considered is the fact that the prediction horizon cannot be taken infinitely long. The reason for that is that most weather predictions predict around 2 days ahead. Predictions further than 2 days ahead can be extremely unreliable. However, heavy rain events that cause floods typically last longer than 2 days. A problem that can arise is that given this prediction of 2 days ahead it is possible that the optimal solution consists of filling the water reservoirs up to their flood level at the end of the horizon because the controller does not know that it might still rain after the prediction horizon. This means that there is no storage capacity available for the rain that might fall during the period beyond the prediction horizon. Therefore, conservativeness must be added to the control strategy. Another problem that can arise is that during periods of normal regulation the water reservoirs are being used in order to improve setpoint regulation. The problem with this is that the total available storage capacity of the water reservoirs reduces which can cause floods afterwards during periods of heavy rainfall. In order to avoid these problems the following constraint strategy is applied in this work:

- For each water level a guard level is defined by the local water administration. As long as the water levels do not violate their corresponding guard level, it is not allowed to fill the water reservoirs.
- Water reservoirs can be used to avoid violation of the guard levels. However, it is not allowed to use the complete storage volume available in the reservoirs. For each reservoir a safety limit is defined. Once this safety limit is reached the reservoirs may not be used anymore and the guard levels might get violated. If it is impossible to satisfy the guard levels, these guard levels will be replaced by their corresponding flood levels.
- If it continues raining and NMPC cannot keep the water levels beneath their flood level within the prediction window, it is allowed to further fill the reservoirs until the water level in the reservoirs reach their corresponding flood level.
- If the rainfall is really excessive then flooding is unavoidable and some or all water levels will violate their corresponding flood levels and the optimization proposed in step 3 will be infeasible. In order to be able to get a useful solution from NMPC flood levels need to be removed from the optimization until a feasible solution is obtained.

5.4. Controllability

Typically in river control the discharges through the hydraulic control structures are chosen as the input variables to be determined by the NMPC. Once the NMPC has computed the optimal discharge trajectory for all the gates, local controllers (PI, PID) situated at the control structures try to follow the optimal discharge trajectory as close as possible. However, by doing this the nonlinearities of the control structure equations are not taken into account explicitly. Because these nonlinearities play an important role in flood prevention, in this work a different approach is used. In this work the gate levels at the hydraulic structures are considered as inputs for the NMPC. A difficulty that arises with this approach is the non-controllability of the gates in the river system. In the gate equations there are some modes in which the discharge over the gate is independent of the gate itself.

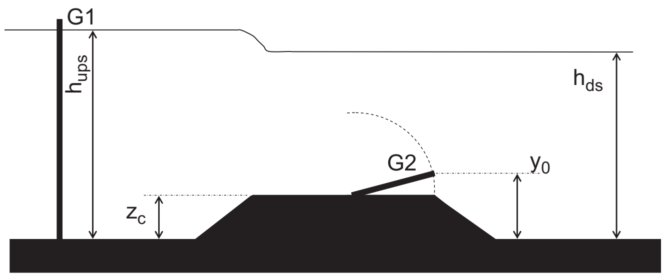

Fig. 3. Uncontrollable gate modes.

If a gate reaches such a mode it possibly gets trapped in this mode and the NMPC controller cannot control it. In Fig. 3 these uncontrollable modes are depicted. The first uncontrollable mode arises when the gate $G1$ is completely closed. In this case the discharge over the gate is zero. By moving the gate by a small amount the discharge is still going to be zero. This is reflected in the linearized model used in the optimization by system matrices where the discharge over the gate is independent of the corresponding gate level. The second uncontrollable mode arises when the gate $G2$ is positioned at a very sharp angle with respect to the bottom. In that case the discharge over the gate is determined by the bottom elevation z_c and the water levels up- and downstream (h_{ups} and h_{ds}) of the gate and therefore independent of the gate position. (for the gate equations we refer to InfoWorks-RS, 2008). In both cases linearization of the discharge equation leads to the following linearized system equations:

$$\begin{pmatrix} h_{ups}(k+1) \\ h_{ds}(k+1) \\ q(k+1) \end{pmatrix} = A \begin{pmatrix} h_{ups}(k) \\ h_{ds}(k) \\ q(k) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} y_0(k) \tag{48}$$

