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ARMA SPECTRAL ESTIMATION BY AN ADAPTIVE IIR FILTER

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Abstract

An ARMA spectral estimation method by an adaptive infinite impulse response (IIR) filter is proposed. The adaptive IIR filter consists of conventional feed-forward tapped delay line and an extra feedback tapped delay line. The feedback is taken from the residual error between an input time series and its estimate (the output of the adaptive filter). A least mean square (LMS) algorithm for the adaptation of the coefficients of the adaptive IIR filter is derived. The adaptive IIR filter functions as a whitening filter. Therefore the coefficients of the filter correspond to the parameters of the ARMA model and once the algorithm converges the spectral estimation of the input time series can be calculated from these coefficients. Computer simulation results are given in comparison with the conventional FFT method.

1 INTRODUCTION

Spectral estimation is a frequently encountered problem in digital signal processing. Conventional methods of spectral estimation, such as in [1,2] are based on fast Fourier transform (FFT) techniques. They are generally efficient in computation and produce reasonable results for stationary signals. These FFT based approaches, however, have several disadvantages as follows: a) The limitation of the ability to distinguish the spectral responses of two or more signals. The frequency resolution in Hz is roughly the reciprocal of the time interval in seconds over which sampled data is available. b) The problem of "leakage" in frequency domain, i.e., the energy in the main lobe of a spectral response "leaks" into the sidelobes, masking adjacent spectral responses that are present. This is due to the implicit windowing of the data when processing. Careful selection of windows can reduce the sidelobe leakage, but always at the expense of reduced resolution. c) The limitation of tracking time-varying spectrum due to the block processing of the FFT.

In order to avoid the problems resulting from the FFT, modern spectral estimation techniques have been proposed [3,4]. The basic principles and results many modern methods are shown in an excellent survey paper [5]. Among them autoregressive moving-average (ARMA) methods are more generalized and produce even better results than others.

The ARMA model assumes that a time series y_n can be modeled as the output of a p pole and q zero filter excited by white noise:

$$y_n = -\sum_{k=1}^{p} a_k y_{n-k} + \sum_{k=0}^{q} c_k n_{n-k} \tag{1}$$

Where n_n is white noise and c_0=1.

Once the parameters of the ARMA model are identified the spectral estimation of the time series y_n can be calculated:

$$P_y(\omega) = \frac{\sigma^2 \Delta t |1 + \sum_{k=1}^{q} c_k exp(-j\omega k \Delta t)|^2}{|1 + \sum_{k=1}^{p} a_k exp(-j\omega k \Delta t)|^2} \tag{2}$$

Where σ^2 is the variance of the white noise and $-1/2\Delta t \leq f \leq 1/2\Delta t$.

Many ARMA parameter estimation techniques have been proposed which usually involve many matrix computations and iterative optimization techniques. They are normally not practical to real-time processing. In the following sections a suboptimal method with considerably less computations is proposed.

2 Spectral estimation by an adaptive IIR filter

The structure of an adaptive IIR filter which was first proposed in [3] is shown in figure 1. y_n is an input time series and $\hat{y}$ is the estimate of y_n by the adaptive filter. The set a_k and c_k are the feed-forward and feedback coefficients of the filter, respectively. They are iteratively adjusted according to the least mean square (LMS) algorithm to be derived in the following. Using vector notation and assuming that the time series y_n is wide-sense stationary and has zero mean the LMS algorithm can derived in analogy with that in [6].

$$\hat{y}_n = -\sum_{k=1}^{p} a_{kn} y_{n-k} + \sum_{k=1}^{q} c_{kn} n_{n-k} \tag{3}$$

