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Vanvuchelen P., De Moor B., ``A Multi-Objective Optimization Approach for Parameter Setting in System and Control Design", in Preprints of the 16th IFIP Conference on system modeling and optimization, Compiegne, France, Jul. 1993, pp. 63-66., Lirias number: 180470.

A Multi-Objective Optimization Approach for Parameter Setting in System and Control Design

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Extended abstract, IFIP 93

Abstract

In applications, system-and-control engineers are often confronted with the problem of tuning some free parameters of a fixed structure, while a variety of statical and dynamical performance specifications is under consideration. For practical problems, it is easy to verify that trade-offs arise spontaneously when the parameters are varied. However, to find solutions on the trade-off boundary, the problem needs to be handled as a multi-objective optimization problem.

1 From real life to a mathematical multi-objective optimization problem (MOOP)

Three different phases are needed to transform a practical problem into a MOOP (ref. [6]).

Phase 1: Parametrization: In this phase, the fixed structure is parametrized, which means that the unknowns are chosen. Let $\vec{X} \in R^n$ be the vector containing the n unknown parameters.

Phase 2: Classifying the dynamical design specifications:
Phase 2a: Trivial specifications and quality specifications: When analyzing the different dynamical design specifications in practical problems, two main categories arise immediately:

- Trivial specifications include requirements such as realizability, stability and limitations on the parameter size.
- Quality specifications must guarantee a certain performance level. Rise time, overshoot and energy are quality measures for time-domain performance, while gain margin and phase margin measure the frequency-domain behaviour. In practice, it is preferred to express a system's quality using a quality vector instead of one single quality measure.

Phase 2b: Hard and soft specifications: After gathering all the dynamical design specifications, the complete set of specifications, found in phase 2a, is splitted into two subsets:

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- The set of hard specifications contains all specifications that need to be satisfied to have a feasible design. In practice, it contains all trivial specifications and those quality measures for which no trade-off is allowed by the designer.
- The set of soft specifications includes all the performance measures that must be traded-off.

Phase 3: Formulation as a multi-objective optimization problem: Once the phases 1 and 2 are completed, it is simple to express the designer's wish: minimize all the soft specifications simultaneously, without violating the hard specifications. The general (vector) minimization problem can be stated as

$$\min_{\vec{X} \in \mathcal{F}} \vec{f} = (f_1, \ldots, f_k)'$$

with $\mathcal{F}$ representing the set of feasible designs.

2 Solving the MOOP

In the literature, a variety of techniques is available for finding solutions on the trade-off boundary. We only use methods based on function scalarization. These methods transform the vector minimization problem into a function minimization problem. Three methods are available in our software:

Vector norm optimization: Using the p-norm as defined in [5], the transformed problem becomes

$$\min_{\vec{X} \in \mathcal{F}} \parallel (\lambda_1 f_1(\vec{X}), \ldots, \lambda_k f_k(\vec{X})) \parallel_p$$

The weights λ_i allow the designer to influence the trade-off to be found.

The ϵ constraint method: The transformed problem (ref. [1]) is stated as

$$\min_{\vec{X} \in \mathcal{F}} f_i(\vec{X}) \; s.t. \; f_j \leq \epsilon_j, \; \forall j \neq i$$

In this method, the constraint values ϵ_j allow the designer to influence the trade-off to be found.

The goal attainment method: Let $\vec{f}^*$ be the designer's goal and $\vec{\lambda}$ be the desired search direction from this goal (ref. [4]), then the problem is transformed in

$$\min \; \gamma \; s.t. \; \vec{X} \in \mathcal{F}, \; f_i(\vec{X}) - \lambda_i \gamma \leq f_i^*, \; \forall i$$

The general problem that shows up after transforming the MOOP, is a non-linear programming problem, stated as

$$\min_{\vec{X}} \; f(\vec{X}) \; s.t. \; \vec{g}(\vec{X}) = (g_1(\vec{X}), \ldots, g_l(\vec{X})) \geq 0$$

To solve this problem, the technique of sequential programming is used with a logarithmic barrier function, as proposed in [3]. The problem reduces to

$$\min_{\vec{X}} \; L(\vec{X}, r) = f(\vec{X}) - r \sum_{i=1}^{l} \log(g_i(\vec{X}))$$

where r is a positive scalar.

3 How to reduce the design time?

Although the designer can influence the trade-off to be found, it should be clear that he almost never finds the 'ideal' trade-off immediately. In practice, this means that the time to find an acceptable design is not equal, but proportional to the time to find one trade-off point. To make the approach useful for applications, this design time should be reduced as much as possible. Many factors influence the time to solve the non-linear programming problem, but three of them, of extreme importance, must be analyzed carefully.

- The nature of the problem: convex or non-convex ?
- Local low order approximations: available or estimated?
- The time needed for a function evaluation

Generically, there are very few problems in dynamical design that guarantee convexity, that allow exact calculation of local approximations and that allow all functions to be evaluated analytically.

4 An example

Consider the problem of designing a dynamical control law $K(s)$ for the following one-degree-of-freedom (1DOF) configuration (see figure 1).

