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A GEOMETRICAL APPROACH FOR THE IDENTIFICATION OF STATE SPACE MODELS WITH SINGULAR VALUE DECOMPOSITION

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Abstract

In this paper, some geometrically inspired concepts are studied for the identification of models for multivariable linear time invariant systems from noisy input-output observations. Starting from a fundamental highly structured input-output matrix equation, it is shown how the singular value decomposition allows to estimate the order of the observable part of the system and its state space model matrices. Moreover, conditions for persistency of excitation of the inputs and the behavior of the algorithm when the data are perturbed by noise, can easily be studied from a geometrical point of view. The singular values allow to quantify these concepts. An example with an industrial plant identification is presented.

Keywords : Linear and Total linear least squares, canonical correlation identification.

1 Introduction

Selection and identification of appropriate mathematical representations are of central importance in the analysis, design and control of multivariable systems. With access only to the external input-outputbehavior of a multivariable dynamical process, the internal structure (other than a priori assumed time invariance and linearity) being unknown, the problem of constructing a model is a highly non-trivial task. Because of this complexity, reliable and robust general purpose identification schemes have not yet become a standard tool. In most cases, (experimental) observations on the input-output behavior of the system under normal operating conditions are readily available. The most obvious choice for a mathematical model is in a lot of cases a state space representation since the major part of modern system and control theory, such as the design of observers, filters and optimal controllers regards this very efficient and compact representation.

In this paper, new geometrically inspired identification schemes will be presented. They make use of the numerically reliable key technique of the singular value decomposition and allow to estimate the order of the system under study and to identify its state space model matrices, from possibly noise corrupted multiple input-output measurements. No a priori parametrization, which may be ill-conditioned with respect to identification, is required.

In section 2, some general properties of dynamic systems are stated, including the fundamental input-output equation, wherefrom in section 3 three different identification approaches are deduced : a linear least squares (section 3.1) and a total linear least squares (section 3.2) approach and finally a canonical correlation approach (section 3.3). Finally section 4 gives an example of an industrial plant identification.

2 Dynamic systems

We now state some general properties that will be used throughout the sequel.

2.1 State space model

We consider linear, discrete time, time-invariant systems with m inputs and l outputs, with state space representation :

$$x[k+1] = A.x[k] + B.u[k] \quad (1)$$
$$y[k] = C.x[k] + D.u[k] \quad (2)$$

Vectors $u[k], y[k]$ and $x[k]$ denote the input,output and state at time k, the dimension of $x[k]$ being the minimal system order n. A, B, C and D are the unknown system matrices to be identified, making use only of measured I/O-sequences $u[k], u[k+1], \ldots$ and $y[k], y[k+1], \ldots$

2.2 Input-output equation

Equation (2) can easily be extended to a general structured I/O-equation :

$$Y_h = \Gamma_i.X + H_t.U_h \quad (3)$$

Y_h is a block Hankel matrix (i block rows, j columns) containing the consecutive outputs :
($y[k]$ is a $l \times 1$ vector, where l is the number of outputs)

$$Y_h = \begin{bmatrix} y[k] & y[k+1] & \ldots & y[k+j-1] \\ y[k+1] & y[k+2] & \ldots & y[k+j] \\ y[k+2] & y[k+3] & \ldots & y[k+j+1] \\ \ldots & \ldots & \ldots & \ldots \\ y[k+i-1] & y[k+i] & \ldots & y[k+j+i-2] \end{bmatrix}$$

Γ_i is an extended observability matrix :

$$\Gamma_i = \begin{bmatrix} C \\ CA \\ CA^2 \\ \ldots \\ CA^{i-1} \end{bmatrix}$$

X contains consecutive state vectors :

$$X = [x[k]\ x[k+1]\ x[k+2]\ \ldots\ x[k+j-1]]$$

H_t is a lower triangular Toeplitz matrix containing the Markov parameters :

$$H_t = \begin{bmatrix} D & 0 & 0 & \ldots & 0 \\ CB & D & 0 & \ldots & 0 \\ CAB & CB & D & \ldots & 0 \\ CA^2B & CAB & CB & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ CA^{i-2}B & CA^{i-3}B & CA^{i-4}B & \ldots & D \end{bmatrix}$$

Finally U_h is a block Hankel matrix with the same block dimensions as Y_h, but now containing the consecutive inputs. ($u[k]$ is a $m \times 1$ vector, where m is the number of inputs)

$$U_h = \begin{bmatrix} u[k] & u[k+1] & \ldots & u[k+j-1] \\ u[k+1] & u[k+2] & \ldots & u[k+j] \\ u[k+2] & u[k+3] & \ldots & u[k+j+1] \\ \ldots & \ldots & \ldots & \ldots \\ u[k+i-1] & u[k+i] & \ldots & u[k+j+i-2] \end{bmatrix}$$

Proof : straightforward by repeated substitution.

□

Instead of going into details, we loosely state that i and j should be chosen "sufficiently large" so as to satisfy certain conditions, and in particular $j \gg \max(mi, li)$ ("very rectangular" block Hankel Matrices).

