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Control of a single reach with model predictive control

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ABSTRACT: Many control strategies can be found in literature for controlling a reach. Most of these methods focus on set-point control such that the most upstream or downstream water level track a certain reference trajectory. One control strategy often applied in the literature is the Linear Quadratic Regulator (LQR). However as good as LQR is for set-point control, it cannot be used for preventing a river from flooding when large disturbances take place. In this paper we present a more advanced control strategy already used in other fields that can be used for set-point control as well as for flood control: Model Predictive Control (MPC). Simulation results show that MPC outperforms LQR for three different test cases.

1 INTRODUCTION

Several studies can be found in literature where control strategies are used to control reaches or irrigation canals, such as PI controllers, heuristic controllers, predictive controllers and optimal controllers (Malaterre et al. 1998; Burt et al. 1998; Litrico et al. 2006). The dynamics of such a reach are often described by two nonlinear partial differential equations, the so-called Saint-Venant equations. Because of computational reasons these controllers do not work directly with these equations but use approximating models, e.g. models based on the linearized Saint-Venant equations. One control strategy which works with such a linear model is the Linear Quadratic Regulator (LQR). This technique has been successfully applied in different studies for set-point control (Clemmens and Schuurmans 2004; Clemmens and Wahlin 2004; Malaterre 1998; Malaterre 1994; Balogun et al. 1988; Balogun 1985; Reddy 1990; Reddy et al. 1992). However, this controller cannot be used for flood control at the same time because the controller is not aware of the existence of flood levels.

This problem does not exist with Model Predictive Control (MPC). MPC (Rossiter 2003; Mayne et al. 2000) is a control strategy originating from the process industry and is used in various applications going from chemicals and food processing to automotive and aerospace applications (Qin and Badgwell 2003). Just as for LQR, one can find many studies in literature where MPC is used for controlling reaches, irrigation canals or river systems towards a certain set-point (van Overloop 2006; Ruiz and Ramirez 1998; Xu and van Overloop 2010; Xu et al. 2011; Wahlin and Clemmens 2006; Wahlin 2002). However these works do not focus on flood control. In previous work (Breckpot et al. 2010a; Breckpot et al. 2010b) we have been using a simplified conceptual model which models the water levels of a river system only at a very limited number of points for set-point control and flood control. In this work we use a linear version of the Saint-Venant equations with a very fine spatial discretization for designing the controllers.

The paper is organized as follows. Section 2 discusses the equations describing the dynamics of a single reach, their discretization and numerical implementation. Section 3 presents the linear approximation of the model equations. Section 4 explains LQR and Section 5 describes how MPC can be used for set-point control and flood control. Section 6 compares the performance of LQR and MPC for set-point control, disturbance rejection and flood control. Section 7 ends the paper with conclusions and future work.

2 THE SAINT-VENANT EQUATIONS

2.1 *Single reach dynamics*

The Saint-Venant equations are often used in practice to model the dynamics of the water levels and discharges in river reaches. The derivation of these equations are based on a set of assumptions which can be found in (Chaudry 2008; Sturm 2001; Chow 1959). Given these assumptions, the following two Partial

Differential Equations (PDEs) describe the dynamics of the water levels and discharges in a single reach:

$$\frac{\partial A}{\partial h}\frac{\partial h}{\partial t}+\frac{\partial Q}{\partial x}=0, \tag{1}$$

$$\frac{\partial Q}{\partial t}+\frac{\partial}{\partial x}\frac{Q^2}{A}+gA\frac{\partial h}{\partial x}+gA(S_{\mathrm{f}}-S_0)=0, \tag{2}$$

with A the cross-sectional flow area (m^2), Q the water discharge (m^3/s), h the water depth (m), g the gravity acceleration (m/s^2), S_0 the bed slope and S_{f} the friction slope. Equation 1 describes the conservation of mass and Equation 2 the conservation of momentum. S_{f} is an (empirical) resistance law approximating the head losses. The resistance law used in this study is the Manning relation (Chaudry 2008; Chow 1959):

$$S_{\mathrm{f}}=\frac{n_{\mathrm{mann}}^2 Q|Q|}{A^2R^{1/3}} \tag{3}$$

where n_{mann} is the Manning coefficient ($\mathrm{s}/\mathrm{m}^{1/3}$), $R=A/P$ is the hydraulic radius (m) and P is the wetted perimeter of the cross section (m).