with y_0 the position of the gate as indicated in Fig. 3. Note that the system matrix related to the inputs consists of zeros only and therefore the controllability matrix of this system is not of full rank which means the system is not controllable. In Barjas Blanco et al. (2008) this problem was discussed for the control of the upstream part of the river Demer and solved by using a fuzzy model for the calculations of the linearized time-varying system. The drawback of this approach is of course that the model used to calculate the derivatives and the model used for the simulation are different which can lead to decrease in performance. Therefore, in this work a different approach is adopted. At the beginning of each sampling time a first simulation is performed in order to approximate the nonlinear behavior of the river model by a linear time variant model, as outlined in Section 5. Afterwards a check is done in order to detect all the uncontrollable gates and at which moments within the prediction horizon the gates are uncontrollable. This is done by analyzing the columns of the corresponding matrix B_k. If all the elements of a column in the matrix B_k are zero, then the corresponding gate is uncontrollable at time step k. Next, it is checked for each uncontrollable gate if the controllability is a result of the gate being completely closed or if it is a result of the gate being positioned at a sharp angle. In case the uncontrollability is a result of the gate being completely closed the reference level for that gate at the moments in time, where the gate is uncontrollable, is set a little bit lower than the highest neighbouring water level. In the other case the reference level is increased in order to increase the angle of the gate. By doing this, the controller tries to steer gates to modes where they are controllable.

6. Simulation results

In this section semi-condensed NMPC with line search is compared with the three-position controller of the local water administration by comparing the performance of both control strategies for the historical rainfall of 1998. The rainfall of 1998 caused the largest flood in the Demer basin of the last 20 years and is therefore the most suited event for making a comparison between the two strategies. NMPC uses rain predictions in order to make a prediction of the future water levels and based on these predictions a suitable control action is performed. An important point that needs to be considered is the fact that these rain predictions never coincide with the real rainfall. So the predictions made by NMPC are always going to deviate from the real future water levels. However, one of the crucial features of NMPC is its receding horizon strategy. This means that at each sampling time the new state of the system is measured or estimated; based on this measured/estimated state a new prediction is made. Because of this information feedback at every sampling time NMPC is able to suppress disturbances and model deviations, hence is inherently robust against uncertainties in the rainfall predictions. In order to show this the simulation for NMPC is going to be performed by adding a realistic amount of uncertainty to the rainfall-runoff discharges predictions.

The uncertainty of the rainfall-runoff discharges prediction varies within the prediction horizon. The further ahead in time the larger the uncertainty on the rainfall-runoff predictions. In these simulations it is assumed that the rainfall-runoff uncertainty at the current time step is 10% and that this uncertainty increases with 0.2% for each hour further ahead in the prediction. This means that if the prediction horizon is 30 h then at the end of the prediction window the rainfall-runoff uncertainty is equal to 16%. These values were estimated based on historical data by means of the techniques described in Timbe (2007). In this section it is assumed that the rainfall-runoff predictions used by NMPC for calculating an optimal input sequence is an overestimation of the real rainfall-runoff of 1998. This is done in the following way:

- At time step $k+i$ within the prediction horizon N the maximum uncertainty Δ_{max} is calculated as p% of the real rainfall-runoff discharge q_{up} of 1998. The variable p satisfies $p=10+0.2i$, which means that at time step k, p is equal to 10 and increases with 0.2 each hour further ahead in the future.
- Based on this value for the maximum uncertainty Δ_{max} the uncertainty Δ is determined by performing a random sampling from a Gaussian distribution with mean 0 and standard deviation σ satisfying $2 \times \sigma = \Delta_{max}$.
- In order to ensure that an overestimation of q_{up} is obtained, the absolute value of Δ is added to q_{up}.

In Table 2 a summary is given of the NMPC parameters used in this simulation. The control horizon and the prediction horizon are equal and at demand of the VMM set to 30 h. The sampling time is 1 h. The matrices **Q** and **R** in the cost function (47) are variable. The values of these matrices depend on the feasible constraint set (see Section 5.3). The result of this simulation with NMPC is plotted in Figs. 6 and 7. In Figs. 4 and 5 the same water

Table 2
NMPC parameters.

