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How Can Different Interpretations and Use of Bond Graph Parameters be Avoided?

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Abstract

Since its invention by H.M. Paynter in the late 50s, a bond graph has become a powerful tool in simulation, analysis and design of multidisciplinary physical systems. Still, there exists no full standardization of the graphical representation of a bond graph. As long as these different representations do not result in misinterpretations of the meaning of a bond graph, no further standardization is needed. However, the success of the bond graph method urges for an elimination of some ambiguities. This paper is mainly devoted to the use and interpretation of bond graph *parameters*. It will be shown that the so called *common use* of some bond graph parameters is ambiguous. A non-ambiguous use and interpretation of all conventional bond graph symbols is proposed in this paper.

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List of Figures

1 Introduction

In the bond graph bible [5] and in early literature [6, 10], the bond graph language was defined *ad hoc*. It was the time that few people used this formal language, and only few research centers believed in the power of this rather new tool. Today hundreds of people are interested in this structured modeling method. Consequently some lacks in the language definition result in different interpretations. These interpretations should be avoided. To be more specific, it is a well known fact that not all bond graph parameters are treated alike. Especially transducer parameters tend to be interpreted in an ambiguous way, as illustrated in for example [8]. But beside stating that problem, no proposition is made in the existing literature to solve it. In this paper it will be shown that simple pre-causal definitions facilitate an unambiguous treatment of all bond graph parameters. This results in a more structured and comprehensive bond graph.

The paper is organized as follows: In Section 2, the problem statement is explained through examples. In Section 3, a possible solution, based on a generalization of the definition of bond graph parameters, is proposed. In Section 4, the proposed generalization is used in a simple mechanical example and exploited in a new structured bond graph modeling method, and in the last Section some concluding remarks are given.

2 Problem statement

2.1 The transformer

A bond graph transformer component, shown in *normal form* in Figure 1 (a), can be defined as

$$e_1 \overset{\text{def}}{=} m \cdot e_2 \tag{1}$$

with e_1 and e_2 the effort variables at both power ports, and m the *transformer modulus*. In the bond graph bible [5] the same equation is used to notify the behavior of a transformer, not to define it [7]. Since it also obeys *power conservation*, i.e.

$$\sum P_i \overset{\text{def}}{=} 0 \longrightarrow P_1 - P_2 = 0 \tag{2}$$

(observe the power arrows at the transformer *node*) it turns out that

$$f_2 = m \cdot f_1 \tag{3}$$

with f_1 and f_2 the corresponding flow variables at the power ports. The use of these two *definitions* and the *normal* graphical representation of Figure 1 (a) has two consequences. First, clearly only one transformer modulus exists for describing both the effort and flow behavior. To represent this, the transformer modulus m is put on the semicolon at the TF symbol. According to Eqns. (1) and (3) the parameter m can be described as *an effort and flow gain* of the transformer. Second, every two-port transformer has *always* exact one power arrow (harpoon) pointing towards the transformer field, and exact one power arrow pointing outwards of the transformer field. In Subsection 3.2. this aspect will be of crucial importance.

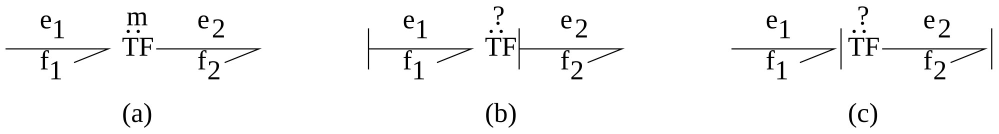

Figure 1: A transducer of the transformer type, shown in a-causal and some causal forms

Problems arise when Eqn. (1) isn't used as a definition or when the transformer is not in normal form, as will be shown in the sequel.

2.2 Invoking causality

Due to the definition of Eqn. (1), a transformer is causally constrained. Figures 1 (b) and (c) show some possible causal bond graphs of the two-port transformer. The causal arrow (effort bar) shows explicitly the direction of the effort variable of the corresponding bond. The question can be raised what kind of parameter symbol do we have to put on the semicolon of the causal TF. Many people use the following two statements for that:

1. In Figure 1 (b) e_1 is a calculated variable according to the causality involved. Following the definition of Eqn. (1), this results in

$$\begin{cases} e_1 = m \cdot e_2 \\ f_2 = m \cdot f_1 \end{cases} \tag{4}$$

and hence a parameter m, the (effort and flow) gain, is put on the semicolon of the TF.

2. In Figure 1 (c) e_2 is a calculated variable according to the causality involved. Following the same definition of Eqn. (1), this results in

$$\begin{cases} e_2 = \frac{1}{m} \cdot e_1 = n \cdot e_1 \\ f_1 = \frac{1}{m} \cdot f_2 = n \cdot f_2 \end{cases} \tag{5}$$

and hence a parameter n (the inverse of m) is put on the semicolon of the TF as new (effort and flow) gain.

