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Comparison of SVD methods to extract the foetal electrocardiogram from cutaneous electrode signals

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Abstract—*The paper presents and compares three methods making use of the singular value decomposition (SVD) of a matrix to extract the foetal electrocardiogram (FECG) from cutaneously recorded electrode signals. The first method constructs a set of orthogonal foetal signals (the so-called principal foetal signals) from the recordings, but needs electrode positions far from the foetal heart, in addition to the abdominal electrodes that pick up a mixture of maternal and foetal electrocardiogram. An online adaptive algorithm has been developed such that a real-time implementation becomes feasible. The second method is a new online approach to a technique presented by van Oosterom. Although this method has some important drawbacks and is suboptimal as far as foetal signal-to-noise ratio is concerned, it is still very useful when only a foetal trigger is required, as the signal obtained is not a complete FECG. Finally, a third method is proposed, based on the generalised SVD and interpreted with the new concept of oriented signal-to-signal ratio. An online version is also presented for this method and some results are shown.*

Keywords—*Abdominal foetal electrocardiogram, Foetal principal signals, GSVD, Oriented energy, Oriented signal-to-signal, SVD*

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Introduction

IN THE SAME way as for the adult ECG it should be possible to obtain a set of potential tracings, describing the electrical activity of the foetal heart. Most accurate methods in this field are invasive, as they use an electrode placed on the foetal scalp. Because this technique is only feasible during delivery, when the foetal scalp is accessible, it is important to look for noninvasive techniques, such that an earlier diagnosis becomes possible.

The task of obtaining a foetal electrocardiogram from recordings with electrodes on the mother's skin is fundamentally equivalent to the adult ECG problem, but some specific practical difficulties arise in this case. The main interference is formed by the omnipresent maternal ECG (MECG), but also, compared with the low voltage of the FECG, other noise levels become relatively high (maternal muscle noise, net interference, noise from electrodes and amplifiers etc.). Another problem is the large variability of the way cutaneous potentials are generated by the internal bioelectric sources due to differences between subjects. But also, for an individual subject, changes in the configuration of the volume conductor depending on the stage of pregnancy can play an important role (foetal growth, creation of new conducting paths, changes in the characteristics of the amniotic fluid and placenta, the development of the vernix caseosa, the changing geometry of the abdomen during pregnancy etc., OOSTENDORP, 1989). The position of the electrodes is also of capital importance, because well chosen electrode locations can become completely useless in the course of an experiment, due to foetal movements.

The earliest attempts and the most straightforward approaches to solve the FECG problem have involved a direct scaled subtraction of a thoracic or near-thoracic MECG from an abdominally measured composite of maternal and foetal heart signals (LONGINI *et al.*, 1977; SUREAU and TROCELLIER, 1961; WHEELER *et al.*, 1978). The main difficulty with these presented 'subtraction' methods is the fact that the shapes of sequential MECG complexes are not always identical due to changing configurations and breathing.

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More complex methods to reduce the MECG use signal processing techniques such as averaging, matched filters, auto- and crosscorrelation and estimation (VAN BEMMEL, 1968; FERRARA and WIDROW, 1982; WIDROW *et al.*, 1975). Typical for all these methods mentioned is the fact that the FECG contribution in the output is never larger than the portion that was already present in the input abdominal recording. This is due to the fact that they try to eliminate all MECG contribution in one signal only, in some way or another.

Finally, one can consider a group of methods that try to obtain a 'clean' FECG by a linear combination or weighted sum of a number of electrode signals. This is a kind of 'spatial filtering'. The methods differ in the way they determine the (weighting) coefficients in this linear combination (BERGVELD and MEIJER, 1981; VANDERSCHOOT *et al.*, 1983; VAN OOSTEROM and ALSTERS, 1984).

In this paper we will show that the singular value decomposition (SVD) and the generalised singular value decomposition (GSVD), which are very robust, numerically reliable and efficient techniques known in linear algebra, play a key role in extracting the FECG from cutaneously recorded potential signals.

2 Modelling the FECG problem

The task of formulating the FECG problem in mathematical terms consists of a two-step procedure. First, the electrical activity of the heart has to be modelled, and then the second part of the analysis concerns the question of how the potential distribution on the skin due to the current generation by all the bioelectric sources present is obtained.

The solution to the first problem is based on the observation that, at a considerable distance from the heart, its electrical activity can be represented in first-order approximation by an equivalent current source dipole vector with fixed position (PLONSEY, 1969). The magnitude and direction of this dipole vector may change with time. This variation can be described by three orthogonal components of the vector in some orthogonal frame of reference.

Because these orthogonal components are generally functions of time, one can associate with each bioelectric current source $s_i(t)$ three orthogonal signals $s_{ix}(t)$, $s_{iy}(t)$ and $s_{iz}(t)$, called the source signals. The electrical activity of the foetal heart, as well as other electrical phenomena, can then be described by the concept of orthogonal source signals. Assume that in general r source signals suffice to represent the activity of all internal bioelectric current sources. Then a signal vector $s(t)$, called the source signal vector, can be defined as

$$s(t) = (s_1(t) \cdots s_r(t))^T \tag{1}$$

However, the main problem is that one cannot measure these internal source signals directly. Instead, the potential differences between electrode pairs placed on the body surface are measurable, and are called the measurement signals $m_i(t)$. Suppose that p measurement signals from cutaneous electrode pairs are measured and arranged in a vector $m(t)$, called the measurement signal vector, as

$$m(t) = (m_1(t) \cdots m_p(t))^T \tag{2}$$

Owing to the resistive and quasi-static behaviour of the body as a conducting medium in the frequency range of interest (between about 0·5 Hz and 100 Hz, PLONSEY 1969), there exists a linear relationship between $m(t)$ and $s(t)$. Indeed, each measurement signal $m_i(t)$ can be written as a linear combination of r source signals $s_j(t)$, plus additive noise, represented by a signal $n_i(t)$.

