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Model Predictive Control applied to a river system with two reaches

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***Abstract*— Many control strategies can be found in literature for controlling a river system. Most of these methods focus on set-point control such that the most upstream or downstream part of each reach tracks a certain reference trajectory while minimizing the effect of disturbances. However many of the control techniques suitable for set-point control cannot be used at the same time for preventing a river from flooding when large disturbances take place. In this paper we show that Model Predictive Control can be used for set-point control and flood control of a river system consisting of two reaches and one gate. For this we use a linearized version of the Saint-Venant equations with special attention to the gate dynamics.**

I. INTRODUCTION

Many different types of control strategies can be found in literature to control reaches or irrigation canals. Examples are PI controllers, heuristic controllers, predictive controllers and optimal controllers [1], [2], [3]. The dynamics of a single reach can be accurately described by two nonlinear partial differential equations, the so-called Saint-Venant equations. These equations have to be combined for every reach with the dynamics of the interconnecting gates to get a mathematical model of the entire river system. Because of computational reasons these controllers do not work directly with these equations but they use approximate models.

The control purpose influences which kind of approximate model is appropriate. E.g. if the only goal is to keep the most downstream water level of each reach close to a set-point, the integrator delay (ID) model can be a good approximation [4]. However since this model simulates the water levels of the reach at only one point, this model is not the right choice for the application of flood control where we want to keep all water levels below their flood levels. Since we are interested in set-point control as well as flood control, another model is used in this study: a model based on the linearized Saint-Venant equations along the entire river.

A control strategy suited for the task of set-point control and flood control at the same time is Model Predictive Control (MPC) [5], [6]. MPC is a control strategy originating from the process industry and is used in various applications going from chemicals and food processing to automotive and aerospace applications [7]. One can find many studies in literature where MPC is used for set-point control of river systems based on the ID model ([8], [9], [10]) and on the linearized Saint-Venant equations [11]. However these works do not focus on flood control.

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In previous work we have been using a very simplified conceptual model which models the water levels of a river system only at a very limited number of points for set-point control in combination with flood control [12]. In recent work we switched to using a linear version of the Saint-Venant equations with a very fine spatial discretization for designing and implementing a control strategy for only a single reach without gates [13]. In this work we test MPC applied to a two reach system with a gate in between. The controller combines the linear Saint-Venant equations together with the nonlinear gate equations.

The paper is organized as follows. Section II discusses the equations describing the dynamics of a single reach as well as its numerical implementation together with the gate equations. Section III presents two linear approximations of the full nonlinear model used by MPC. Section IV describes how MPC can be used for set-point control and flood control at the same time. Section V compares the performance of MPC for the two types of approximate models for set-point control, disturbance rejection and flood control. Section VI ends the paper with conclusions and future work.

II. RIVER MODELLING

A. Reach dynamics

The Saint-Venant equations model the dynamics of the water levels and discharges in a reach by the following partial differential equations (PDEs) [14], [15]:

$$\frac{\partial A}{\partial h}\frac{\partial h}{\partial t} + \frac{\partial Q}{\partial x} = 0, \tag{1}$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\frac{Q^2}{A} + gA\left(\frac{\partial h}{\partial x} + S_\mathrm{f} - S_0\right) = 0, \tag{2}$$

with Q the water discharge ($\mathrm{m^3/s}$), h the water depth (m), A the cross-sectional flow area ($\mathrm{m^2}$), g the gravity acceleration ($\mathrm{m/s^2}$), S_0 the bed slope and S_f the friction slope. Equation (1) describes the conservation of mass and (2) the conservation of momentum. S_f is an (empirical) resistance law given by the Manning relation:

$$S_\mathrm{f} = \frac{n_\mathrm{mann}^2 Q|Q|}{A^2 R^{1/3}} \tag{3}$$

where n_mann is the Manning coefficient ($\mathrm{s/m^{1/3}}$), $R = A/P$ is the hydraulic radius (m) and P is the wetted perimeter of the cross section (m).