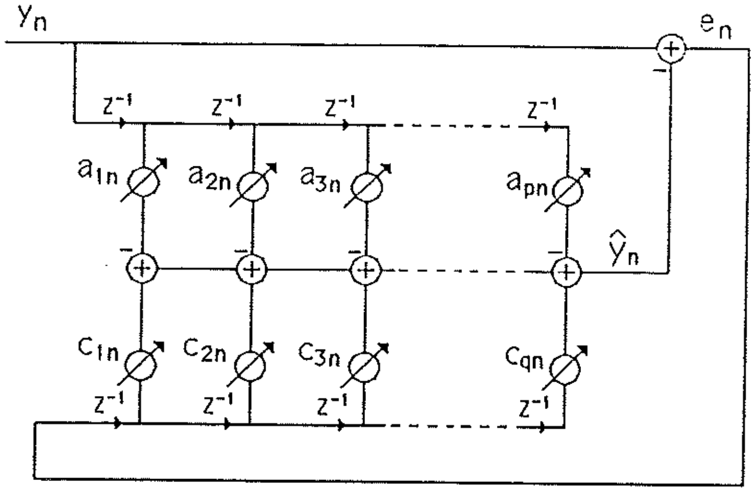

Figure 1: The Structure of an Adaptive IIR Filter

$$= A_n^T Y_n + C_n^T \mathbf{e}_n \tag{4}$$

$$e_n = y_n - \hat{y}_n \tag{5}$$

Where,

$$A_n^T = [a_{1n}, a_{2n}, ..., a_{pn}]$$
$$C_n^T = [c_{1n}, c_{2n}, ..., c_{qn}]$$
$$Y_n^T = [y_{n-1}, y_{n-2}, ..., y_{n-p}]$$
$$\mathbf{e}_n^T = [e_{n-1}, e_{n-2}, ..., e_{n-q}]$$

Let E[.] denote the expectation operation and replace A_n, C_n by A, C for simplicity then the mean square error $E[e_n^2]$ can be expressed as:

$$E[e_n^2] = E[y_n^2] + A^T RA + C^T R'C - 2A^T P - 2C^T P' + 2A^T QC \tag{6}$$

Where,

$$R = E[Y_n Y_n^T], R' = E[\mathbf{e}_n \mathbf{e}_n^T]$$
$$P = E[y_n Y_n], P' = E[y_n \mathbf{e}_n]$$
$$Q = E[Y_n \mathbf{e}_n^T]$$

The solution of the adaptive filter coefficients that minimize the mean square error is obtained by setting the gradient vector with respect to the filter parameters equal to zero.

$$\nabla_A[E(e_n^2)] = 2RA - 2P + 2QC = 0$$

$$A = R^{-1}(P - QC) \tag{7}$$

$$\nabla_C[E(e_n^2)] = 2R'C - 2P' + 2QA = 0$$

$$C = R'^{-1}(P' - QA) \tag{8}$$

The Wiener solution of the filter parameters can be obtained if all the second-order statistics are known. In most of real applications, however, these statistics are not known as a priori. The steepest descent method is applied for the adaptation of the filter parameters:

$$A_{n+1} = A_n - \mu_1 \nabla_A [E(e_n^2)] = A_n - 2\mu_1(RA_n - P + QC_n) \tag{9}$$

$$C_{n+1} = C_n - \mu_2 \nabla_C [E(e_n^2)] = C_n - 2\mu_2(R'C_n - P' + Q^T A_n) \tag{10}$$

According to the LMS algorithm [7,8] the above unknown matrices are replaced by instantaneous estimates of their values, i.e., R is estimated by $Y_n Y_n^T$, R' by $\mathbf{e}_n \mathbf{e}_n^T$, P by $y_n Y_n$, P' by $y_n \mathbf{e}_n$ and Q by $Y_n \mathbf{e}_n^T$. Finally, the LMS algorithm for the adaptation of the filter coefficients is obtained as follows:

$$A_{n+1} = A_n - 2\mu_1(Y_n Y_n^T A_n - y_n Y_n + Y_n \mathbf{e}_n^T C_n)$$

$$= A_n + 2\mu_1 e_n Y_n \tag{11}$$

$$C_{n+1} = C_n - 2\mu_2(\mathbf{e}_n \mathbf{e}_n^T C_n - y_n \mathbf{e}_n + Y_n^T \mathbf{e}_n A_n)$$

$$= C_n + 2\mu_2 e_n \mathbf{e}_n \tag{12}$$

Where μ_1, μ_2 are constants which control the convergence speed of the algorithm. Their values must be chosen less than the reciprocal of the total input energy to the adaptive filter. Small values result in better performance at the expense of slower convergence. The determination of the order of the ARMA model, i.e., the values of p and q can be found from the literature [9,10]