2.2 *Discretization and numerical implementation*

Because in general an analytical solution cannot be found for the Saint-Venant equations, a numerical simulator is needed which approximates the infinite dimensional variables on a finite grid (Strelkoff and Falvey 1993; Xu et al. 2011). In this study, the partial derivatives are approximated with finite differences while the θ-method, e.g. $f(t_j+\theta\Delta t)=\theta f(t_j+\Delta t)+(1-\theta)f(t_j)$ with $\theta\in[0,1]$, is used for the time integration. The spatial grid used in this paper is a staggered grid:

where $2N-1$ is the total number of unknown variables (N water levels and $N-1$ discharges) and Q_{up} and Q_{down} are the (known) boundary discharges. The derivatives in Equation 1 are approximated by (note $h(x_i,t_j)=h_i^j$)

$$\frac{\partial h_i^j}{\partial t}\simeq\frac{h_i^{j+1}-h_i^j}{\Delta t}, \tag{4}$$

$$\frac{\partial Q_i^j}{\partial x}\simeq\frac{\theta(Q_i^{j+1}-Q_{i-1}^{j+1})+(1-\theta)(Q_i^j-Q_{i-1}^j)}{\Delta x}, \tag{5}$$

$$\frac{\partial A_i^j}{\partial h}\simeq\theta\frac{\partial A_i^{j+1}}{\partial h}+(1-\theta)\frac{\partial A_i^j}{\partial h}. \tag{6}$$

A similar approach is used for the terms $\partial Q/\partial t$, A, $\partial h/\partial x$ and S_{f} in Equation 2. The advection term $\partial(Q^2/A)/\partial x$ is approximated with an upwinding approach:

$$\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right)_i^j\simeq\begin{cases}\omega\left(\frac{Q^2}{A},i+1,j+1\right) & Q_i^j<0\\ \omega\left(\frac{Q^2}{A},i,j+1\right) & Q_i^j\geq 0\end{cases} \tag{7}$$

with

$$\omega\left(\frac{Q^2}{A},i,j\right)=\frac{\theta}{\Delta x}\left(\left(\frac{Q^2}{A}\right)_i^j-\left(\frac{Q^2}{A}\right)_{i-1}^j\right)+$$
$$+\frac{1-\theta}{\Delta x}\left(\left(\frac{Q^2}{A}\right)_i^{j-1}-\left(\frac{Q^2}{A}\right)_{i-1}^{j-1}\right). \tag{8}$$

In this way the two PDEs are transformed into a system of nonlinear equations:

$$\boldsymbol{f}\left(\boldsymbol{h}^{j+1},\boldsymbol{h}^j,\boldsymbol{Q}^{j+1},\boldsymbol{Q}^j\right)=0, \tag{9}$$

$$\boldsymbol{g}\left(\boldsymbol{h}^{j+1},\boldsymbol{h}^j,\boldsymbol{Q}^{j+1},\boldsymbol{Q}^j\right)=0, \tag{10}$$

with $f:\mathbb{R}^{4N+2}\to\mathbb{R}^N$, $g:\mathbb{R}^{4N+2}\to\mathbb{R}^{N-1}$, $h=(h_1,\dots,h_N)^T$ and $Q=\left(Q_{\mathrm{up}},Q_1,\dots,Q_{N-1},Q_{\mathrm{down}}\right)^T$, which has to be solved for $h_1^{j+1},h_2^{j+1},\dots,h_N^{j+1}$ and $Q_1^{j+1},Q_2^{j+1},\dots,Q_{N-1}^{j+1}$. The upstream and downstream discharges Q_{up} and Q_{down} are known at every time. They are given by the controller or are disturbance signals. Equations 9 and 10 can be solved with Newton's method.