This leads to the following relationships:

$$\begin{cases} m_1(t) = t_{11}\, s_1(t) + \cdots + t_{1r}\, s_r(t) + n_1(t) \\ \quad\vdots \\ m_p(t) = t_{p1}\, s_1(t) + \cdots + t_{pr}\, s_r(t) + n_p(t) \end{cases} \tag{3}$$

or in signal vector notation

$$m(t) = Ts(t) + n(t) \tag{4}$$

In this equation, only $m(t)$ is known. The matrix T, which contains all coefficients in the linear combinations, is called the transfer matrix and is completely determined by the geometry of the body, the electrode and source positions, and the conductivities of the body tissues. A very detailed and interesting study concerning the modelling of body volume conductor is performed by OOSTENDORP (OOSTENDORP *et al.*, 1986; OOSTENDORP, 1989).

The separation of the MECG and FECG can now be accomplished by finding estimates for the source signals $s_j(t)$. Actually, characterising the electrical activity of the foetal and adult heart is a so-called inverse problem: it consists in estimating the internal source signals that have generated a given set of external measurement signals.

Digital processing of these measured analogue signals requires sampling, a process by which a continuous-time signal is transformed into a series of digital numbers. This makes a matrix formalism for eqn. 4 possible as

$$M_{p,q} = T_{p,r} S_{r,q} + N_{p,q} \tag{5}$$

where p is the number of electrode signals, r is the number of observed source signals and q is the number of sampling instants.

3 Three methods for the separation of MECG and FECG

3.1 *Method 1 SVD of a set of p cutaneously measured potential signals*

In this method (CALLAERTS *et al.*, 1986*a*; *b*); VANDERSCHOOT *et al.*, 1984; 1987) p potential signals are recorded from electrodes placed on the maternal skin. After sampling, a $p \times q$ data matrix M is constructed, where each row of M consists of the q samples of one signal. Earlier work had already proved that the SVD of the data matrix M, given by $M = U\Sigma V^T$, provides, under certain conditions of electrode placement, an efficient way to construct an MECG-free foetal electrocardiogram.

If the maternal ECG is sufficiently strongly present in the recordings, compared with the presence of the foetal ECG, then the singular spectrum Σ of M can be partitioned into three groups

$$\Sigma = \begin{pmatrix} \Sigma_M & 0 & 0 \\ 0 & \Sigma_F & 0 \\ 0 & 0 & \Sigma_0 \end{pmatrix} \tag{6}$$

where Σ_M contains r_M singular values, associated with the maternal heart, Σ_F contains r_F singular values, associated with the foetal heart and Σ_0 contains $r_0 = p - r_M - r_F$ singular values, associated with other possible sources of bioelectric activity and noise. A number of orthonormal singular directions correspond to each group of singular values, to form a subspace of the p-dimensional column space of the data matrix M, spanned by the columns of U. This results in a maternal subspace (dimension r_M), a foetal subspace (dimension r_F) and a subspace for other sources

of electrical activity (dimension r_O), such that the matrix U can be partitioned into

$$\begin{matrix} r_M & r_F & r_O \end{matrix}$$
$$U = (U_M \quad U_F \quad U_O) \tag{7}$$

Because these three subspaces are orthonormal to each other, the solution of the separation problem is presented as an orthonormal projection of the data onto the foetal subspace, whereby the maternal ECG contribution is eliminated. Formally, this can be written as

$$\hat{S} = U^T M = \begin{pmatrix} U_M^T & M \\ U_F^T & M \\ U_O^T & M \end{pmatrix} = \begin{pmatrix} \hat{S}_M \\ \hat{S}_F \\ \hat{S}_O \end{pmatrix} \tag{8}$$

where $\hat{S}$ is a $p \times q$ matrix containing p estimates of the source signals of eqn. (1).

Once an estimation of the foetal source signals is found in $\hat{S}_F$, one can construct a matrix F that contains only the FECG contribution in each measured signal, as follows:

$$F = U_F \hat{S}_F = \sum_{i=r_M+1}^{r_M+r_F} \sigma_i u_i v_i^T \tag{9}$$

where u_i (v_i) is the ith column of $U(V)$ and σ_i is the ith singular value of M. Therefore, the signals in $\hat{S}_F$ are called the principal foetal signals: each foetal contribution in the recordings can be found as a linear combination of the r_F principal foetal signals only. This set of principal signals has the same meaning as an orthonormal reference frame in an n-dimensional vector space. The principal foetal signals have the additional advantage that they do not depend upon the physical orientation of the foetus and of the foetal heart, with respect to the electrodes. This makes an easy comparison between different subjects possible. The construction of this matrix F is very important for the interpretation by the obstetrician, because the signals in F come from the respective electrode pairs on the maternal skin. In that way the r_F signals obtained in $\hat{S}_F$ are 'virtual', as they cannot be associated with specific electrode pairs.