B. Gate equations

The gates used in this paper are underflow-vertical sluices. In general the gate dynamics can be modelled by:

$$Q = z(C_\mathrm{D}, w, c, h_\mathrm{up}, h_\mathrm{down}) \tag{4}$$

with Q the discharge through the gate, C_D the discharge coefficient, w the width of the gate (m), c the gate opening (m), h_up the upstream water level and h_down the downstream water level. In this study the following equation is used:

$$Q = C_\text{D} w c \sqrt{2 g h_\text{up}} \tag{5}$$

with different equations for C_D depending on the flow condition ([16], [17]):

$$\text{free flow: } C_\text{D} = \frac{C_\text{C}}{\sqrt{1+\eta}}, \tag{6}$$

submerged flow:

$$C_\text{D} = C_\text{C} \frac{\left[\xi - \sqrt{\xi^2 - \left(\frac{1}{\eta^2}-1\right)^2\left(1-\frac{1}{\lambda^2}\right)}\right]^{1/2}}{\frac{1}{\eta}-\eta}. \tag{7}$$

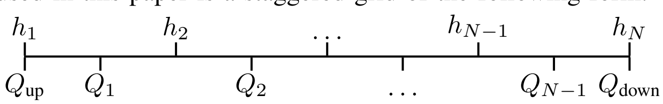
C_C is the contraction coefficient, $\eta = C_\text{C} c / h_\text{up}$, $\lambda = h_\text{up}/h_\text{down}$ and $\xi = (1/\eta - 1)^2 + 2(\lambda - 1)$. C_C can vary from 0.598 to 0.74 but for most applications a value of 0.611 is usually taken ([18], [19]). The gate is considered to be in free flow if h_down is below the limit $h_\text{down,max}$ given by:

$$h_\text{down,max} = \frac{C_\text{C} c}{2}\left(\sqrt{1 + \frac{16}{\eta(1+\eta)}}\right). \tag{8}$$

Otherwise the gate is in submerged flow condition.

C. Discretization and numerical implementation

Since in general there is no analytical solution for the Saint-Venant equations, the infinite dimensional variables will be approximated on a finite grid [20]. The partial derivatives are approximated with finite differences while the θ-method, e.g. $f(t_j + \theta\Delta t) = \theta f(t_j + \Delta t) + (1-\theta) f(t_j)$ with $\theta \in [0,1]$, is used for the time integration. The spatial grid used in this paper is a staggered grid of the following form:



$2N-1$ is the total number of unknown variables (N water levels and $N-1$ discharges) and Q_up and Q_down are the (known) boundary discharges. The derivatives in (1) are approximated by (note $h(x_i, t_j) = h_i^j$)

$$\frac{\partial h_i^j}{\partial t} \simeq \frac{h_i^{j+1} - h_i^j}{\Delta t}, \tag{9}$$

$$\frac{\partial Q_i^j}{\partial x} \simeq \frac{\theta(Q_i^{j+1} - Q_{i-1}^{j+1}) + (1-\theta)(Q_i^j - Q_{i-1}^j)}{\Delta x}, \tag{10}$$

$$\frac{\partial A_i^j}{\partial h} \simeq \theta \frac{\partial A_i^{j+1}}{\partial h} + (1-\theta)\frac{\partial A_i^j}{\partial h}. \tag{11}$$

A similar approach is used for the terms $\partial Q/\partial t$, A, $\partial h/\partial x$ and S_f in (2). The advection term $\partial(Q^2/A)/\partial x$ is approximated with an upwinding approach:

$$\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right)_i^j \simeq \begin{cases} \omega\left(\frac{Q^2}{A}, i+1, j+1\right) & Q_i^j < 0 \\ \omega\left(\frac{Q^2}{A}, i, j+1\right) & Q_i^j \geq 0 \end{cases} \tag{12}$$

with

$$\omega\left(\frac{Q^2}{A}, i, j\right) = \frac{\theta}{\Delta x}\left(\left(\frac{Q^2}{A}\right)_i^j - \left(\frac{Q^2}{A}\right)_{i-1}^j\right) + \\ + \frac{1-\theta}{\Delta x}\left(\left(\frac{Q^2}{A}\right)_i^{j-1} - \left(\frac{Q^2}{A}\right)_{i-1}^{j-1}\right). \tag{13}$$

