91-145

INT. J. CONTROL, 1992, VOL. 55, NO. 1, 257–259

Correspondence

Comments on 'State-space model identification with data correlation'

MARC MOONEN† and BART DE MOOR†

In a recent paper, Hou and Hsu (1991) derive a state-space identification method which apparently improves upon the results of Moonen *et al.* (1989). Here we point out that neither one of these methods really applies to the examples given in the paper, and we give an outline of a method which should be used instead.

In their paper, Hou and Hsu (1991) consider general state space models of the form

$$x_{k+1} = A \cdot x_k + B \cdot u_k + w_k$$
$$y_k = C \cdot x_k + D \cdot u_k + v_k$$

where $y_k \in \Re^l$ is the observed output and $u_k \in \Re^m$ is the observed deterministic input. The process noise w_k and measurement noise v_k are unknown. The aim is to identify the system matrices A, B, C, D—up to a similarity transformation—by means of recorded I/O-sequences $\{u_1\ u_2\ \dots\ u_N\}$ and $\{y_1\ y_2\ \dots\ y_N\}$.

The original method of Moonen *et al.* (1989) works with two matrices

$$H_1 = \left[\begin{array}{c} U_1 \\ \hline Y_1 \end{array}\right], \quad H_2 = \left[\begin{array}{c} U_2 \\ \hline Y_2 \end{array}\right]$$

where U_1, U_2, Y_1, Y_2 are certain block Hankel matrices, with in- and output vectors. The key property here is that

$$\text{span}_{\text{row}}\{X_2\} = \text{span}_{\text{row}}\{H_1\} \cap \text{span}_{\text{row}}\{H_2\}$$

for a certain state vector sequence $X_2 = [x_{k+i}\ x_{k+i+1}\ \dots]$. Once X_2 is known, the system matrices are computed from a set of linear equations. The computations are based on a singular value decomposition of the compound matrix

$$\left[\begin{array}{c} H_1 \\ \hline H_2 \end{array}\right]$$

It is shown that consistent results are obtained when the number of columns in H_1 and H_2 tends to ∞ and when the in- and outputs are only corrupted by independent additive white noise sequences, with the same variance, i.e.

$$x_{k+1} = A \cdot x_k + B \cdot (u_k + w_k)$$
$$(y_k + v_k) = C \cdot x_k + D \cdot (u_k + w_k)$$
$$E\left\{\begin{bmatrix} w_k \\ v_k \end{bmatrix} \cdot [w_{k+\Delta}^{\mathrm{T}}\ \ v_{k+\Delta}^{\mathrm{T}}]\right\} = \begin{cases} \sigma \cdot I, & \text{for } \Delta = 0 \\ 0, & \text{for } \Delta \neq 0 \end{cases}$$

Received 17 May 1991.
† ESAT-KUL, K. Mercierlaan 94-3001 Heverlee, Belgium.

In the coloured noise case, or for instance when only some of the in- and/or outputs are subject to noise, a QSVD is used instead of an SVD, as pointed out by Moonen and Vandewalle (1990), with a similar consistency result.

Hou and Hsu try to apply much the same method to the general state space model. Their 'correlation method' has an additional, and clever, prefiltering of u_k and y_k. A moving average filter is used with filter coefficients $y_k, y_{k+1}, \ldots, y_{k+h-1}$ (hence the relation with correlation methods). A crucial observation here is that —and this is not stated explicitly in the paper—the frequency response of the filter is a replica of the spectrum of $y_k, y_{k+1}, \ldots$ Hence, the filtering does an amazingly good job in reducing the effect of the noise in for example, the first example given in the paper, where only the output is corrupted by noise:

$$x_{k+1} = \begin{bmatrix} 1\cdot 5 & -0\cdot 7 \\ 1\cdot 0 & 0\cdot 0 \end{bmatrix} \cdot x_k + \begin{bmatrix} 1\cdot 0 \\ \hline 0\cdot 0 \end{bmatrix} \cdot u_k$$

$$y_k = [1\cdot 0 \quad 0\cdot 5] \cdot x_k + v_k$$

The point is that, although the pre-filtering clearly reduces the effect of the noise—and the choice of the filter coefficients is maybe the best one could every make—it does not turn the problem into one where the identification procedure is known to give consistent results, not even when the number of columns in H_1 and H_2 tends to ∞. In other words, the effect of the noise is reduced by the pre-filtering, but not nullified afterwards by the identification procedure. Even worse, in some cases—such as with the above 'additive white noise model'—the pre-filtering would introduce a noise colouring, such that the identification procedure is no longer applicable, even though it was before.