This method can easily be automated and an online adaptive algorithm to compute the U-matrix in the SVD of M has been designed (CALLAERTS *et al.*, 1986*a*; *b*); VANDERSCHOOT *et al.*, 1987). At the moment a PC-based real-time implementation for bedside monitoring is in development (CALLAERTS *et al.*, 1987).

3.1.1. *Results:* As an example the method is applied to a set of six recordings (Fig. 1). The first three signals were recorded on the maternal thorax (far from the foetal heart); the last three signals were measured on the abdomen of the mother. The singular spectrum Σ of M contains three large singular values ($r_M = 3$), corresponding to the maternal subspace, two singular values ($r_F = 2$)

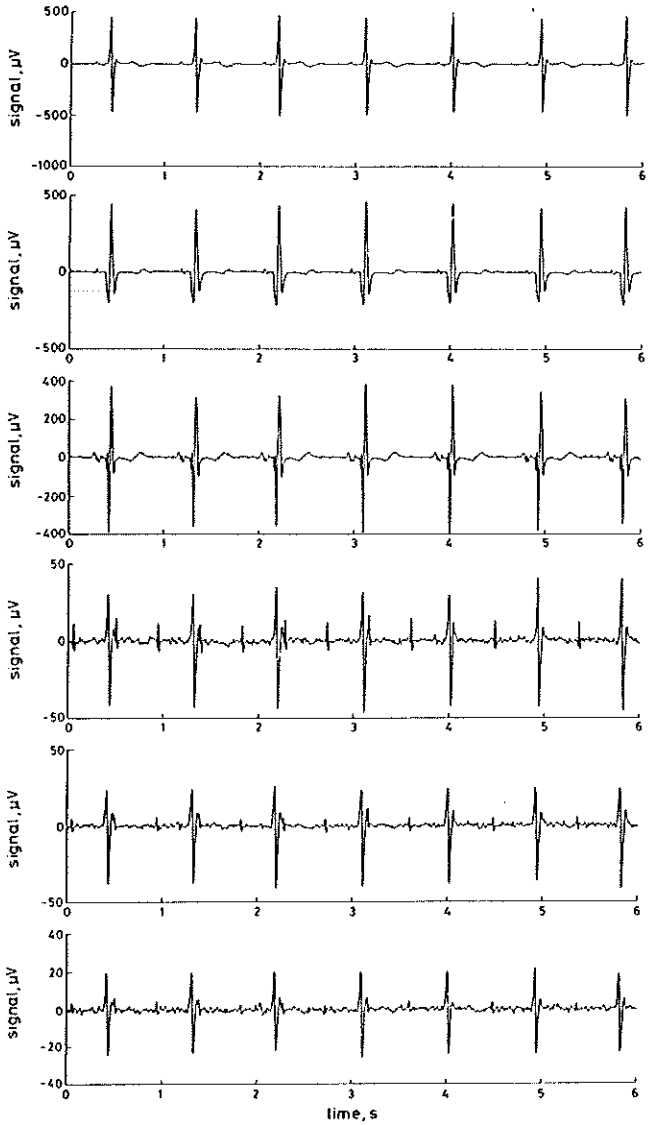

Fig. 1 *Six-channel measurement set 6 s long: the first three signals were taken from the thorax, the last three signals were recorded abdominally*

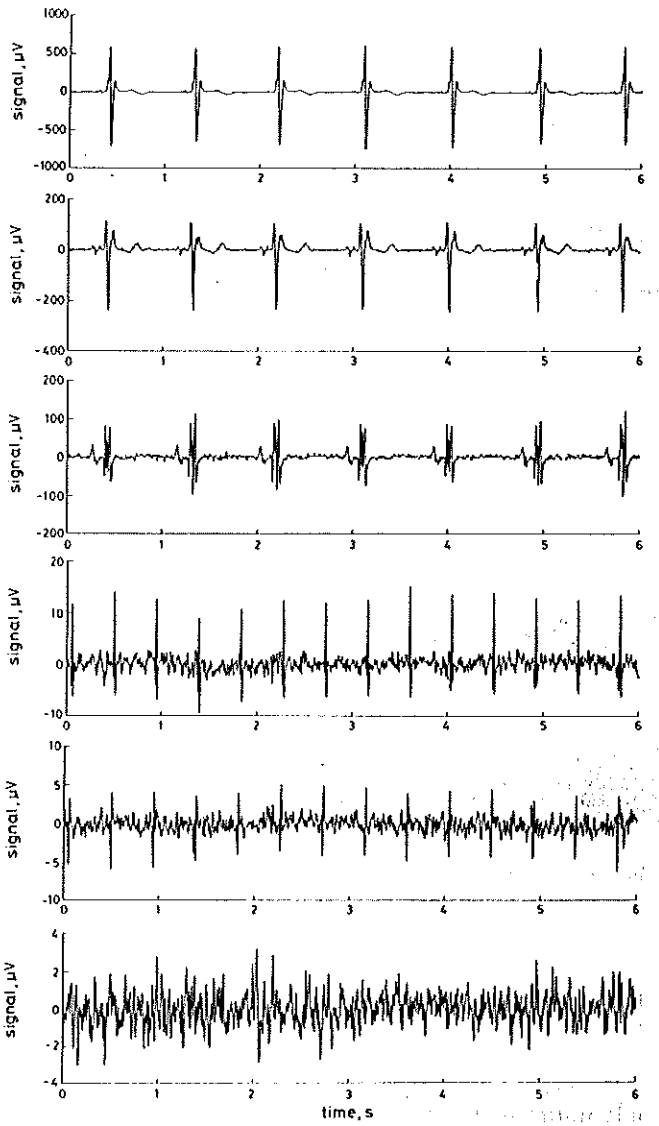

Fig. 2 *The six signals that result from a projection of the data of Fig. 1 onto the U-matrix in the SVD of M. The fourth and fifth signals are foetal signals*