In this way the two PDEs for each reach are transformed into a system of nonlinear equations:

$$\boldsymbol{f}\left(\boldsymbol{h}^{j+1}, \boldsymbol{h}^j, \boldsymbol{Q}^{j+1}, \boldsymbol{Q}^j\right) = \boldsymbol{0}, \tag{14}$$

$$\boldsymbol{g}\left(\boldsymbol{h}^{j+1}, \boldsymbol{h}^j, \boldsymbol{Q}^{j+1}, \boldsymbol{Q}^j\right) = \boldsymbol{0}, \tag{15}$$

with $\boldsymbol{f} : \mathbb{R}^{4N+2} \to \mathbb{R}^N$, $\boldsymbol{g} : \mathbb{R}^{4N+2} \to \mathbb{R}^{N-1}$, $\boldsymbol{h} = (h_1, \ldots, h_N)^T$ and $\boldsymbol{Q} = (Q_\text{up}, Q_1, \ldots, Q_{N-1}, Q_\text{down})^T$, which has to be solved for $h_1^{j+1}, h_2^{j+1}, \ldots, h_N^{j+1}$ and $Q_1^{j+1}, Q_2^{j+1}, \ldots, Q_{N-1}^{j+1}$. Boundary conditions (BCs) for the upstream and downstream discharges Q_up and Q_down of every reach are needed to be able to solve the nonlinear system of equations. These discharges can be given by the gate dynamics (5) or can be disturbances. Given these BCs, (14) and (15) can be solved with Newton's method.

A discussion about the choice of Δt and θ can be found in [21]. In this paper θ is set equal to 0.6.

D. Test case

In this study we will test our controllers on a river system consisting of two consecutive trapezoidal reaches with a gate in between (Fig. 1). The most downstream discharge Q_out can be directly controlled (e.g. with a pump), the most upstream discharge Q_in is either a disturbance signal or also an input variable. A gate controls the discharge going from reach one to reach two. The mathematical model describing the system dynamics can be summarized as follows:

$$\text{BC: } Q_1(0,t) = Q_\text{in}(t), \tag{16}$$

$$\text{reach: } \frac{\partial A_1}{\partial h}\frac{\partial h_1}{\partial t} + \frac{\partial Q_1}{\partial x} = 0, \tag{17}$$

$$\frac{\partial Q_1}{\partial t} + \frac{\partial}{\partial x}\frac{Q_1^2}{A_1} + g A_1\left(\frac{\partial h_1}{\partial x} + S_\text{f} - S_0\right) = 0, \tag{18}$$

$$\text{gate: } Q_2(0,t) = z(C_\text{D}, w, c(t), h_1(L_1,t), h_2(0,t)), \tag{19}$$

$$Q_1(L_1, t) = Q_2(0,t), \tag{20}$$

$$\text{reach: } \frac{\partial A_2}{\partial h}\frac{\partial h_2}{\partial t} + \frac{\partial Q_2}{\partial x} = 0, \tag{21}$$

$$\frac{\partial Q_2}{\partial t} + \frac{\partial}{\partial x}\frac{Q_2^2}{A_2} + g A_2\left(\frac{\partial h_2}{\partial x} + S_\text{f} - S_0\right) = 0, \tag{22}$$

$$\text{BC: } Q_2(L_2, t) = Q_\text{out}(t) \tag{23}$$

The parameters of the river system are given in Table I.

III. APPROXIMATE MODELS

In this section we will discuss two linear models which can be used by MPC for controlling river systems consisting of multiple reaches and gates. The explanation will be given for the river system discussed in Section II-D.

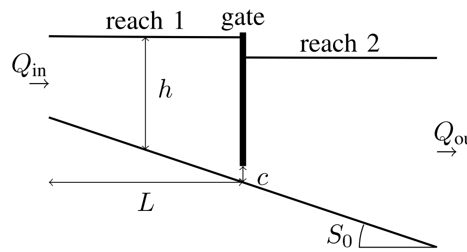


Fig. 1. Schematic structure of the river system with Q_{in} and Q_{out} the discharges at the boundaries, c the gate position, h the water levels, L the length of each reach and S_0 the channel slope.

	parameters	values	
		reach 1	reach 2
reaches:	N	10	40
	S_0	0.0004	0.0001
	reach length L	1000 m	4000 m
	side slope S	0.5	0.5
	n_{mann}	0.014 s/m$^{1/3}$	0.014 s/m$^{1/3}$
	bottom width B	4 m	4 m
gate:	w	4 m	

TABLE I
PARAMETER VALUES OF THE RIVER REACHES AND THE GATE.