A discussion about the choice of Δt and θ can be found in (Clemmens et al. 2005). However, we will not elaborate on this in this paper and set θ equal to 0.6.

3 THE LINEAR MODEL

The controllers used in this paper need a linear approximation of the Saint-Venant equations. One approach is to linearize Equations 9 and 10 around a steady-state point ($h_{\mathrm{ss}}\in\mathbb{R}^N$ for the water levels, $Q_{\mathrm{ss}}\in\mathbb{R}^{N-1}$ for the discharges, $u_{\mathrm{ss}}\in\mathbb{R}^{n_u}$ for the control inputs and $d_{\mathrm{ss}}\in\mathbb{R}^{n_d}$ for the disturbances with n_u and n_d the number of inputs and disturbances) resulting in the following model:

$$\Delta\boldsymbol{x}(k+1)=\boldsymbol{A}\Delta\boldsymbol{x}(k)+\boldsymbol{B}\Delta\boldsymbol{u}(k)+\boldsymbol{D}\Delta\boldsymbol{d}(k), \tag{11}$$

with $\Delta x(k)=\left(\Delta h(k),\Delta Q(k)\right)^T$, $\Delta h(k)=h(k)-h_{\mathrm{ss}}$, $\Delta Q(k)=Q(k)-Q_{\mathrm{ss}}$, $\Delta u(k)=u(k)-u_{\mathrm{ss}}$, $\Delta d(k)=d(k)-d_{\mathrm{ss}}$, $A\in\mathbb{R}^{(2N-1)\times(2N-1)}$, $B\in\mathbb{R}^{(2N-1)\times n_u}$ and $D\in\mathbb{R}^{(2N-1)\times n_d}$. The input vector $u(k)$ contains the controllable upstream and/or downstream river discharges at time instant k while the disturbance vector $d(k)$ contains the uncontrollable ones. E.g. if only the upstream discharge can be controlled, then we have that $u(k)=Q_{\mathrm{up}}(k)$ and $d(k)=Q_{\mathrm{down}}(k)$.

4 LINEAR QUADRATIC REGULATOR

4.1 *Set-point control*

It is desired to keep all or some of the water levels h and the discharges Q close or equal to its steady-state values h_{ss} and Q_{ss} with a minimum of control effort. This goal can be translated into the following cost function or objective function which we want to minimize

$$\sum_{k=0}^{\infty} \Delta \boldsymbol{x}(k)^T \underbrace{\begin{pmatrix} \boldsymbol{W}_h & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{W}_Q \end{pmatrix}}_{\boldsymbol{W}} \Delta \boldsymbol{x}(k) + $$

$$+ \sum_{k=0}^{\infty} \Delta \boldsymbol{u}(k)^T \boldsymbol{R} \Delta \boldsymbol{u}(k) \tag{12}$$

with $W_h \in \mathbb{R}^{N \times N}$, $W_Q \in \mathbb{R}^{N-1 \times N-1}$ and $R \in \mathbb{R}^{n_u \times n_u}$ three positive semi-definite diagonal penalty matrices. The higher the value of a diagonal element in any of these three matrices, the more important the corresponding variable is. E.g. if all the elements in W_h are much larger than the elements in W_Q and R, then the controller will mainly focus on steering the water levels as close as possible to its steady-state values.

The solution of the optimization problem with Equation 12 as cost function and with the linear model

$$\Delta \boldsymbol{x}(k+1) = \boldsymbol{A} \Delta \boldsymbol{x}(k) + \boldsymbol{B} \Delta \boldsymbol{u}(k) \tag{13}$$

as constraint, is the linear feedback law (Althans 1971; Kwakernaak and Sivan 1972; Franklin et al. 1997):

$$\Delta \boldsymbol{u}(k) = -\boldsymbol{K} \Delta \boldsymbol{x}(k) \tag{14}$$

where the feedback matrix K is given by

$$\boldsymbol{K} = \left(\boldsymbol{R} + \boldsymbol{B}^T \boldsymbol{S} \boldsymbol{B}\right)^{-1} \boldsymbol{B}^T \boldsymbol{S} \boldsymbol{A} \tag{15}$$