corresponding to the foetal subspace, and a last singular value ($r_O = 1$) corresponding to other weaker influences.

The signals obtained in the matrix $\hat{S}$ are shown in Fig. 2. The interpretation that can be given to these estimated source signals is the following: any contribution of the maternal heart in the p measured potential signals can be found as an appropriate linear combination of only r_M orthonormal signals, obtained in $\hat{S}_M = U_M^T M$ (the first three signals in Fig. 2). A similar interpretation is valid for the estimated foetal source signals. In Fig. 3 the matrix F of eqn. 9 has been formed for the example in Fig. 1, using the two estimated foetal source signals $\hat{S}_F$ obtained in Fig. 2. One can notice that even the first three recordings, measured on the maternal thorax, contain a portion of FECG, which is very small compared with the portion in the abdominal recordings.

If one is interested in the exact waveform of the foetal QRS-complex in the two estimated foetal source signals of Fig. 1, a mean foetal QRS-complex is computed by averaging the occurring complexes. This results in the two waveforms shown in Fig. 4. A very interesting observation is that, by averaging the noise in each of the obtained signals, the P-wave of the FECG becomes clearly visible. This is important for a cardiologist, as the P-R interval contains important information about the growth and health of the foetal heart.

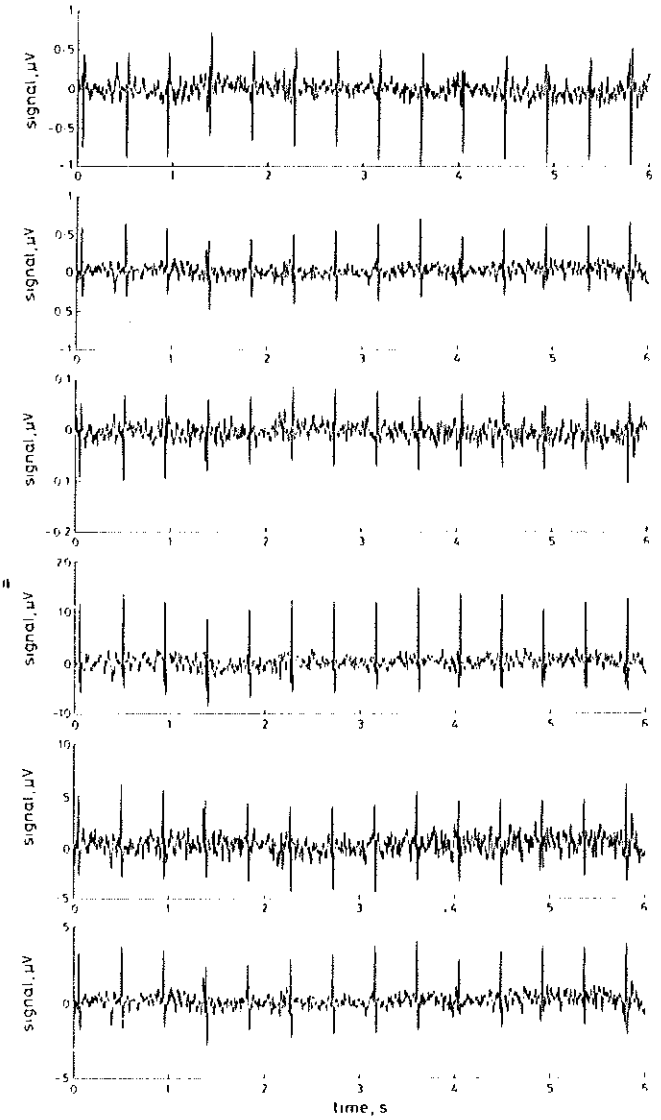

Fig. 3 *The contribution of FECG in each of the six measured potential signals of Fig. 1. Notice the small contribution in the first three signals, which were recorded far from the foetal heart. All MECG influence present in the recordings is eliminated here*

The method has shown to be successful in several cases. But it is important to stress that the success of the separation of MECG and FECG depends on appropriate electrode positioning. Indeed, the method requires not only abdominal potential signals, but also signals that pick up strong MECG signals only, close to the maternal heart (see the first three signals in Fig. 1). It is thereby based on the spatial separation of both sources of electrical activity. The reason why the method did not succeed for some of the recorded data sets was precisely the inappropriate choice of the electrode positions.

3.1.2 *Advantages:*

(*a*) The method and the presented solution are very elegant.
(*b*) The resultant signals are optimal with respect to extremal oriented energy.
(*c*) The resultant foetal signals are orthogonal and form a set of principal foetal signals, independent of the physical orientation of the foetal heart.
(*d*) The method is suitable for PC-based real-time implementation, due to the development of an online adaptive algorithm.
(*e*) Reconstruction of the original foetal ECG contributions in each of the measured signals is possible.