Now comparing the expression of the filter output (3) with the ARMA model in equation (1) we can find that the filter output is indeed an ARMA estimation of the input time series y_n if the residual error e_n is a white process. The whiteness of the e_n has been proved in [7]. Therefore once the algorithm converges the power spectral estimation can be calculated from the filter parameters by equation (2).

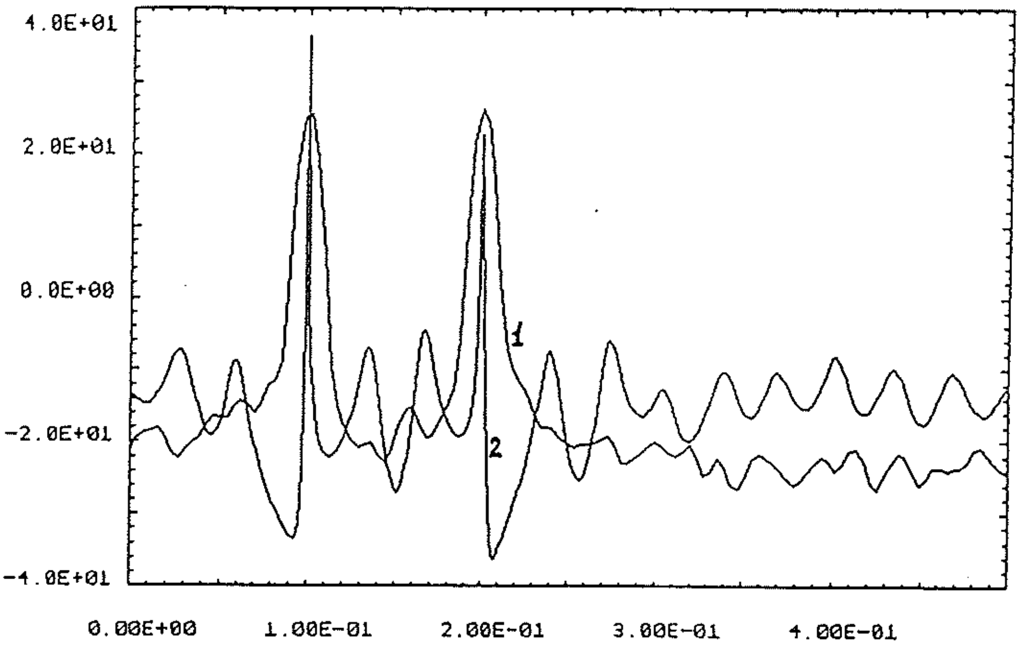

Figure 2: Results of Spectral Estimation. Real frequencies: $f_1 = 0.1Hz, f_2 = 0.2Hz$. Curve 1 : Spectral estiamtion by Periodogram. Curve 2 : bý the proposed method.

3 Results

For investigating the performance of the proposed method in the previous section, computer simulations have been conducted in comparison with a conventional FFT based Periodogram method. In the first simulation the input time series y_n consists of two sinusoids in white noise. The signal (two sinusoids) to noise ratio equals 17 db. The frequencies of these two sinusoids are $0.1Hz$ and $0.2Hz$, respectively. For the proposed algorithm the lengths of filter feed-forward and feedback coefficients, p, q, are chosen to be 30 and 20. The results are shown in figure 2 where the vertical axis stands for power spectrum and horizontal one for frequency. Curve 1 is the spectral estimation by the periodogram method and curve 2 is that by the proposed method. Comparing these two results one can see that the proposed method produces higher resolution, i.e., two peaks in the curve 2 are much narrower than those in curve 1.