A. The linear model

After discretizing the model equations (16)-(23) and linearizing the resulting equations around a nominal operating point ($\boldsymbol{h}_{\text{ss}} \in \mathbb{R}^{N_1+N_2}$ for the water levels, $\boldsymbol{Q}_{\text{ss}} \in \mathbb{R}^{N_1+N_2-2}$ for the discharges, $\boldsymbol{u}_{\text{ss}} \in \mathbb{R}^{n_u}$ for the control inputs and $\boldsymbol{d}_{\text{ss}} \in \mathbb{R}^{n_d}$ for the disturbances with N_1 and N_2 the number of water levels for reach one, resp. reach two, n_u and n_d the number of inputs and disturbances) the following linear model can be found:

$$\Delta \boldsymbol{x}(k+1) = \boldsymbol{A}\Delta \boldsymbol{x}(k) + \boldsymbol{B}\Delta \boldsymbol{u}(k) + \boldsymbol{D}\Delta \boldsymbol{d}(k), \quad (24)$$

with $\Delta \boldsymbol{x}(k) = (\Delta \boldsymbol{h}(k), \Delta \boldsymbol{Q}(k))^T$, $\Delta \boldsymbol{h}(k) = \boldsymbol{h}(k) - \boldsymbol{h}_{\text{ss}}$, $\Delta \boldsymbol{Q}(k) = \boldsymbol{Q}(k) - \boldsymbol{Q}_{\text{ss}}$, $\Delta \boldsymbol{u}(k) = \boldsymbol{u}(k) - \boldsymbol{u}_{\text{ss}}$, $\Delta \boldsymbol{d}(k) = \boldsymbol{d}(k) - \boldsymbol{d}_{\text{ss}}$, $\boldsymbol{A} \in \mathbb{R}^{(2N_1+2N_2-2)\times(2N_1+2N_2-2)}$, $\boldsymbol{B} \in \mathbb{R}^{(2N_1+2N_2-2)\times n_u}$ and $\boldsymbol{D} \in \mathbb{R}^{(2N_1+2N_2-2)\times n_d}$. The input vector $\boldsymbol{u}(k)$ contains the gate position c together with the controllable upstream Q_{in} and/or downstream Q_{out} discharges at time instant k while the disturbance vector $\boldsymbol{d}(k)$ contains the uncontrollable ones. For example, if only the downstream discharge can be controlled, then we have that $\boldsymbol{u}(k) = (c(k), Q_{\text{out}}(k))^T$ and $\boldsymbol{d}(k) = Q_{\text{in}}(k)$. We will refer to this model as the L-model.

B. The linear-nonlinear model

However as we have shown in [22] more accurate results can be obtained if we use a linear model which treats the nonlinearities of the gate equations in a separate way. In this model the gate equations are pulled out of the linear model and we work with the gate discharge as input variable instead of the gate position for the linear part. More specifically for our test example, equations (16)-(18) and (20)-(23) remain unchanged and equation (19) is replaced with

$$Q_1(L_1, t) = Q_{\text{gate}}(t). \quad (25)$$

Discretizing and linearizing this new set of equations around the operating point results in the following model:

$$\Delta \boldsymbol{x}(k+1) = \bar{\boldsymbol{A}}\Delta \boldsymbol{x}(k) + \bar{\boldsymbol{B}}\Delta \bar{\boldsymbol{u}}(k) + \bar{\boldsymbol{D}}\Delta \boldsymbol{d}(k), \quad (26)$$

with $\Delta \bar{\boldsymbol{u}}(k) = \bar{\boldsymbol{u}}(k) - \bar{\boldsymbol{u}}_{\text{ss}} \in \mathbb{R}^{n_u}$, $\bar{\boldsymbol{A}} \in \mathbb{R}^{(2N_1+2N_2-2)\times(2N_1+2N_2-2)}$, $\bar{\boldsymbol{B}} \in \mathbb{R}^{(2N_1+2N_2-2)\times n_u}$ and $\bar{\boldsymbol{D}} \in \mathbb{R}^{(2N_1+2N_2-2)\times n_d}$. The control variables $\bar{u}$ are the discharge controlled by the gate Q_{gate} together with the controllable upstream and/or downstream river discharges. When using this model, first a conversion is needed from the gate position to the gate discharge before the linear model can be used. We will refer to this model as the LN-model.