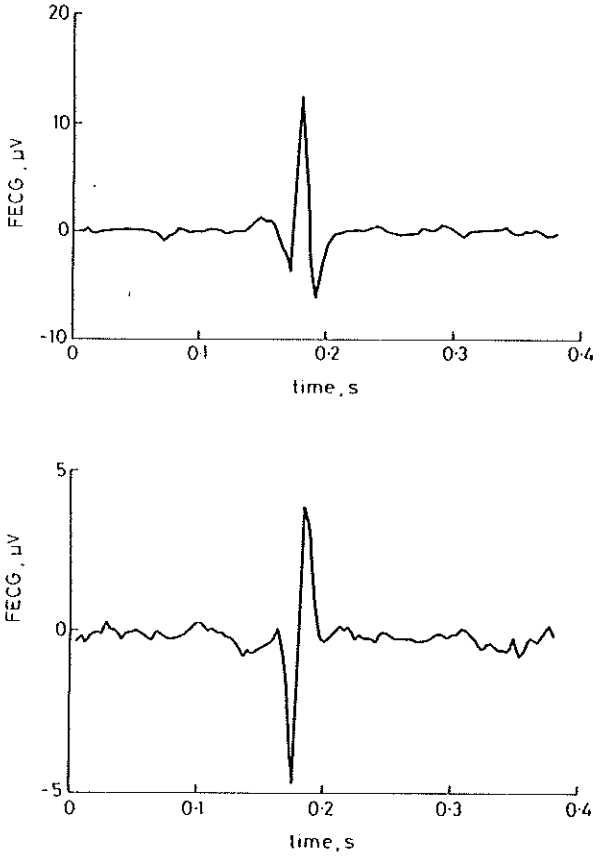

Fig. 4 *Mean FECG waveforms for the two estimated foetal source signals obtained in Fig. 2*

3.1.3 *Disadvantages:* No general electrode placement strategy can be used, because for each subject the most ideal electrode positions have to be looked for. When using bipolar electrode signals (no reference electrode), this problem has many degrees of freedom. The realisation of a real-time signal separation unit will, however, eliminate this problem, because an immediate evaluation of each chosen electrode pair can be performed.

3.2 *Method 2 Two-step SVD procedure*

Another way to illustrate the conceptual flexibility of the SVD is presented by the following method. It is essentially based on a method presented by VAN OOSTEROM (VAN OOSTEROM and ALSTERS, 1984; VAN OOSTEROM, 1986) but modified such that the online adaptive algorithm designed for the first method can be applied for this case too. A slight modification concerning the initialisation of the algorithm is then the only required change.

Before we present the second method, we will discuss the method as presented by VAN OOSTEROM. This method has been shown to be successful, as some MECG-free foetal signal is obtained in most cases, but has in our opinion some important drawbacks. First, visual inspection of the data signals for the selection of those intervals that contain only a maternal and a foetal QRS-complex hampers an efficient automation of the procedure. Secondly, the procedure comprises two separate steps, an MECG-elimination step and an FECG-detection step. In the second step an MECG-free foetal signal may be found. However, this signal is only optimal with respect to the restricted complementary subspace, found from the first step, and not 'the' optimal MECG-free foetal signal that could possibly be found from the recordings. It can indeed happen that the foetal heart has contributions in the subspace, assigned to the maternal heart (see first step of the procedure), such that together with the MECG-elimination, a part of the foetal signal disappears. Thirdly, the determination of the dimension of the maternal subspace is also very critical, as an overdetermination of it can cause an important reduction of the complementary subspace, in which one searches for an optimal linear combination afterwards. And finally, the method is very sensitive to the choice of the selected intervals. Especially, the selection of a maternal QRS-interval is quite difficult, because of the more frequent occurrence of a foetal QRS-complex.

We will now present a somewhat different approach to the same method, which will allow us to design an online version for it. Compose a matrix M_M as a sequence of several (5–10) maternal QRS-intervals, selected from the data matrix M. Some of these intervals may contain a P-wave, while others also contain a T-wave, such that the subspace associated with the maternal heart is described much better than before, when only one maternal QRS-complex was selected. Then compute the SVD of M_M and partition the left singular matrix U_M into

$$U_M = \begin{matrix} r_M & p - r_M \\ (U_1 & U_2) \end{matrix} p \qquad (10)$$

with r_M the estimated dimension of the maternal subspace.

Instead of selecting a foetal QRS-interval, we now project the whole data matrix M onto U_2, the complement of the maternal subspace

$$A_{(p-r_M),\,q} = U_2^T M \qquad (11)$$

This matrix A then contains no MECG contribution any more, and if the foetal signals are sufficiently strong (relative to all other unwanted signals present in the recordings), then the direction of maximum oriented energy of A, found as u_{A1}, provides the MECG-free linear combination of the $p - r_M$ signals in A.

This slight modification of the algorithmic scheme allows the application of the same online strategy as developed in the first method. In a first step the p recorded signals in M are reduced to $p - r_M$ MECG-free signals in A by projection onto U_2. For the second step the online adaptive algorithm can then be applied to the matrix A to meet time-variance due to foetal movements. Experiments furthermore showed that the subspace, defined by the maternal heart, is nearly time-invariant, such that the matrix U_2 has to be computed only once at the beginning of the experiment. This $p \times (p - r_M)$ matrix U_2 can then be used to initialise the U-matrix in the online algorithm, as $U_0 = U_2$.