with S the positive definite solution of the discrete algebraic Riccati equation

$$\boldsymbol{S} = \boldsymbol{A}^T \left(\boldsymbol{S} - \boldsymbol{S}\boldsymbol{B} \left(\boldsymbol{R} + \boldsymbol{B}^T \boldsymbol{S} \boldsymbol{B}\right)^{-1} \boldsymbol{B}^T \boldsymbol{S}\right) \boldsymbol{A} +$$

$$+ \boldsymbol{W}. \tag{16}$$

If there is a rate of change constraint for the inputs or if the inputs have some operational limits, then they have to be enforced using a saturator such that $\Delta u(k)$ does not violate these limits.

The control actions given by Equation 14 will always try to steer the water levels and discharges back to their steady state values. However in practice, it can be important that the water levels and/or discharges follow a certain reference signal $r(k)$. In this paper this reference signal will only contain step changes. To ensure a zero steady-state error to this step input $r(k)$, it can be shown that the feedback control law has to be modified as follows:

$$\Delta \boldsymbol{u}(k) = -\boldsymbol{K} \Delta \boldsymbol{x}(k) + (\boldsymbol{N}_u + \boldsymbol{K} \boldsymbol{N}_x) \Delta \boldsymbol{r}(k) \tag{17}$$

with $\Delta r(k) = r(k) - r_{ss}$, $r_{ss} = \left(h_{ss}, Q_{ss}\right)^T$ and

$$\begin{pmatrix} \boldsymbol{N}_x \\ \boldsymbol{N}_u \end{pmatrix} = \begin{pmatrix} \boldsymbol{A} - \boldsymbol{I} & \boldsymbol{B} \\ \boldsymbol{I} & \boldsymbol{0} \end{pmatrix}^{-1} \begin{pmatrix} \boldsymbol{0} \\ \boldsymbol{I} \end{pmatrix} \tag{18}$$

and I the identity matrix.

4.2 *Flood control*

Just as the control law in Equation 14 does not know the limits on the control actions, the controller is also not aware of any possible upper limit (flood levels) on the water levels. Therefore it is really difficult to tune the weighting matrices W and R such that an LQR controller will keep the water levels below their flood levels for different disturbance signals. This problem can be avoided by using MPC.

5 MODEL PREDICTIVE CONTROL

5.1 *The working procedure of MPC*

Model Predictive Control (MPC) is an optimization based control strategy which makes use of a process model to predict the future process outputs within a specified prediction horizon. MPC solves an optimization problem over this horizon to determine the optimal inputs for the process taking into account input and output constraints, future disturbances and the process model. Only the first input sample of the complete optimal sequence is applied to the process, new samples are taken to determine the new current state of the system and the entire procedure is repeated.

5.2 *Optimization problem*

At every time step, the controller solves an optimization problem. In practice this optimization problem will often be a Quadratic Programming problem (QP). This is a convex minimization problem with a quadratic cost function and linear equality and linear inequality constraints. In our study, it has the following form:

$$\min_{\boldsymbol{u}(k),\boldsymbol{\zeta}} \quad \sum_{k=1}^{N_P} \|\boldsymbol{x}(k) - \boldsymbol{r}(k)\|^2_{\boldsymbol{W}} + \sum_{k=1}^{N_P - 1} \|\boldsymbol{u}(k) - \boldsymbol{u}(k-1)\|^2_{\boldsymbol{R}} + \|\boldsymbol{\zeta}\|^2_{\boldsymbol{V}} \tag{19}$$

$$\text{s.t.} \qquad \boldsymbol{x}(0) = \hat{\boldsymbol{x}}, \tag{20}$$

$$\Delta \boldsymbol{x}(k+1) = \boldsymbol{A} \Delta \boldsymbol{x}(k) + \boldsymbol{B} \Delta \boldsymbol{u}(k) + \boldsymbol{D} \Delta \boldsymbol{d}(k), \tag{21}$$