2.3 Rank property

Let H denote the concatenation of Y_h and U_h :

$$H = \begin{bmatrix} Y_h \\ U_h \end{bmatrix}$$

then, generically (see below), the following theorem holds.

Theorem 1

$$\mathrm{rank}(H) = \mathrm{rank}(U_h) + n \quad (4)$$

where n is the system order (observable part). Or, when U_h has full row rank :

$$\mathrm{rank}(H) = mi + n \quad (5)$$

For a proof see [1].

2.4 Persistent excitation

The singular values of U_h serve as quantitative measures for the degree of persistency of excitation of the input sequence. Loosely speaking, the input sequence has to be persistently exciting in order to 'excite' all modes of the systems. When the matrix U_h is (nearly) rank deficient (some singular values are small) the input sequence is 'poor' in that it (almost) consists of a finite number of complex exponentials. When the singular values are all (almost) equal, the input sequence tends to be 'white' noise . Also for an impulsive input, the singular values are all equal (SISO).

3 Identification strategies

Three different identification approaches can now be derived from the input-output equation 3 : a linear least squares (section 3.1) and a total linear least squares (section 3.2) approach and finally a canonical correlation approach (section 3.3).

3.1 Linear least squares identification

Let $U^\perp$ be any $j \times (mi - \mathrm{rank}(U_h))$ matrix satisfying $U_h.U^\perp = 0$. I/O-equation 3, postmultiplied by $U^\perp$, then reveals

$$Y_h.U^\perp = \Gamma_i.K.U^\perp$$

Consider the SVD of $Y_h.U^\perp = P.S.Q^t$. Under mild conditions, $\mathrm{rank}(S) = n$ and there exists a non-singular $n \times n$ matrix R such that: $P = \Gamma_i.R$. This implies that a realization of the state transition matrix and the output matrix of the form $R^{-1}.A.R, C.R$ can be performed in a similar way as in Kung's realization algorithm by exploiting the so-called shift structure [5,1]. The matrices $R^{-1}.B$ and D follow from a set of linear equations [1,6]. In [1] it is shown that this identification approach corresponds to a linear least squares version for identification problems where the input is noisefree while the output is noisy. The row space of the output block Hankel matrix is orthogonalized with respect to the input block Hankel row space . Examples can be found in [1,6,2].

3.2 Total linear least squares identification

In a very similar way (the I/O-equation should now be premultiplied by Γ_i's orthogonal complement) the following result can be obtained (proof omitted for brevity). Let the SVD of

$$\begin{bmatrix} Y_h \\ U_h \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} S_1 & 0 \\ 0 & 0 \end{bmatrix} Q^t$$

where $\mathrm{rank}(S_1) = \mathrm{rank}(U_h) + n$ and the partitioning of the left singular matrix is such that P_{11} is a $(li) \times (mi+n)$ matrix. Then, there exists a non-singular nxn matrix T such that :

$$P_{11}.P_{21}^\perp = \Gamma_i.T$$

where $P_{21}^\perp$ is any $(mi+n) \times n$ matrix satisfying $P_{21}.P_{21}^\perp = 0$. This implies that a realization of the state transition matrix and the output matrix of the form $T^{-1}.A.T, C.T$ can be performed in a similar way as in Kung's realization algorithm. The matrices $T^{-1}.B$ and D follow from a set of linear equations [1,6]. Contrary to the previous versions, this corresponds to a total linear least squares approximation of the multivariable identification problem, which applies when both input and output are corrupted by the same amount of noise [3,1]. Considerable insight has been gained into the behavior of the algorithm in noisy industrial applications. More details are found in [1,6,2].

3.3 Canonical correlation identification

The canonical correlation approach to the identification of a state space model, is based upon the following fundamental observation readily deduced from the I/O-equation (for a proof see [2,7]). Let Y_1, U_1 be a output - input block Hankel pair (block dimensions $i \times j$) containing output- input measurements on a linear dynamic system up to time k and let Y_2, U_2 be another output input block Hankel pair of block dimensions $i \times j$, containing measurements from time $k+1$ on. If the rows of the matrix Z (with j columns) form a basis for the intersection of the row spaces of

$$\begin{bmatrix} Y_1 \\ U_1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} Y_2 \\ U_2 \end{bmatrix}$$

then:

- $\dim(\mathrm{span}_{\mathrm{row}} Z) = \mathrm{rank}(Z) = n$
- there exists a non-singular $n \times n$ matrix R such that

$$Z = R.[x[k+1] \;\; x[k+2] \;\; \ldots \;\; x[k+j]]$$

Hence, the matrix Z is nothing but a state vector sequence realization. Once such a sequence is available, the model matrices A, B ,C ,D follow from the set of linear equations :

$$\begin{bmatrix} x[k+1] \\ y[k] \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x[k] \\ u[k] \end{bmatrix}$$

that can be solved with TLLS or one of its variations.

Hence, the 'difficult' problem of identification of a linear state space model has now been reduced to 2 SVD steps, that may be implemented in a very streamlined identification algorithm. Adaptive versions for updating and downdating the QR- and SVD factorizations via a gliding window approach are actually being implemented, taking into account the specific structure of the matrices. For more detail, the reader may wish to consult [2].