Control horizon	Prediction horizon	Sampling time	**Q**	**R**
30 h	30 h	1 h	Variable	Variable

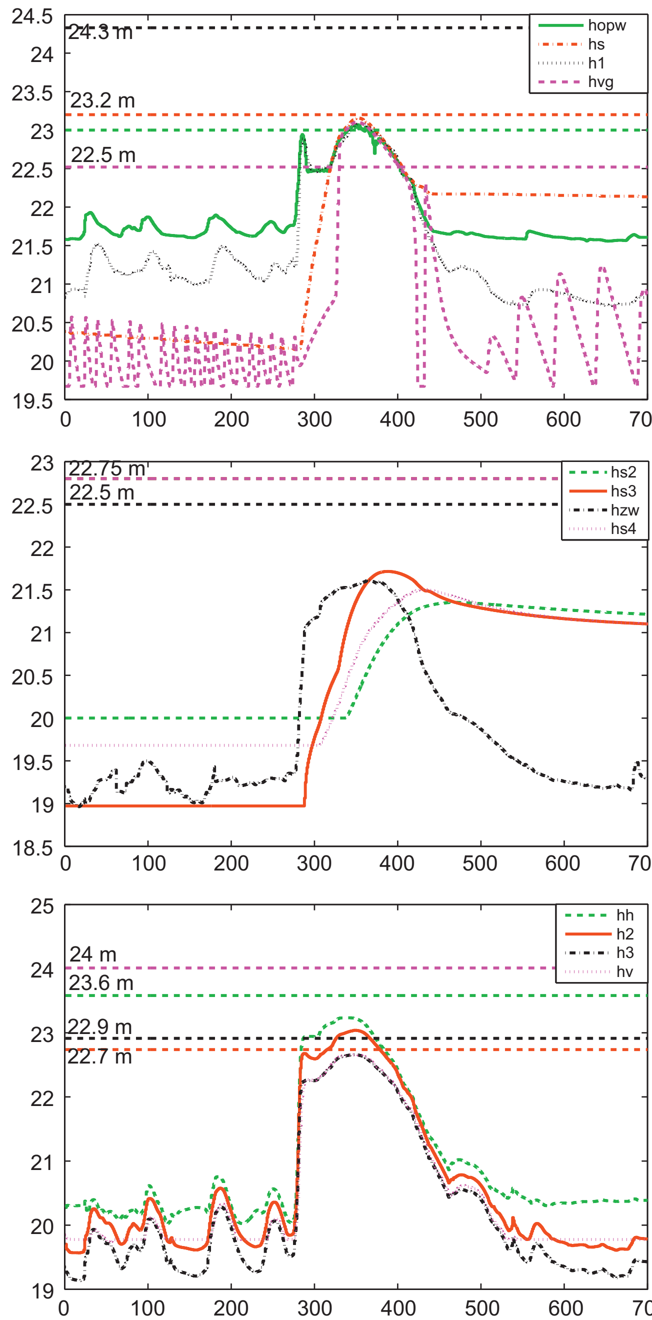

Fig. 4. Regulation with three-position controller.