IV. MODEL PREDICTIVE CONTROL

MPC is an optimization based control strategy which makes use of a process model to predict the future process outputs within a specified prediction horizon. MPC determines the next inputs for the process by solving an optimization problem over this horizon taking into account input and output constraints, future disturbances and the process model. Only the first element of the complete optimal sequence is applied to the process, the new current state of the system is measured or estimated and the entire procedure is repeated.

If we let the controller minimize the deviation of the water levels from their set-points, MPC can be used for set-point control. If we add at the same time the flood levels as upper limits on the water levels, the same controller can also be used for flood control.

A. MPC based on the L-model

The optimization problem that has to be solved at every time step is the following quadratic problem (QP):

$$\min_{\boldsymbol{u},\boldsymbol{x},\boldsymbol{\xi}} \sum_{k=1}^{N_{\text{P}}} \| \boldsymbol{x}(k) - \boldsymbol{r}(k) \|_{\boldsymbol{W}}^2 + \sum_{k=1}^{N_{\text{P}}} \| \boldsymbol{\xi}(k) \|_{\boldsymbol{S}}^2 + \sum_{k=0}^{N_{\text{P}}-1} \| \boldsymbol{u}(k) - \boldsymbol{u}(k-1) \|_{\boldsymbol{R}}^2 \quad (27)$$

$$\text{s.t. } \boldsymbol{x}(0) = \boldsymbol{x}_0, \quad (28)$$

$$\Delta \boldsymbol{x}(k+1) = \boldsymbol{A}\Delta \boldsymbol{x}(k) + \boldsymbol{B}\Delta \boldsymbol{u}(k) + \boldsymbol{D}\Delta \boldsymbol{d}(k), \quad (29)$$

$$\boldsymbol{u}_{\text{min}} \le \boldsymbol{u}(k) \le \boldsymbol{u}_{\text{max}}, \quad (30)$$

$$|\boldsymbol{u}(k) - \boldsymbol{u}(k-1)| \le \Delta \boldsymbol{u}_{\text{max}}, \quad (31)$$

$$\boldsymbol{h}(k) \le \boldsymbol{h}_{\text{max}} + \boldsymbol{\xi}(k), \quad (32)$$

$$\boldsymbol{\xi}(k) \ge 0, \quad (33)$$

with N_{P} the prediction horizon, $\boldsymbol{W}$, $\boldsymbol{R}$ and $\boldsymbol{S}$ three positive semi-definite diagonal weighting matrices of appropriate dimension, $\boldsymbol{r}(k)$ the reference signal for the states, $\boldsymbol{x}_0$ the current state of the process, $\boldsymbol{h}_{\max}$ the flood levels, $\boldsymbol{u}_{\min}$ and $\boldsymbol{u}_{\max}$ the operational limits on the inputs, $\Delta\boldsymbol{u}_{\max}$ the maximal allowed rate of change for the inputs and $\boldsymbol{\xi}$ a vector of slack variables (one slack variable for each water level).

It can be shown that for positive semi-definite weighting matrices, the QP has only one (global) solution [23]. In this study we use Mosek [24] for solving the QPs. From now on this controller will be referred to as L-MPC.

The flood limits for the water levels are implemented as soft constraints together with the positivity constraints on the slack variables (33) and the inclusion of the slack variables in the objective function (27). By using soft constraints, the QP will always be feasible. The first goal of the controller is to keep the water levels below their flood limits. Therefore the diagonal elements in the matrix $\boldsymbol{S}$ are taken much larger than the diagonal elements in $\boldsymbol{W}$ and $\boldsymbol{R}$. This will force the optimizer to find control actions such that ideally all the slack variables are equal to zero. If this is not possible, the controller will try to minimize the violations of these constraints and hence to reduce the flood risk. If there is no flood risk, the controller needs to focus on set-point control of the most important water levels. This is achieved by choosing large elements in the matrix $\boldsymbol{W}$ corresponding with these water levels. With the elements in the matrix $\boldsymbol{R}$ we can influence the control effort of the different input variables.

$\boldsymbol{u}_{\min}$ and $\boldsymbol{u}_{\max}$ are the operational limits on the inputs. For the gate they correspond to the minimal and maximal gate position. For the controllable upstream or downstream discharges they are the minimal and maximal discharge.