As long as one knows which parameter (m or n) is used in the bond graph, no problems occur. In fact errors are made when people tend to mix up the two interpretations, i.e. when the parameter n is used above the semicolon of the TF in Figure 1 (b).

3 How can we avoid different interpretations?

3.1 An unambiguous solution

One possible solution to avoid misinterpretations of this bond graph parameter is to write down, beside the bond graph, the meaning of the transformer parameter by means of its two assignment statements.

This has been shown in figure 2 for one type of causality. Such a visualization defines the transformer parameter in an unambiguous way. However there are several drawbacks. First of all, causality has to be imposed before the gain t can be defined properly. This causality of the TF can change during design, and consequently the accompanying assignment statements has to be rewritten to avoid misinterpretations of the parameter t. Second, people tend to forget to write down these accompanying assignment statements, and hence the parameter t becomes ambiguous and has to be re-interpreted afterwards. Third, interpretation difficulties

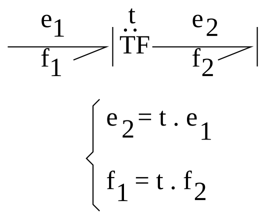

Figure 2: Explicit notation of the transformer behavior and parameter after causality assignment

occur when the reciprocal of t is used either in the bond graph or in the equations. And last but not least, the used method isn't compact at all, where precisely compactness is one of the main powers of a bond graph representation. So, if one tries to avoid writing down equations beside the bond graph, a definition must be given of (1) *what* parameter will be written down on the semicolon of the TF, and (2) what the *meaning* of that parameter is. This is explained in some detail below.

3.2 In search for general and simple pre-causal definitions

3.2.1 The linear transformer case

First, the parameters of a transformer, m and n, have to be seen as *symbols* obeying a different definition:

$$\begin{cases} m \stackrel{\text{def}}{=} \dfrac{e_1}{e_2} \\ n \stackrel{\text{def}}{=} \dfrac{1}{m} = \dfrac{e_2}{e_1} \end{cases} \tag{6}$$

Consequently their use is constrained in some way. Note that similar things can be said of the effort and flow *symbol*, e resp. f. We don't use e instead of an f ....

Second, in order to be consistent and general, we must treat *all* bond graph parameters alike. The proposed generalization is based on the power port equivalence, i.e. every bond graph parameter will be determined by observing the power port characteristics. Therefore we first observe a storage component and a dissipation component.

3.2.2 The linear storage component case

A 1D translation inertia is shown in Figure 3.

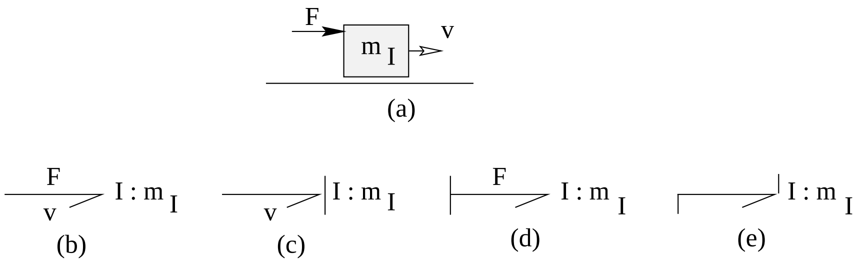

Figure 3: A 1D mechanical inertia and its several possible bond graph representations

The constitutive relation of such an inertia (assume no relativistic effects) is given by Newton's law

$$F = m_I \cdot \dot{v} \tag{7}$$

with m_I the mass of the inertia. The bond graph representation of this field is shown in Figure 3 (b). An important remark is that the reference power direction shown in the bond graph is directed towards the inertia field. This is due to the fact that the reference power variables have the same direction in the IPM (Ideal Physical Model) of Figure 3 (a). In [5] this is called the *inward sign convention* since the reference power is flowing *to* the component. This representation remains the same for every possible causal form of the inertia field, some of which are shown in Figure 3 (c) to (e). The first one represents the integral form, the second one the differential form, and the third one is a bicausal form [4]. The integral form can be used in the derivation of block diagrams and state space models. The second one is effective in transfer function generation by means of the Mason loop rule, or in (inverse) system analysis. The bicausal form has benefits in parameter estimation applications. The reason for such an identical parameter representation is simple: Bond graph modeling is observing energy interactions and constitutive relations. These things aren't changing when a causal construct is imposed on the bond graph, hence there is no need to change a parameter or symbol or direction of the reference power (variables) during causal analysis. The only thing that changes is the assignment statement of the inertia (i.e. the way of calculating the behavior of the inertia in a certain application). For the integral causal form, Figure 3 (c), this is