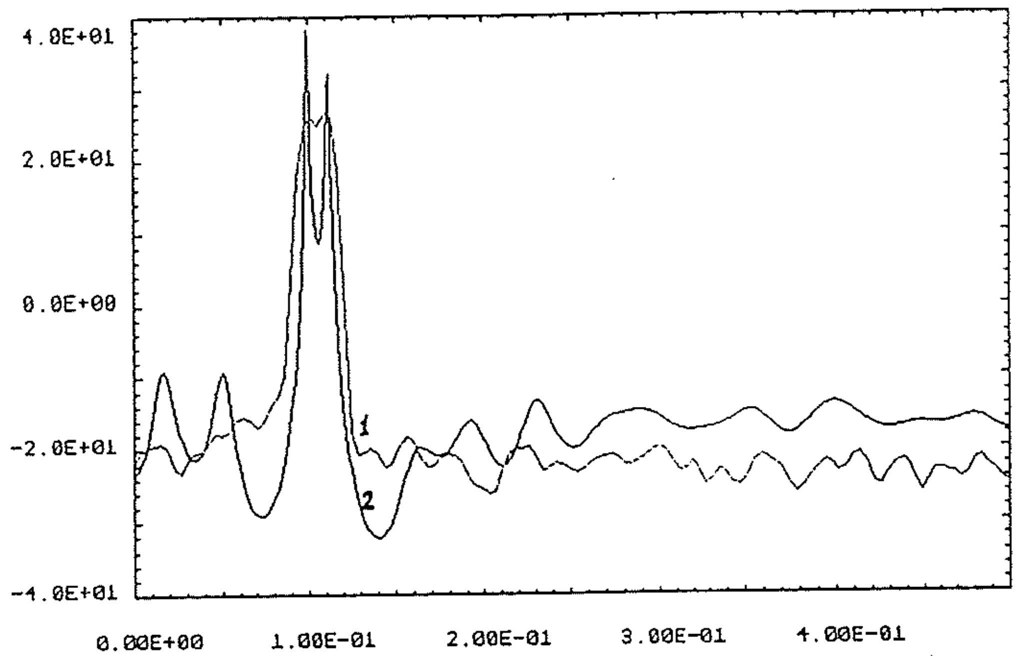

Figure 3: Results of Spectral Estimation. Real frequencies: $f_1 = 0.1Hz, f_2 = 0.11Hz$. Curve 1 : Spectral estiamtion by Periodogram. Curve 2 : by the proposed method.

The second simulation is designed to investigate the ability of distinguishing two very close spectral lines. The input time series has the same composition as the previous simulation but the frequencies of the two sinusoids are equal to $0.1Hz$ and $0.11Hz$, respectively. Figure 3 shows the results. The spectral estimation by the periodogram method is shown in curve 1 from which one can see that two spectral responses merged into one. The estimation by the proposed method is shown in curve 2 where two different spectral responses are well recognizable.

4 Conclusion

In this paper an ARMA spectral estimation by an adaptive IIR filter is described. The LMS algorithm for the adaptation of the filter coefficients is derived. The proposed adaptive IIR filter produces an ARMA estimation of the input time series and whole system functions as a whitening filter. Once the algorithm converges, the spectral estimation of the input time series can be calculated from the filter coefficients.

The proposed method has been compared with the FFT based periodogram method by computer simulations. It has advantages of producing higher resolution and of resulting in higher ability of distinguishing close spectral lines. It is simple and has low computational complexity, so it can be used for real-time processing. It is adaptive and so can be used for tracking the instantaneous frequency of the time series. It produces better results than autoregressive methods with the same complexity of computation such as the method in [4] because it is based on the ARMA model.

Since the convergence of the LMS algorithm is relatively slow it is not efficient for the application to a time series with only short data available.

The proposed method has been applied to biomedical signal processing with success[11]. Recently a recursive least square algorithm has been developed.

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