$$\boldsymbol{h}(k) \leq \boldsymbol{h}_{\max} + \boldsymbol{\zeta}, \tag{22}$$

$$\boldsymbol{u}_{\min} \leq \boldsymbol{u}(k) \leq \boldsymbol{u}_{\max}, \tag{23}$$

$$|\boldsymbol{u}(k) - \boldsymbol{u}(k-1)| \leq \Delta_u, \tag{24}$$

$$|\boldsymbol{u}(0) - \boldsymbol{u}_{\text{prev}}| \leq \Delta_u, \tag{25}$$

$$\boldsymbol{\zeta} \geq 0, \tag{26}$$

with N_P the prediction horizon, W, R and $V \in \mathbb{R}^{N\times N}$ three positive semi-definite diagonal weighting matrices, $\hat{x}$ the current state of the process, $h_{\max}$ the flood levels, $u_{\min}$ and $u_{\max}$ the operational limits on the inputs, Δ_u the maximal allowed rate of change for the inputs, u_{prev} the input applied to the reach in the previous time step and $\boldsymbol{\zeta} \in \mathbb{R}^N$ a vector of slack variables (one slack variable for each water level). It can be shown that for positive semi-definite weighting matrices, the QP has only one (global) solution (Nocedal and Wright 2000). In this study we use Mosek (Andersen et al. 2003) for solving the QPs.

Note that we did not implement the flood limits as hard constraints ($h(k) \leq h_{\max}$), but we used the vector of slack variables $\boldsymbol{\zeta}$ to implement them as soft constraints (Equation 22) together with a positivity constraint on $\boldsymbol{\zeta}$ (Equation 26) and the weighting matrix V. This makes sure that the QP will always be feasible (the hard constraints can sometimes be too restrictive for large disturbances). It is important to choose the elements in V larger than the elements in W and R. This forces the controller to keep the violations of the flood limits as small as possible or if possible equal to zero when a large disturbance takes place (flood control). If there is no risk of flooding, the controller sets $\boldsymbol{\zeta}$ equal to zero and the third term in the objective function is eliminated. Hence the controller will focus on steering the different states towards their reference signal (set-point control).

One big difference with LQR besides the incorporation of the flood levels is the linear model used in MPC (Equation 21). If the disturbances can be predicted in advance (e.g. based on rainfall predictions), this allows the controller to react on the future rainfall preventively which limits the risk of flooding. A last difference is that MPC knows the limits on the inputs (Equations 23–25) such that a saturator is not needed.

6 SIMULATION RESULTS

In this study we will control a single reach with LQR and MPC and compare their performance for three different test cases: set-point control, disturbance rejection and flood control. The reach has a trapezoidal cross section with a side slope of 0.5, it has a length L of 4000 m, the channel slope S_0 is equal to 0.0001, the bottom width is equal to 4 m and the Manning coefficient n_{mann} is taken equal to 0.014 s/m$^{1/3}$. Figure 1 visualizes some of these parameters. In all test cases the reach is initially in steady-state with a downstream water level equal to 3 m and a discharge of 1 m^3/s at every point along the reach. The linear model is derived based on this steady state condition. The reach is approximated with a grid structure with $N = 81$ water levels. For the inputs we have that $-u_{\min} = u_{\max} = 7$ m^3/s and $\Delta_u = 2$m^3/s. The controllers works with a sampling time of 5 min and the size of the prediction horizon N_P is 15. In all the three test cases we have assumed that we know at every time step the current state $\hat{x}$ of the process. The control actions defined by both controllers will be applied to the full nonlinear model discussed in Section 2.

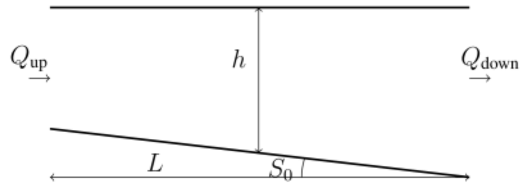

Figure 1. Schematic structure of the reach with Q_{up} and Q_{down} the discharges at the boundaries, h the water levels measured from the bottom of the reach, L the length of the reach and S_0 the channel slope.