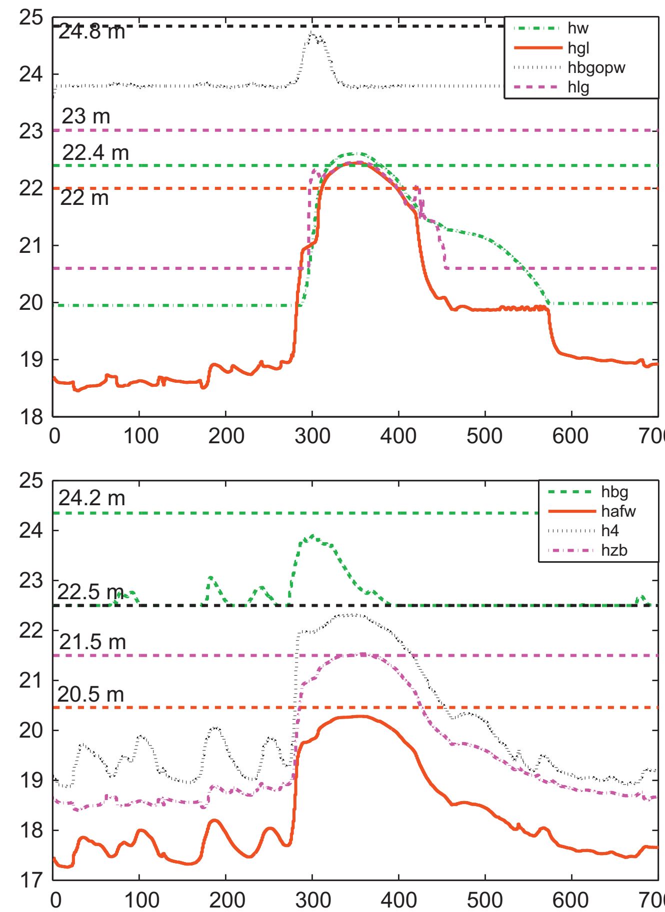

Fig. 5. Regulation with three-position controller.

levels are plotted for simulation with the three-position controller. Comparing both figures leads to the following observations:

- NMPC clearly improves the regulation of the upstream water level h_{opw} during periods of moderate rainfall.
- NMPC steers hbg_{opw} to the desired setpoint but the three-position controller performs a better and smoother regulation of hbg_{opw}.
- The three-position controller is not capable of keeping h_w under its flood level. The violation of h_w is very severe and lasts for 60 h. On the other hand, NMPC is capable of keeping h_w under its flood level. This is a significant improvement as water level h_w corresponds to a water reservoir. Water reservoirs are typically very large so a small change in water level corresponds with a large amount of water volume.
- Water levels h_2, h_{gl} and h_{vg} violate their corresponding flood level for both control strategies. However, from the figures it can be seen that these violations are smaller for NMPC. Moreover, the period of time for which these water levels violate their flood level is shorter for NMPC.

Besides a visual comparison based on Figs. 4–7 in Tables 3 and 4 a quantitative assessment of both control strategies is made by comparing both control strategies by means of a cost function. The cost function in Table 3 is referred to as the regulation cost and is defined as

$$J_{reg}(h) = \sum_{i=1}^{N_b} e_{k+i}(h) + \sum_{i=1}^{N_{sim}-N_a} e_{N_a+i}(h) \tag{49}$$

with N_b the time instant where the severe flow event starts (here: hour 300), N_a the time instant where the flow event ends (here: hour 500), N_{sim} the time instant where the simulation ends (here: hour 700) and with $e_k(h)$ defined as

$$e_k(h) = h_k - h_r \tag{50}$$

with h_k the value of the water level h at time instant k, and h_r the desired setpoint for water level h. With other words, the cost function $J(h)$ is the sum of all the deviations of water level h from the setpoint. From Table 3 it can be seen that NMPC regulates h_{opw} much better than the three-position controller. On the other hand, the three-position controller performs a slightly better regulation