3.2.1 *Results:* This new online approach to VAN OOSTEROM's method has not yet been tested extensively, but for all of the examined data sets it gave very satisfying results. The method has been applied to a set of five abdominal recordings, shown in Fig. 5. The resultant MECG-free foetal signal is shown in Fig. 6.

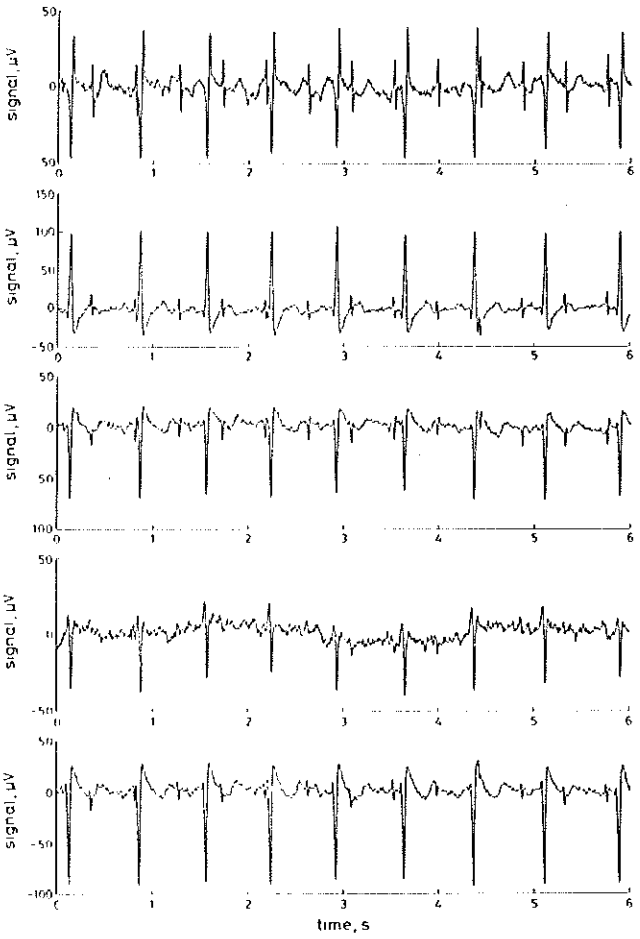

Fig. 5 *Set of five recorded abdominal signals (interval of 6 s), containing a mixture of MECG and FECG*

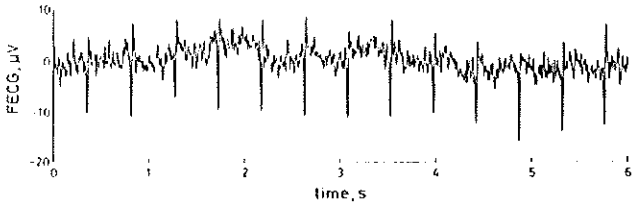

Fig. 6 *The MECG-free foetal signal obtained by applying the two-step SVD-procedure to the five abdominal signals of Fig. 5*

3.2.2 *Advantages:*

(*a*) The application of the on-line adaptive algorithm in the second step becomes possible now.

(*b*) The subspace associated with the maternal heart is

described better by taking more than one maternal QRS-complex into account.
(c) Recordings with a reference electrode and standard electrode positioning can be used.
(d) Good method if a foetal trigger has to be obtained.

3.2.3 *Disadvantages:*

(a) There is a more complicated initialisation of the online procedure than for Method 1.
(b) Concerning the foetal signal-to-noise ratio, it is expected that this method is suboptimal in the same way as VAN OOSTEROM's method is.
(c) No principal signal is obtained.
(d) Only one signal (not a complete electrocardiogram) is obtained.
(e) The determination of r_M still poses important problems.

3.3 *Method 3 GSVD-based signal separation method*

From Appendix 1 we know that the GSVD of the pair (A, B) looks for non-orthogonal directions x_i of extremal oriented signal-to-signal ratio. Let α_i/β_i be the generalised singular values of A and B, arranged in non-increasing order. Then the column x_i of X in the GSVD of (A, B) (see eqn. 19) is a vector for which the oriented energy of matrix A is α_i/β_i times larger than the oriented energy of matrix B (cf. concepts of signal-to-signal ratio, DE MOOR *et al.*, 1987).

Therefore, arrange the p recorded potential signals in a $p \times q$ data matrix M and compose a matrix B as a sequence of several maternal QRS-intervals, not coinciding with foetal complexes. This means that the matrix B only contains contributions from the unwanted signal, the MECG. The GSVD of the matrix pair (M, B) can then be written as

$$M = X^{-1} D_M U_M^T$$
$$B = X^{-T} D_B U_B^T \tag{12}$$

All columns of X provide, after normalisation, a linear combination of the p recorded potential signals, in which the oriented signal (in M)-to-signal (in B)-ratio is extremal. Only some of them contain an MECG-free foetal heart signal. Concerning the computation of the X matrix in the GSVD of a matrix pair (A, B), a very important observation is stated in the following theorem.