The same weighting matrices are used for all the three test cases:

$$\boldsymbol{W}_h = \begin{pmatrix} 1 & & & \\ & \ddots & & \\ & & 1 & \\ & & & 100 \end{pmatrix}, \boldsymbol{W}_Q = 10^{-2} \times \boldsymbol{I}, \tag{27}$$

$$\boldsymbol{R} = \boldsymbol{I}, \boldsymbol{V} = 10^4 \times \boldsymbol{I}. \tag{28}$$

The elements of the different matrices are chosen such that flooding will be prevented (V) and that the controllers will mainly focus on steering the most downstream water level towards its reference value, and if possible also the other water levels.

6.1 *Set-point control*

The ability to track a reference trajectory for the water levels is tested for both controllers. There is no risk of flooding ($h_{\max}$ is set to ∞) and the controllers can control both the upstream and the downstream discharges. There are no disturbances.

Figure 2 shows the results for LQR on the left and for MPC on the right. On top we have the evolution of the water levels in space (x-axis) and time (y-axis) relative to the bottom of the reach (the plane in the bottom). The reference trajectory for all the water levels is shown by the thick lines with a step change after 3000 s and 13,000 s. The bottom figures show the applied control actions. The upstream discharge is the full line and the downstream discharge is the dashed line. Their limits are the dotted lines.

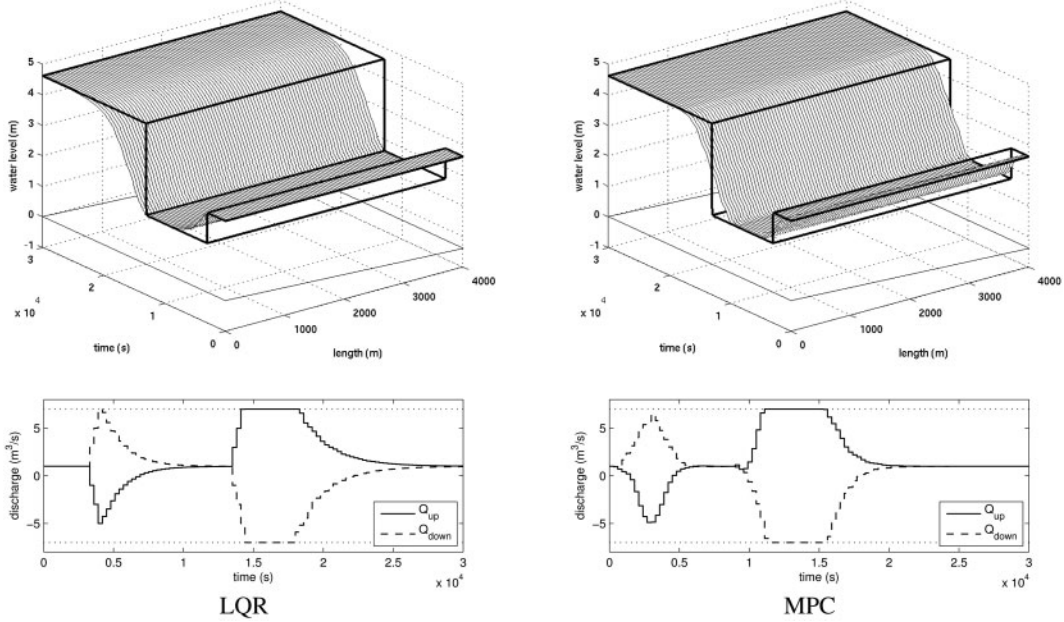

Figure 2. Simulation results of LQR (left) and MPC (right) for set-point control. The top plots show the evolution of the water levels (thin line) with the reference trajectory (thick line). The bottom plots present the control actions that correspond to the upstream (full line) and downstream (dashed line) discharges together with their limits (dotted lines).

We can conclude that both controllers succeed in tracking the reference trajectory. However because MPC sees the step changes in the reference trajectory earlier because of its prediction horizon, it reaches the new set-point much earlier. LQR only reacts when the step change effectively takes place. Both controllers satisfy the limits on the inputs, LQR because of using a saturator and MPC because it incorporates these limits in its formulation.