$$v := \int \frac{1}{m_I} \cdot F \cdot dt + v(0) \tag{8}$$

For the derivative causal form, Figure 3 (d), this is

$$F := m_I \cdot \dot{v} \tag{9}$$

And for the bicausal form, Figure 3 (e), this is

$$m_I := \frac{F}{\dot{v}} \tag{10}$$

No other symbol than m_I is used in the bond graph because it simply does not appear in the constitutive relation, Eqn. (7). Consequently this equation can be seen as a definition of the inertia parameter m_I. However, regarding the conservation laws (change of stored kinetic energy and change of linear momentum impose time aspects -integration!-) it seems a better choice to prefer the integral form of Eqn. (8) as a definition. Hence the following is proposed

$$\underbrace{v}_{\substack{\text{intensive}\\ \text{variable}}} \stackrel{\text{def}}{=} \frac{1}{m_I} \cdot \underbrace{\int F \cdot dt}_{\substack{\text{extensive}\\ \text{variable}}} + v(0) \tag{11}$$

The inertia parameter can then be defined as the reciprocal of the factor before the integral in Eqn. (11). The integral definition of a linear storage parameter can be generalized towards all other types of linear storage components. Taking into account the thermodynamic assignment of the power and energy variables, one can define the general *storage_parameter* as

$$storage\_parameter \stackrel{\text{def}}{=} \frac{extensive\_variable}{intensive\_variable} \tag{12}$$

The generalized extensive variables are p (momentum) and q (displacement). The corresponding generalized intensive variables are f (flow) and e (effort).

3.2.3 The linear dissipation component case

Another typical example is the electrical resistor, shown in Figure 4.

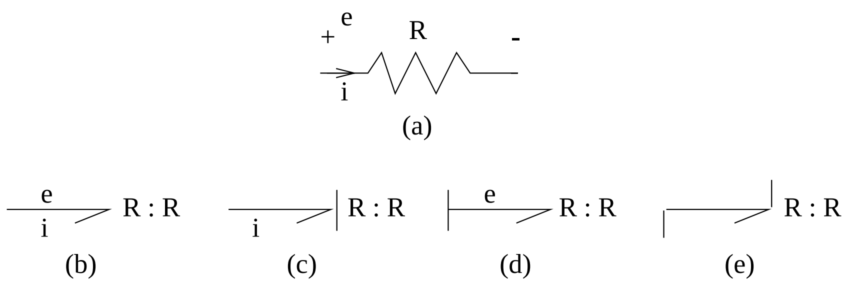

Figure 4: An electrical resistor and its several possible bond graph representations

Due to the *inward sign convention* there is a one-to-one resemblance between the directions of the reference power variables in the IPM and the reference power in the bond graph. The constituent relation of this resistor is defined as

$$e \stackrel{\text{def}}{=} R \cdot i \tag{13}$$

and consequently, the dissipation parameter is fixed as

$$dissipation\_parameter \stackrel{\text{def}}{=} \frac{effort\_variable}{flow\_variable} \tag{14}$$

The assignment statement depends on the causal construct imposed on the resistor. For the resistance causal form shown in Figure 4 (c), this is

$$i := \frac{1}{R} \cdot e \tag{15}$$

For the conductance causal form shown in Figure 4 (d), this is

$$e := R \cdot i \tag{16}$$

And for the bicausal form, shown in Figure 4 (e), this is

$$R := \frac{e}{i} \tag{17}$$

Again, no other direction of the reference power variables or other parameter symbol than R is used in the bond graph. Even when this effect is calculated as a conductance (Figure 4 (c)) *no* parameter $\frac{1}{R}$ or conductance parameter $G = \frac{1}{R}$ is used in the bond graph. This convention is also observed in electrical circuit theory [2]. The same holds for the bicausal form Figure 4 (e).

3.2.4 The general nonlinear case

If components behave nonlinear, nonlinear generalizations of the parameter definitions can be made. Consider for example the inertia, shown in Figure 3. For the more general nonlinear case the storage parameter becomes a function, and hence its definition is somewhat more difficult. However, for storage components preserving the energy conservation principle internal modulation is the only possible nonlinear effect. Hence Eqn. (12) can be generalized as

$$storage\_parameterfcn(extensive\_variable) = \frac{extensive\_variable}{intensive\_variable} \tag{18}$$

where $storage\_parameterfcn(extensive\_variable)$ has to be seen as a function of the $extensive\_variable$. For convenience the example of the relativistic point mass will be given. Remark first that a nonlinear generalization of Eqn. (7) is not preferable. Therefore it is necessary to observe the more general momentum law $F = \dot{p}$, with $p = m_I \cdot v$ the momentum. In classical mechanics m_I represents the (constant) mass, in relativistic mechanics m_I represents the relativistic mass m_{rel}, see [3]:

$$m_{\mathrm{rel}} = m \cdot \sqrt{1 + \frac{p^2}{m^2 \cdot c^2}} = m_{\mathrm{rel}}(p) \tag{19}$$

where m is the rest mass and c is the velocity of light. Hence Figure 3 can also represent the (nonlinear) relativistic case. The only difference with the linear case is that m_I is no more a constant parameter but a function depending on the extensive variable p of the inertia field, and hence should be replaced with for example $m_I(p)$.