3.3.1 *Theorem 1: Four-step method for GSVD computation*: The computation of the X-matrix in the GSVD of a matrix pair (A, B) can be performed by the following four-step method (PARLETT, 1980):

Step 1: compute the SVD of B

$$B = U_B \Sigma_B V_B^T$$

Step 2: define a transformation $Q = \Sigma_B^{-1} U_B^T$ on the matrix A such that

$$QA = A^* = \Sigma_B^{-1} U_B^T A$$

Step 3: compute the SVD of A^*

$$A^* = (U_A^*)(\Sigma_A^*)(V_A^*)^T$$

Step 4: compute the X-matrix in the GSVD of (A, B) as

$$X = Q^T U_A^* = U_B \Sigma_B^{-1} U_A^*$$

3.3.2 *Proof:* Suppose that the GSVD of a matrix pair (A, B) is given by eqn. (19), then

$$AA^T = X^{-T} D_A^2 X^{-1}$$
$$BB^T = X^{-T} D_B^2 X^{-1} \tag{13}$$

In other words, the $p \times p$ matrix X diagnalises both AA^T and BB^T. The corresponding generalised symmetric eigenvalue problem then has the following formulation:

$$AA^T x_i = \lambda_i BB^T x_i \tag{14}$$

with λ_i the ith generalised eigenvalue. In the first step of the procedure the SVD of B is computed, such that

$$BB^T = U_B \Sigma_B^2 U_B^T = Q^{-1} Q^{-T} \tag{15}$$

Substitution into eqn. (14) then gives

$$AA^T x_i = \lambda_i Q^{-1} Q^{-T} x_i \tag{16}$$

This can also be written as

$$(QAA^T Q^T) Q^{-T} x_i) = \lambda_i (Q^{-T} x_i) \tag{17}$$

and this is nothing more than a normal symmetric eigenvalue problem. Diagonalisation with an orthonormal matrix U_A^* of the symmetric matrix $QAA^T Q^T$ and comparison with the first line of eqn. 13 results in

$$X = Q^T U_A^* = U_B \Sigma_B^{-1} U_A^* \tag{18}$$

As already mentioned, experiments showed that the transfer for the signals coming from the maternal heart is nearly time-invariant, such that the matrix B has to be constructed and decomposed only once as a kind of initialisation. The rest of the method can again be performed, using the online adaptive strategy, but now with an initial projection matrix equal to $U_B \Sigma_B^{-1}$, which is now a non-orthogonal matrix.

3.3.3 *Results:* This method too has still to be tested exhaustively, but in all cases examined up till now, the results were very promising. The same technique is now used for the enhancement of speech signals in the presence

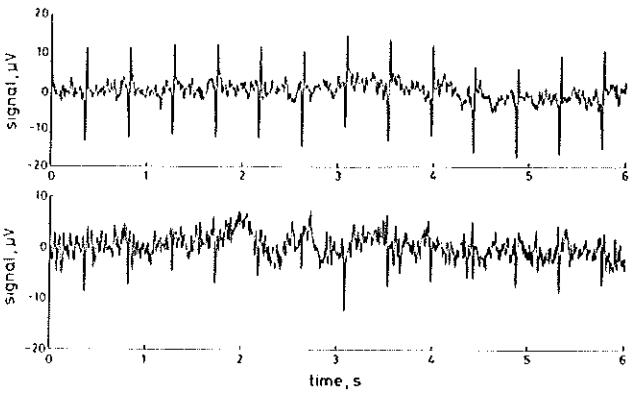

Fig. 7 *Two MECG-free signals obtained with the GSVD-based signal separation method as $x_1^T M$ and $x_2^T M$ respectively, where M is the data matrix of Fig. 4*

of stationary noise. The method has been applied to the five abdominally recorded signals of Fig. 5 and results in two MECG-free foetal signals (Fig. 7).

3.3.4 *Advantages:*

(a) The application of the online adaptive algorithm is possible, such that can be dealt with time-variance due to foetal movements.
(b) The obtained signals are optimal with respect to extremal oriented signal-to-signal ratio.

(c) Recordings with a reference electrode and standard electrode positioning can be used.
(d) Good method if a foetal trigger has to be obtained.

3.3.5 *Disadvantages:*
(a) For this method too a rather complicated initialisation of the online procedure (visual selection of data intervals) is required.
(b) No principal signals are obtained, because the signals are not orthogonal.

4 Conclusion

This paper presented three SVD methods that can be used for the separation of foetal and maternal electrocardiograms recorded from electrodes on the maternal skin. The first method is based on a simple SVD of the data matrix M that contains the measured potential signals. It finds directions that are optimal with respect to extremal oriented energy. One of the big advantages of this method is the possibility for an online computation strategy that can be used in a real-time implementation. In contrast with other methods, this technique also requires, besides abdominal electrode signals, signals measured at a far distance from the foetal heart.

The second technique applies a two-step SVD procedure. This technique is successful as far as MECG-elimination is concerned, but suboptimal with respect to foetal signal-to-noise ratio (only optimal in the limited complementary subspace). After slight modification of its initialisation step, the online adaptive algorithm becomes applicable for this method.

A third new technique based on the signal-to-signal ratio concept has been proposed. Now the generalised SVD is used to find some non-orthonormal directions that are optimal in the sense of extremal signal-to-signal ratio. Also, for this method, the same adaptive online SVD-based algorithm can be used.