6.2 *Disturbance rejection*

This test case checks how the controller reacts when a disturbance takes place. The goal here is to keep the most downstream water level as close as possible to its steady state value. The controllers can only use the downstream discharge, the upstream discharge is the disturbance signal. After 6000 s the upstream discharge jumps from 1 to 3 m^3/s (e.g. a gate upstream is opened). Also here there is no flood level defined.

The results are visualized in Figure 3. On top we have the reference signal for the most downstream water level (thick line), the water level controlled by LQR (dashed line) and the water level controlled by MPC (solid line). The bottom figure shows the disturbance signal (thick) and the control action for both controllers (dashed line for LQR and solid line for MPC). It is evident that the MPC controller has better disturbance rejection capabilities than the LQR controller. At the end we have a deviation of 34 cm from the set-point for the LQR case, and a deviation of only 0.8 cm for the MPC controller.

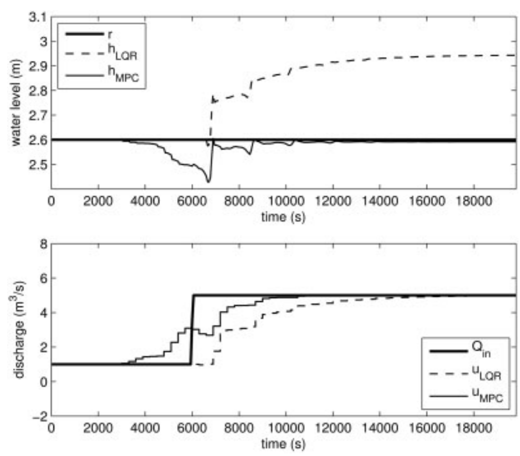

Figure 3. The top plot shows the effect of a disturbance on the evolution of the most downstream water level controlled via LQR (dashed line) and via MPC (full line) as well as its reference value (thick line). The bottom plot shows the control action of the controllers together with the disturbance signal (thick line).

6.3 *Flood control*

The last test case is very similar as the previous one, only now the disturbance signal is so large that there is a risk of flooding. The disturbance is the upstream discharge signal and only the downstream discharge

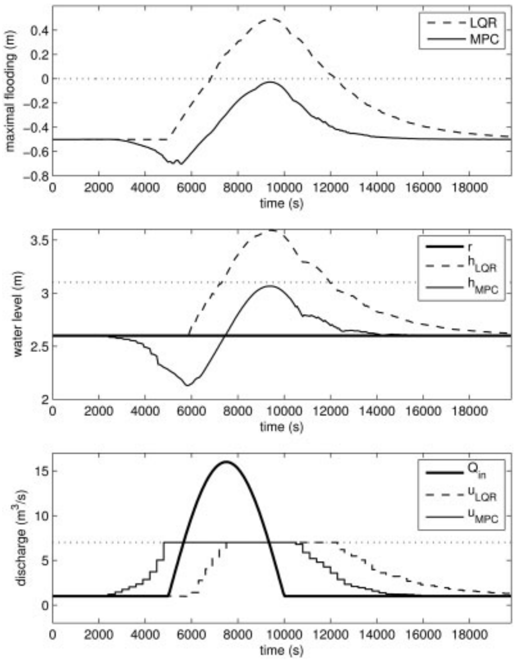

Figure 4. The top plot shows the evolution of $m(k)$ for LQR (dashed line) and MPC (solid line). The second plot shows the evolution of the downstream water level for LQR and MPC together with its reference value (thick line) and flood level (dotted line). The bottom plot shows the control actions as well as the disturbance signal (thick line).

can be controlled. The flood levels are taken equal to $h_{\max} = h_{\text{ss}} + 0.5$, the upstream discharge is equal to

$$Q_{\text{up}}(k) = \begin{cases} 1 \text{ for } k \leq 5000 \text{ or } k \geq 10,000 \\ 1 + 15\sin\left(\pi\frac{k-5000}{5000}\right) \text{ elsewhere} \end{cases}. \tag{29}$$