In a recent report (Moonen *et al.* 1991), a general procedure is derived for the general state-space model. It resembles the original procedure of (Moonen *et al.* 1989), but an additional projection is used. The key result here is that

$$\text{span}_{\text{row}}\{X_2\} = \text{span}_{\text{row}}\{H_1\} \cap \text{span}_{\text{row}}\{H_2 \Pi\}$$

where Π is the orthogonal projection onto the row space of $[H_1^{\text{T}} \; U_2^{\text{T}}]^{\text{T}}$. For the details, we refer the reader to Moonen *et al.* (1991). The table shows a few simulation results, comparing the different methods. We adopt the second example from Hou and Hsu (1991), with

$$x_{k+1} = \begin{bmatrix} 1\cdot 5 & -0\cdot 7 \\ 1\cdot 0 & 0\cdot 0 \end{bmatrix} \cdot x_k + \begin{bmatrix} 1\cdot 0 \\ 0\cdot 0 \end{bmatrix} \cdot u_k + \begin{bmatrix} w_k \\ 0 \end{bmatrix}$$

$$y_k = [1\cdot 0 \quad 0\cdot 5] \cdot x_k + v_k$$

	Identified poles		
	$j = 100$	$j = 200$	$j = 400$
Original	$0\cdot 7458 \pm 0\cdot 3020i$	$0\cdot 6202 \pm 0\cdot 2018i$	$0\cdot 5248 \pm 0\cdot 2410$
Pre-filtered	$0\cdot 7114 \pm 0\cdot 2366i$	$0\cdot 7564 \pm 0\cdot 3053i$	$0\cdot 7283 \pm 0\cdot 2904$
New	$0\cdot 7096 \pm 0\cdot 3137i$	$0\cdot 7126 \pm 0\cdot 3320i$	$0\cdot 7410 \pm 0\cdot 3603$

Table. Original model poles: $0\cdot 7500 \pm 0\cdot 3708i$.

where w_k and v_k are normally distributed random noise sequences with variance 1·0. We took $i = 40$ in all cases and $h = 200$ in the pre-filtered method (see Hou and Hsu 1991). Parameter j denotes the number of columns in H_1 and H_2. The original method (Moonen *et al.* 1989) does not apply, and this is clearly evident. The pre-filtered method (Hou and Hsu 1991) does not apply either, but clearly delivers much better results. Still, it fails to give consistent results when the number of columns in H_1 and H_2 increases. With the new method (Moonen *et al.* 1991), the system poles clearly converge to those of the original model, as the number of columns in H_1 and H_2 tends to ∞.

Acknowledgments

This research was partially sponsored by the ESPRIT Basic Research Action Nr. 3280. Marc Moonen is a senior research assistent with the N.F.W.O. (Belgian National Fund for Scientific Research). Bart De Moor is a research associate with the N.F.W.O.

References

Hou, D., and Hsu, C. S., 1991, State-space model identification with data correlation. *International Journal of Control*, **53**, 181–192.

Moonen, M., De Moor, B., Vanderberghe, L., and Vandewalle, J., 1989, On- and off-line identification of linear state space models. *International Journal of Control*, **49**, 219–232.

Moonen, M., and Vandewalle, J., 1990, A QSVD approach to on- and off-line state space identification. *International Journal of Control*, **51**, 1133–1146.

Moonen, M., Vanoverschee, P., and De Moor, B., 1991, A subspace algorithm for combined stochastic-deterministic state space identification. ESAT-SISTA Report, Electrical Engineering Department, Katholieke Universiteit Leuven, Belgium, in preparation.