B. MPC based on the LN-model

The form of the QP that has to be solved when the LN-model is used by the controller is very similar to the QP of the previous subsection. The biggest difference is the replacement of the model equation (29) with (26). Hence the discharge controlled by the gate is now an optimization variable instead of the gate position. However this means that after solving the QP, we still need to make the conversion from the optimal discharge through the gate Q_{gate} to the corresponding gate position c. Given the current upstream and downstream water level of the gate, (5) can be solved iteratively to find the corresponding c. The advantage of doing this is that the nonlinearities of the gate equations are included within the controller. However a conversion is also needed for the limits $\boldsymbol{u}_{\min}$, $\boldsymbol{u}_{\max}$ and $\Delta\boldsymbol{u}$ on the position of the gate Q_{gate} to the corresponding unknown time-varying limits on the discharge through the gate over the entire prediction horizon. This conversion can be done in the following way. Given the current state of the process $\boldsymbol{x}(k)$ and the last applied gate position $c(k-1)$, the maximal and minimal gate discharge can be found:

$$Q_{\max}(k) = z(C_D, w, c(k-1) + \Delta u_{\max}, h_{\text{up}}(k), h_{\text{down}}(k)), \tag{34}$$

$$Q_{\min}(k) = z(C_D, w, c(k-1) - \Delta u_{\max}, h_{\text{up}}(k), h_{\text{down}}(k)). \tag{35}$$

These limits are the new limits $\boldsymbol{u}_{\max}$ and $\boldsymbol{u}_{\min}$ at time k for the gate discharge. The next step is to use the linear model (26) given $Q_{\text{gate}}(k)$, $Q_{\text{out}}(k)$ and $Q_{\text{in}}(k)$ to estimate $\boldsymbol{x}(k+1)$. Based on these estimates of $h_{\text{up}}(k+1)$ and $h_{\text{down}}(k+1)$ and the optimal gate discharge $Q_{\text{gate}}(k+1)$ found by solving the QP in the previous time step, the corresponding gate position $c(k+1)$ can be found. Given $c(k+1)$ and the state $\boldsymbol{x}(k+1)$, the upper and lower limits on the gate discharge at time $k+1$ can be found in the same way. This procedure has to be repeated until $k+N_{\text{P}}-1$. In general we need to iterate between solving the QP and calculating the new upper and lower limits until convergence, however as it will be shown in the next section very good results can already be achieved by performing this iteration only once. We will refer to this controller as LN-MPC.

V. SIMULATION RESULTS

In this section the performance of the L-MPC and LN-MPC controllers will be evaluated under three different scenarios: set-point control, disturbance rejection and flood control. The test system is described in Section II-D. The controllers can only change the inputs every 15 min, N_{P} is taken equal to 15, $u_{\min}$, $u_{\max}$ and $\Delta u_{\max}$ for Q_{in} and Q_{out} (if controllable) are -7 m^3/s, 7 m^3/s and 5 m^3/s, while the gate has to remain between 0 m and 2 m and can maximally be moved over a distance of 20 cm each time step. As initial condition the river system is considered in steady state with a discharge equal to 4 m^3/s at every point along the river system, the most downstream water level is equal to 3 m and the initial gate opening is 0.4 m. The flood levels are 80 cm above the nominal water levels of reach one and 30 cm above the nominal water levels of reach two. The control actions are applied to the fully nonlinear model. We assume that the disturbances are known in advance. If there is no flooding risk, the controller needs to keep the most downstream water level of each reach as close as possible to its initial value.

The diagonal elements of the matrix $\boldsymbol{W}$ corresponding with the most downstream water level of each reach are taken equal to 10 for L-MPC and LN-MPC, all the other diagonal elements are set equal to 10^{-5}. The other parameters of the controllers are set as follows: $\boldsymbol{R} = \boldsymbol{I}$ and $\boldsymbol{S} = 10^4 \times \boldsymbol{I}$ with $\boldsymbol{I}$ every time a unit matrix of appropriate dimension. The results for L-MPC are shown with dashed lines and the results for LN-MPC with full lines.