Figure 4 shows the results for the two controllers. The top plot shows the evolution of

$$m(k) = \max\left(\boldsymbol{h}(k) - \boldsymbol{h}_{\max}\right). \tag{30}$$

A negative $m(k)$ means that none of the water levels violates the flood limit at time k. However, if $m(k)$ is positive, then the reach is flooding and $m(k)$ indicates the maximal violation of the flood level. The middle plot shows the evolution of the most downstream water levels in combination with its reference value (thick line). The bottom plot shows the disturbance signal (thick line) and the control actions. All the results for LQR are visualized with dashed lines and for MPC with solid lines.

The top plot shows that LQR cannot prevent flooding: m becomes highly positive. The maximal violation of the flood levels is 0.5 m. This is not the case for the MPC controller where m remains always negative. The reason for this difference can be seen in the middle plot. Long before the increase in the upstream discharge takes place, the MPC controller steers the downstream water level below its reference value. MPC performs this preventive control action because the prediction horizon allows MPC to see the big increase in the upstream discharge before it actually happens. The bottom plot clearly shows how MPC reacts before the disturbance takes place, much earlier than LQR. This action decreases all the water levels and makes m more negative: the controller creates extra buffer capacity. After the disturbance has taken place both controllers succeed in steering the downstream water level back to its reference value.

The same conclusions can be made from Figure 5, where the evolution of all the water levels controlled by means of the LQR (left) and MPC (right) controllers are shown. Notice that unlike the LQR controller, MPC first lowers the water levels to increase the buffer capacity such that they remain at all time below their flood limits (the dashed thick lines).

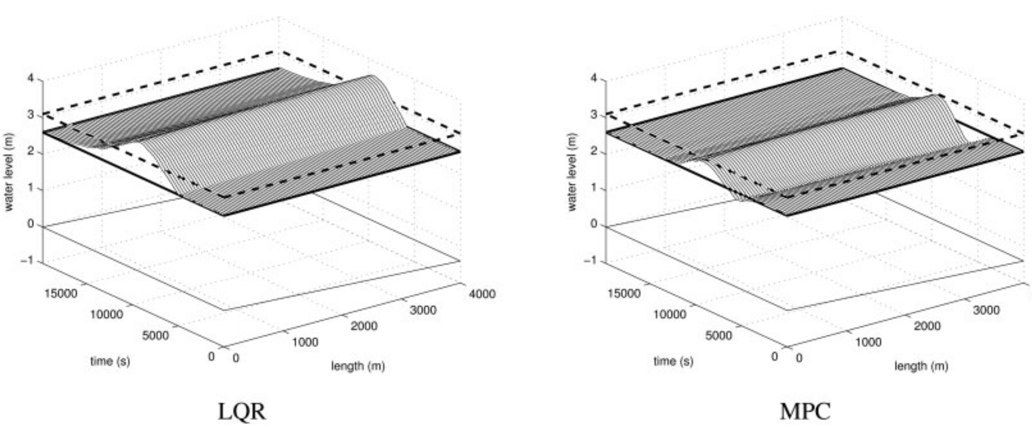

Figure 5. Evolution of the water levels when a large disturbance is applied to the system (test case: flood control). The left plot presents the results obtained with LQR and the right one the results obtained with MPC. The reference is visualized via the thick solid lines and the flood limits via the thick dashed lines.

7 CONCLUSIONS

In this paper we have explained how LQR and MPC can be used for controlling a river reach. The performance of both controllers have been tested and compared. The results showed that the same MPC controller can be used for both set-point control and flood control. For a large disturbance signal the MPC controller prevented the reach from flooding by creating extra buffer capacity. This was not the case for LQR that gave good results for set-point control, but it was not able to prevent the reach from flooding.

In future work we will test MPC on river systems with multiple reaches, gates and junctions. Furthermore, an estimator will be added to the current control scheme in order to estimate the discharges and water levels of a river system from a small number of measurements.

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