4 Real life example

The performance of the algorithm has been evaluated on both simulated and industrial data sets. The following example is due to Prof. R.Guidorzi (University of Bologna, [4]). The I/O-sequence was obtained under normal operating conditions of a 120 MW power plant (Pont sur Sambre - France), a system with 5 inputs and 3 outputs. The identified models (for different system order estimates) were evaluated by comparing original and simulated outputs, using the original input signals and the identified model (see Figures). These simulations demonstrate the remarkable robustness of the identification scheme with respect to over- and underestimation of the system order.

5 Conclusions

In this paper, a survey was given of geometrical concepts for a new identification strategy. The properties of the singular value decomposition are exploited to compute a state space model from noisy input-output observations. It is shown how the condition of persistent excitation can be translated in a geometrical framework.

Future work will be directed to a complete geometrical treatment of the identification scheme and to efficient numerical implementation of adaptive versions of the singular value decomposition for structured matrices.

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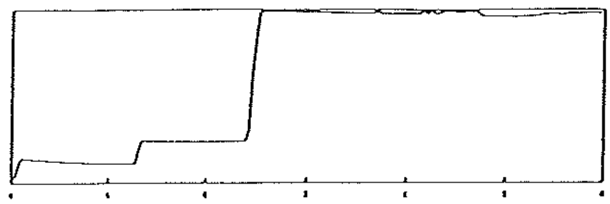
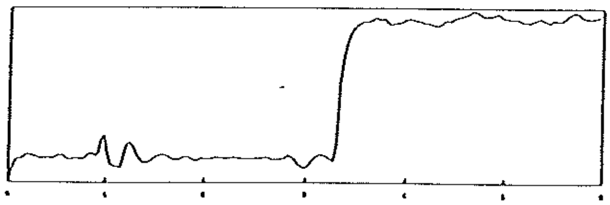
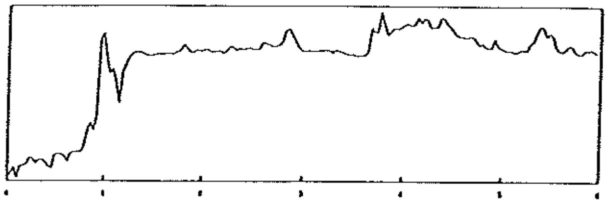


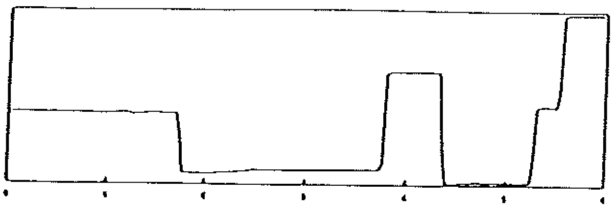
First input, C_b (Gas flow).



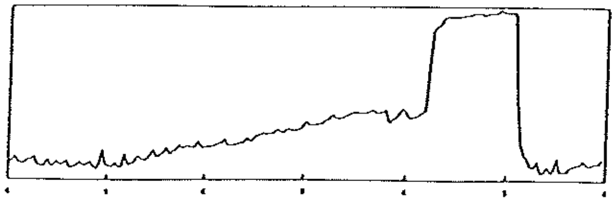
Third input, Q_d (Super heater spray flow).



Second input, O_t (Turbine valves opening).



Fourth input, R_v (Gas dampers).

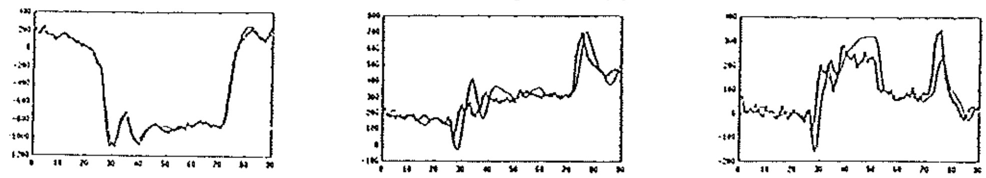


Fifth input, Q_a (Air flow).

First Output: Steam Pressure

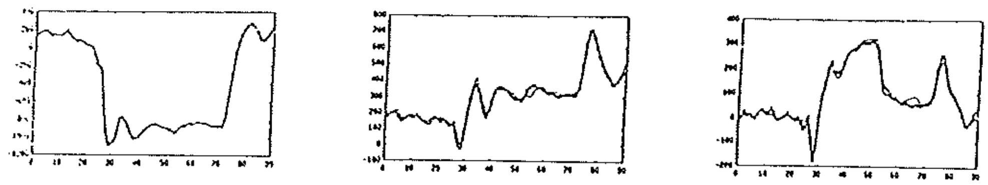
Second Output: Steam Temperature

Third Output: Reheat Steam Pressure



First order model

Third order model



Sixth order model

Figure 1 : Identification of a power plant : original and reconstructed outputs for different system order estimates.