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Appendix 1

Singular value decomposition and the generalised singular value decomposition: basic theorems

This section presents the basic theorems of SVD and GSVD. For this purpose, the following characterisations, restricted to real matrices, will be sufficient. In these characterisations, the 'economical' factorisation, where the pseudodiagonal matrices are replaced by square diagonal ones, is preferred.

Theorem 2 Autonne-Eckhart-Young theorem

For any real $p \times q$ matrix A, there exists a real factorisation

$$A_{p,q} = U_{p,p}\Sigma_{p,p}V^{T}_{p,q} \quad (\text{for } p < q)$$

in which the matrix U is orthogonal ($UU^T = U^TU = I_p$), the matrix V contains p orthonormal columns ($V^TV = I_p$) and $\Sigma_{p,p}$ is a real diagonal matrix with nonnegative diagonal elements, called the singular values σ_i of the matrix A (GOLUB and VAN LOAN, 1983).

Theorem 3 Generalised SVD

Let A be a $p \times q$ and B a $p \times k$ matrix ($p < q$ and $p < k$), then there exist matrices $U_A(q \times p)$ and $U_B(k \times p)$, both with p orthonormal columns, and a non-singular $p \times p$ matrix X such that

$$A = X^{-T}D_A U_A^T$$
$$B = X^{-T}D_B U_B^T \tag{19}$$

where $D_A = \text{diag}(\alpha_1, \ldots, \alpha_p)$ and $D_B = \text{diag}(\beta_1, \ldots, \beta_p)$, ($\alpha_i, \beta_i \geq 0$), are square diagonal $p \times p$ matrices and $\alpha_1/\beta_1 \geq \alpha_2/\beta_2 \geq \ldots \geq \alpha_r/\beta_r$, $r = \text{rank}(B)$

For proofs see GOLUB and VAN LOAN (1983). The elements of the set $\sigma(A, B) = (\alpha_1/\beta_1, \ldots, \alpha_r/\beta_r)$ are referred to as the generalised singular values of A and B.

Appendix 2

Concepts of oriented energy and signal-to-signal ratio

This section introduces the concepts of oriented energy and oriented signal-to-signal ratio of a vector sequence and shows their relations with SVD and GSVD.

Definition 1: Oriented energy:

Consider a sequence of p-vectors $\{a_k\}$, $k = 1, \ldots, q$ and arrange them as the columns of a $p \times q$ matrix A. Then $E_e[A]$, the energy of the vector set in the direction of unit vector $e \in \mathscr{R}^p$, is defined as

$$E_e[A] = \sum_{k=1}^{q}(e^Ta_k)^2 = \|e^TA\|^2 \tag{20}$$

There exists a relationship between the singular values and vectors of the matrix A and its directions of extremal oriented energy as follows:

$$E_{u_i}[A] = \|u_i^TA\|^2 = \sigma_i^2 \tag{21}$$

where u_i is a column vector of U in the SVD of A and σ_i is the corresponding singular value of A. For proof see STAAR (1982), VANDERSCHOOT *et al.* (1983; 1984), DE MOOR *et al.* (1987), DE MOOR (1988). Moreover, we know from linear algebra that each u_i contains the coefficients of a linear combination of the rows of A, such that $\|u_i^TAA^Tu_i\| = \|u_i^TA\|^2$ reaches an extremal value, which equals σ_i^2. In other words, the columns u_i, of the U-matrix in the SVD of A provide directions in the column space of A for which the oriented energy is extremal. Therefore, the SVD of a matrix A finds $r_A = \text{rank}(A)$ orthonormal directions of extremal oriented energy.

Definition 2 Oriented signal-to-signal ratio

The oriented signal-to-signal ratio $R_e[A, B]$ of two sets of p-vectors $\{a_k\}$ and $\{b_k\}$, ($k = 1, \ldots, q$), in the direction of unit vector $e \in \mathscr{R}^p$, is defined as

$$R_e[A, B] = \frac{E_e[A]}{E_e[B]} = \frac{\|e^TA\|^2}{\|e^TB\|^2} \tag{22}$$

In analogy with the oriented energy-SVD relationship, a relationship between the oriented signal-to-signal ratio concept and the GSVD exists: if the GSVD of matrices A and B is given as in Theorem 3 then

$$R_e[A, B] = \left(\frac{\alpha_i}{\beta_i}\right)^2 \quad \text{for } e = \frac{x_i}{\|x_i\|}$$

For proof see DE MOOR *et al.* (1987), DE MOOR (1988), VANDEWALLE and CALLAERTS (1988). Applied to the signal separation problem, this can be interpreted as follows: assume that A contains p signals that are all disturbed by an unwanted signal, while B contains only contributions from the unwanted signal. The GSVD of the matrix pair (A, B) then looks for directions in the column space of A and B for which the oriented ratio of wanted signal to unwanted signal is extremal. This was illustrated for the FECG-MECG separation problem (see Method 3).

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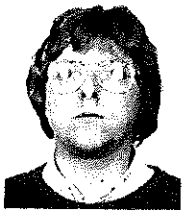
Dirk Callaerts was born in Leuven, Belgium, in 1961. He received the Engineering degree in Applied Sciences from Katholieke Universiteit Leuven, Belgium, in 1984. From then on he has been working on a doctorate in biomedical signal processing, and signal separation.



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