A. Set-point control

In this situation the controllers can control the gate together with Q_{in} and Q_{out}; there is no disturbance. The results for both controllers are visualized in Fig. 2. On top we have the evolution of the most downstream water level for reach one (left) and for reach two (right) together with the reference trajectory (dashed-dotted line). The bottom figures show the control actions for the two controllers: the upstream discharge (left), the gate opening (middle) and the downstream discharge (right). Both controllers succeed in tracking the first change of the reference trajectory. However the second change in the reference trajectory is better tracked with LN-MPC than with L-MPC. With L-MPC there is clearly a steady state error. The reason for this is that the linear approximation of the gate equation is not accurate enough when the difference between the upstream and downstream water levels of the gate increases compared to the nominal

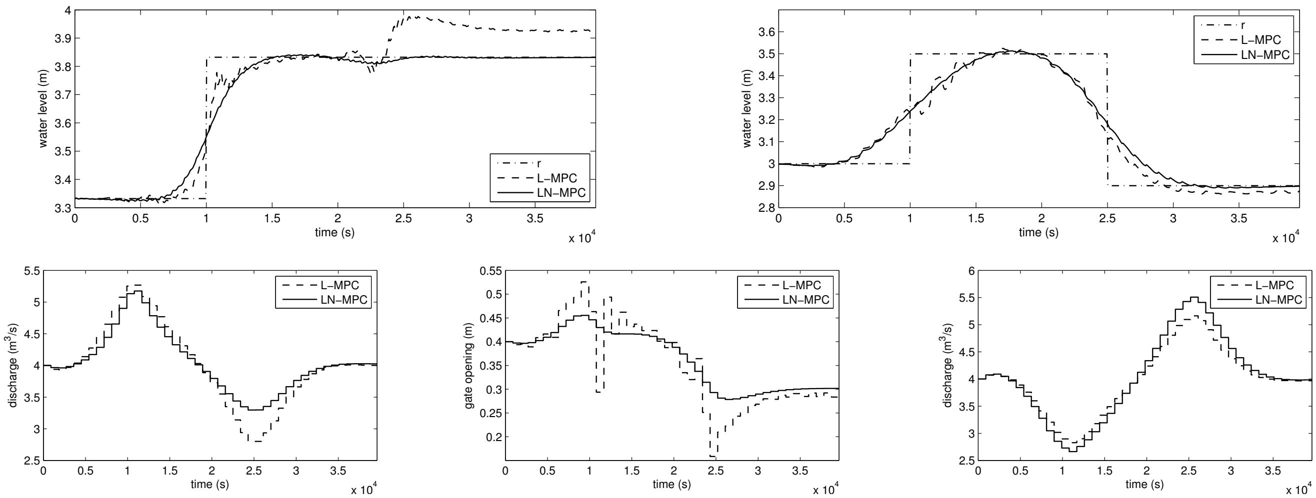


Fig. 2. Simulation results for set-point control. The top plots show the evolution of the most downstream water level for reach one (left) and reach two (right) together with the reference trajectory (dashed-dotted line). The bottom plots present the control actions: the upstream discharge (left), the gate position (middle) and the downstream discharge (right).

values. Since LN-MPC takes the gate equation into account separately, it can deal with set-point changes very well.

B. Disturbance rejection

This scenario checks how the controllers react when a small constant disturbance takes place. The goal here is to keep the most downstream water level of each reach as close as possible to its nominal value. The controllers can only use the downstream discharge and the gate. The upstream discharge is the disturbance signal. After 15000 s the upstream discharge jumps from 4 to 6 m^3/s (e.g. a gate upstream is opened). The results are visualized in Fig. 3. On top we have the results for reach one and below the results for reach two. As we can see the disturbance rejection for LN-MPC is much better than for L-MPC. For the first reach the deviation of the reference signal at the end of the simulation is only 0.08 cm with LN-MPC while it is 3.78 cm with L-MPC. For the second reach the deviation with LN-MPC is 0.41 cm while it is 0.56 cm for L-MPC.