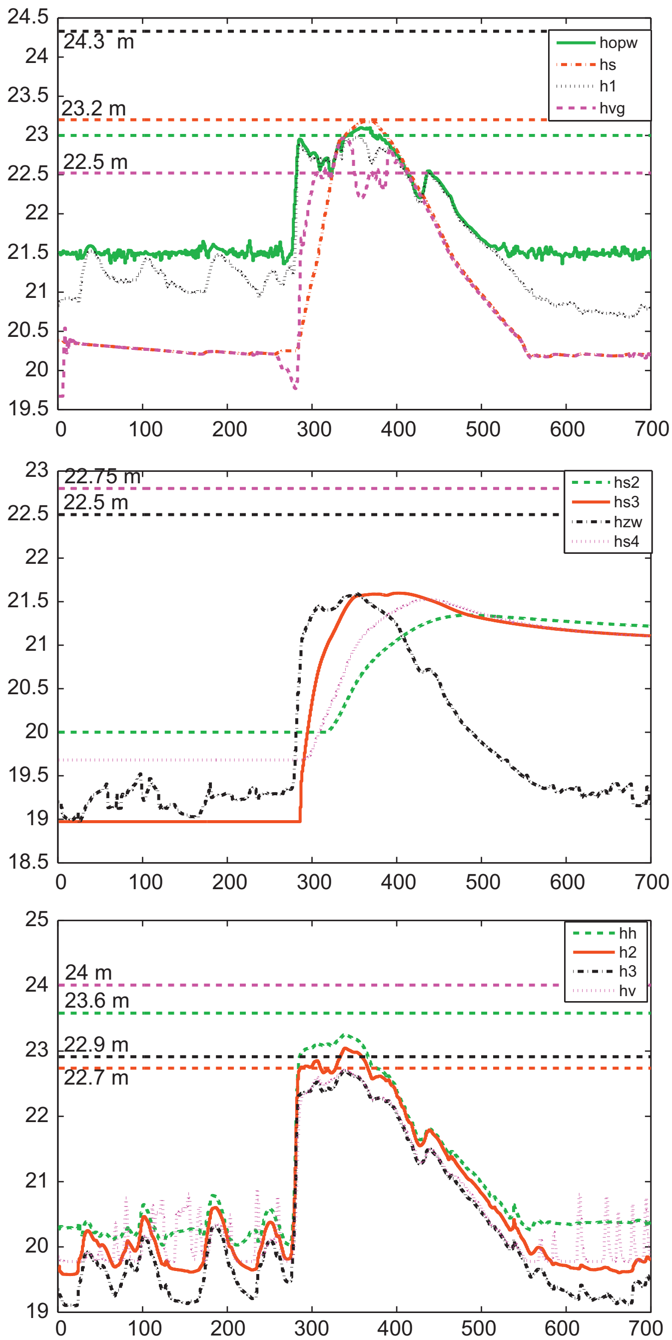

Fig. 6. Regulation with NMPC controller.

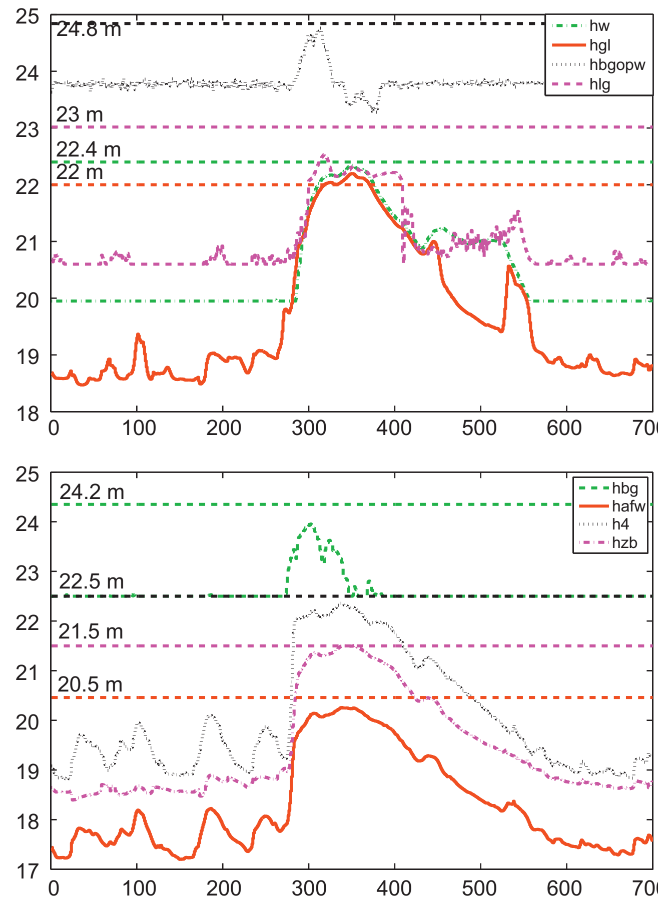

Fig. 7. Regulation with NMPC controller.

of hbg_{opw} than NMPC. Overall, it can be stated that NMPC performs a better setpoint regulation. In Table 4 the flooding cost is shown. The flooding cost is a measure for the amount of flooding that has occurred and is defined as

$$J_f(h) = \sum_{i=1}^{N_{sim}} e^f_{k+i}(h) \tag{51}$$

with

$$e^f_k(h) = \max(h_k - h_f, 0) \tag{52}$$

and h_f the flooding level of water level h. From Table 4 it can be seen that NMPC is capable of avoiding flooding of the water reservoir h_w. Besides that NMPC also significantly reduces the flooding of the other three water levels. Therefore, despite the uncertainty in the rainfall-runoff predictions, NMPC outperforms the current three-position controller.