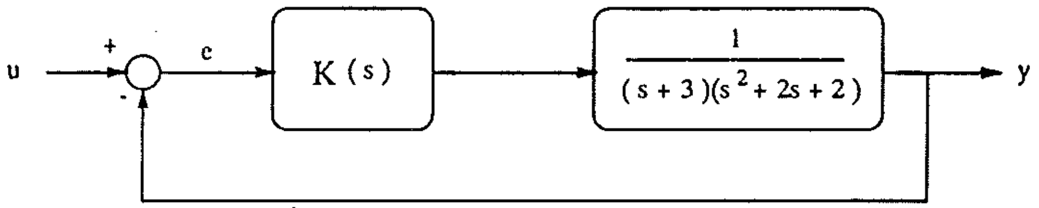

Figure 1: A 1DOF configuration with a control law $K(s)$ to be designed

Four different parametrizations are investigated: Three classical parametrizations of the control law $K(s)$ (a compensator network, a PI-controller and a PID-controller) and a modern controller, based on the Youla-parametrization.

The design specifications of interest:

- As hard specifications (to be satisfied always), we consider stability, realizability, bounds on the design variables and zero steady state error for a step input.
- As soft specifications (to be traded-off), we consider the l_2 norm of the error signal $e(t)$ and the overshoot in the response $y(t)$, for a step input $u(t)$.

The problem is stated as

$$\min_{\vec{X}} \ \lambda_1\Phi_1(\vec{X}) + \lambda_2\Phi_2(\vec{X}) \quad \text{s.t. } \vec{X} \text{ satisfies } \Phi_{hard,j}, \ \forall j$$

with Φ_1 measuring the overshoot and Φ_2 as a measure for the energy (cfr. vector norm optimization for $p = 1$). The transformed problem can be solved with the logarithmic barrier method.

Figure 2 shows the 'optimal' step responses for different weights, while figure 3 shows the trade-off curves of the different parametrizations.

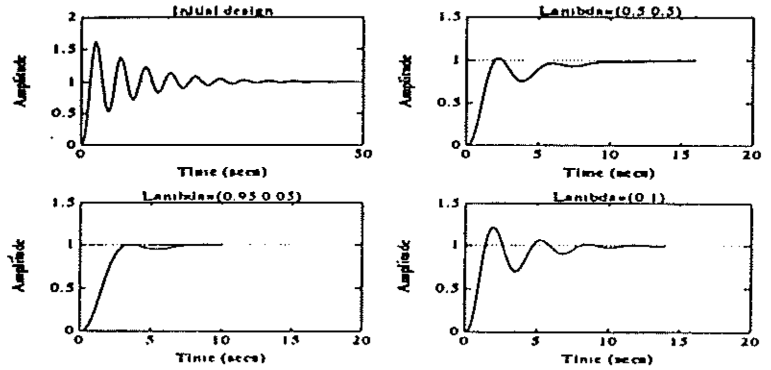

Figure 2: Step responses with a PI-controller: the initial design and designs on the trade-off boundary (designs for various weights)

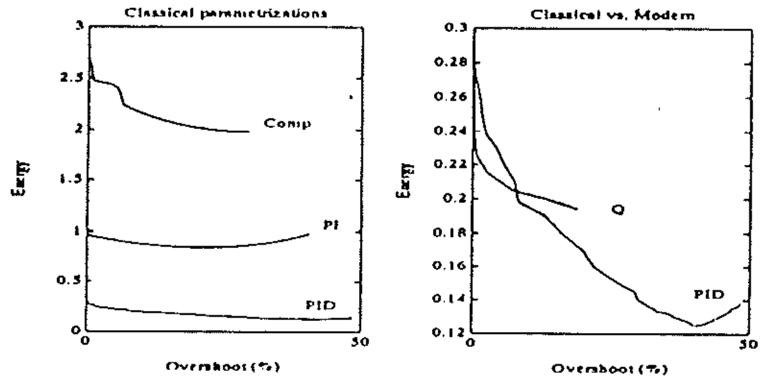

Figure 3: (l) Trade-off curves for classical parametrizations. (r) Classical versus modern parametrization.

References

[1] Boyd S.P., Baratt C. *Linear Controller Design, Limits of performance.* Prentice Hall Inc., 1990.

[2] Brayton R.K., Spence R. *CAD of electrical circuits, 2, Sensitivity and Optimization.* Elsevier Scientific Publishing Company, 1980.

[3] Fiaco A.V., Mc Cormick G.P. *Non-linear programming; Sequential Unconstrained Minimization Techniques.* John Wiley and Sons, 1968.

[4] Fleming P.J., Pashkevich A.P. *Computer-Aided Control System Design Using a Multi-Objective Optimization Approach.* Control '85 Conference, Cambridge, U.K., 174-179, 1985.

[5] Golub G., Van Loan C. *Matrix Computations.* Johns Hopkins Univ. Press, Second edition, 1989.

[6] Vanvuchelen P., De Moor B. *A Multi-Objective Optimization Approach for Parameter Setting in System and Control Design.* ESAT-SISTA report, 1992.