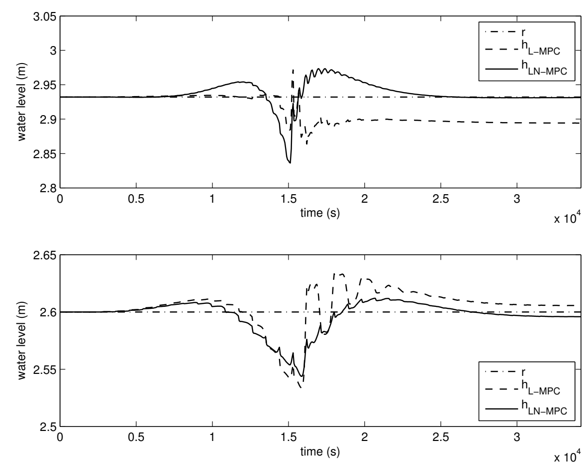


Fig. 3. Evolution of the most downstream water level of the first (top) and second (bottom) reach.

C. Flood control

The last scenario is very similar to the previous one, the only difference is that the disturbance signal is so large that there is risk of flooding. The disturbance is the upstream discharge signal. Fig. 4 shows the evolution of

$$m(k) = \max\left(\boldsymbol{h}(k) - \boldsymbol{h}_{\max}\right) \qquad (36)$$

for reach one (top) and reach two (bottom). A negative $m(k)$ means that none of the water levels violates the flood limit at time k. However, if $m(k)$ is positive, then the reach is flooding and $m(k)$ indicates the maximal violation of the flood level. Fig. 5 shows the control actions for both controllers. The top plot shows the evolution of the gate opening and the bottom plot the discharge at the end of the second reach (Q_{out}) together with the disturbance signal (dashed-dotted line). LN-MPC succeeds in keeping the water levels of both reaches below their flood limits which is not the case for L-MPC. There is a maximal flooding for the second reach of 3.89 cm. At the end of the simulation both controllers succeed in steering the water levels of both reaches back to their nominal condition restoring the buffer capacity.

VI. CONCLUSIONS AND FUTURE WORKS

In this paper we have shown that one single controller based on the principles of MPC can be used for controlling river systems in different scenarios such as reference tracking, disturbance rejection and flood control. We have tested two types of controllers: one controller based on a model where the gate dynamics are linearized around the nominal condition (L-MPC) and another controller where the gate dynamics are treated separately (LN-MPC). Both controllers

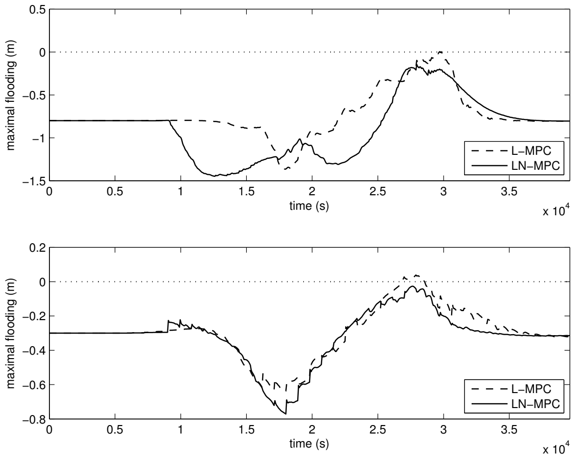


Fig. 4. Evolution of $m(k)$ for the first reach (top) and the second reach (bottom). A positive value indicates a violation of the flood limits.

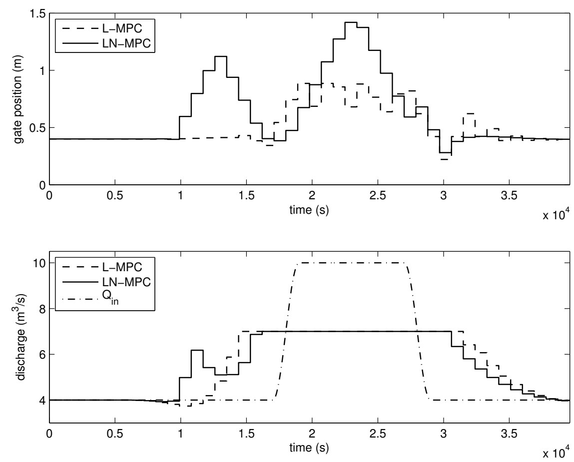


Fig. 5. Evolution of the control actions for the MPC controllers. The top plot shows the gate opening and the bottom plot the discharge Q_{out} at the end of second reach together with the disturbance signal Q_{in}.

achieve a good performance, however by treating the gate equations in a special way, LN-MPC outperforms L-MPC. For future work we will test LN-MPC for larger river systems consisting of multiple reaches, gates and junctions.

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