Table 3
Comparison of the regulation cost of the two controllers during the periods before and after the flood event.

	Three-position controller	NMPC
$J_{reg}(h_{opw})$	16.08	0.90
$J_{reg}(hbg_{opw})$	0.16	1.05

Table 4
Comparison of the flooding cost of the two controllers.

	Three-position controller	NMPC
$J_f(h_2)$	3.00	1.89
$J_f(h_w)$	1.48	0
$J_f(h_{gl})$	9.47	0.58
$J_f(h_{vg})$	13.02	3.91

Table 5
NMPC efficiency.

	CPU time (s)	Variables	Hessian
Uncondensed NMPC	3.5	2610	6 812 100
Semi-condensed NMPC	0.4	735	540 225

To conclude this section in Table 5 a comparison in efficiency is made between the uncondensed NMPC scheme and the new semi-condensed NMPC scheme. The CPU time for 1 iteration is 3.5 s for the uncondensed scheme and just 0.4 s for the semi-condensed scheme. Since several iterations are needed in order to obtain convergence the computational speed of the semi-condensed scheme is obvious. Besides computational efficiency, it can also be seen that the optimization problem of the semi-condensed scheme is significantly smaller than that of the uncondensed scheme. In the semi-condensed scheme the unknown vector consists of only 735 variables whereas in the uncondensed scheme the unknown vector consists of 2610 variables. The consequence of this is that the Hessian matrix of the QP consists of 6 812 100 elements for the uncondensed scheme whereas of only 540 225 elements for the semi-condensed scheme and therefore the uncondensed scheme uses significantly more CPU memory. Therefore, the semi-condensed scheme outperforms the uncondensed scheme w.r.t. CPU time and memory usage. With respect to convergence both schemes solve the same problem and therefore lead to the same solution.

Remark 1. The states added into the optimization problem of the semi-condensed scheme are $\chi=\{x_{k+5},x_{k+10},x_{k+15},x_{k+20},x_{k+25},x_{k+30}\}$.

7. Conclusions

In this work a NMPC scheme was discussed for flood regulation. The NMPC scheme is based on a semi-condensed optimization scheme combined with a line search approach. A constraint strategy is discussed in order to tackle possible infeasibilities during periods of heavy rainfalls. Also controllability problems are discussed and a suitable way to solve them. The resulting scheme is tested on the historical flood event of 1998. In order to simulate uncertainties in the rainfall-runoff prediction the rainfall-runoff model results of 1998 are perturbed by a realistic amount of uncertainty. The simulations show that despite these uncertainties NMPC still outperforms the current three-position controller. The results also show that the newly developed semi-condensed scheme outperforms the uncondensed scheme w.r.t. CPU time and memory usage.

8. Future research

- The work in this paper considers full-state feedback. In practice only a subset of states is measured. Therefore, a state estimator should be added to the control system in order to obtain a full state estimate based on measurements of the water levels. Future work will address the application of a moving horizon estimator (MHE) in combination with NMPC. Note that the absence of a state estimator does not change the conclusions made in this paper. The VMM already implemented a state estimator that gives very accurate estimates of the states in the full hydrodynamic model of the Demer river. The uncertainties arising from this state estimation process are negligible compared to the uncertainties on the rainfall predictions.
- In this paper the model only considered the hydraulic model of the river Demer. In order to be able to incorporate financial damage into the cost function the model should be extended by models of the flood plains such that a prediction can be obtained of the flood that occurs in the populated areas of the Demer basin from which an estimate can be obtained for the accompanying financial damage.

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