3.2.5 The transformer revisited

Now, let's take again the transformer shown in Figure 1. It is obvious that the same conditions hold for that component. To be more specific: At each power port there must be a one-to-one resemblance of the directions of the reference power variables in the IPM and the reference power in the bond graph, and every bond graph representation of the transformer (causal or not) uses the same symbol m as transformer parameter. Never a parameter n or $\frac{1}{m}$ occurs. This way of dealing with bond graph parameters makes it possible to treat every field parameter alike. They are all defined pre-causally and hence are treated alike. Remark that the transformer is depicted with the *through sign convention* and not with the *inward sign convention* [5, 9]. As already been shown in Section 2, this has the benefit of having a single transformer parameter.

Since we don't know yet what side of the two-port transformer has the label 1 and 2 (these labels appear in the definition as indices on the power variables e and f), another problem arises. In fact there is in reality nothing like a power port 1 and a power port 2. A simple example of this is a lever mechanism, shown in Figure 5.

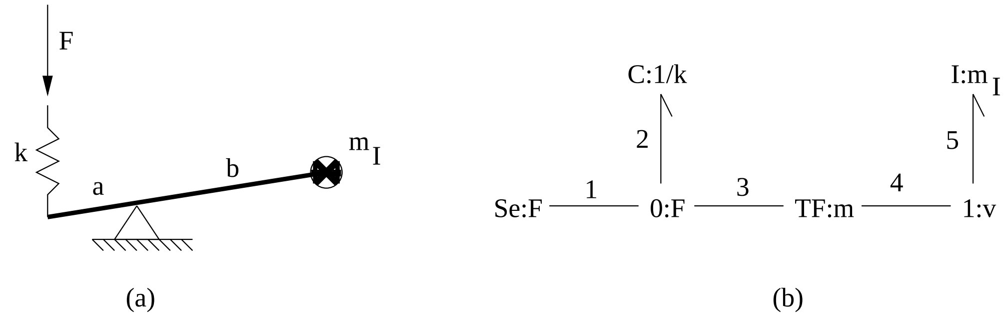

Figure 5: A lever mechanism connecting a mechanical spring with an inertia

Since no reference directions are shown on the IPM, no power arrows on the bonds of the transformer can be drawn [9]. Strictly spoken this is even not possible for the storage components. However, it is assumed that the inward sign convention is applied to these one-ports. In such a bond graph the bond numbers of the transformer power port differ from the indices in the definition of Eqn. (1). However always one can make a physical distinction between the two power ports of the transformer. The *first* power port represents in the definition the one with a (dummy) index 1, the *other* with a (dummy) index 2. Thus, if one defines power port 1 (i.e. with dummy index 1) as the bond with the power arrow directed towards the transformer field and if the transformer is in normal form, then the interpretation difficulties for non-causally augmented and causally augmented transformers are solved. Consequently the only thing one has to know of the power ports of the lever (labeled in the bond graph as 3 and 4) is, which one is the *first* one? This in fact is well defined only when reference directions of the power variables are imposed on the IPM of the lever.

Let's illustrate this by observing the bond graph of a hydraulic cylinder, shown in Figure 6 (a). The two possible normal forms are shown in Figure 6 (b) and (c). In Figure 6 (b) the reference power entering the transformer field stems from the hydraulic side, and the reference power leaving the transformer stems form the mechanical side. Thus, according to Eqn. (1) $e_1 = p$, $e_2 = F$, and $m = \frac{1}{A}$. Similarly in Figure 6 (c), it holds that $e_1 = F$, $e_2 = p$, and $m = A$. However, it should be clear that the IPM of both bond graphs differ in the used reference direction of the power variables. Let's focus on Figure 6 (b). Because the reference power of the hydraulic part is directed towards the transformer, the reference directions of the power variables at the hydraulic side are directed in the same sense. And because the reference power of the mechanical part is directed outwards the transformer, the reference directions of the power variables at the mechanical side are directed in opposite direction. From this point of view the bond graph of Figure 6 (b) can correspond with the IPM shown in Figure 6 (d), (f), (h) or (j). In order to determine which one is the right one, one has to consider the two *embedded constraint equations*, i.e. the effort equilibrium equation

$$\sum e_i = 0 \longrightarrow +p - \frac{F}{A} = 0 \longrightarrow p = \frac{1}{A} \cdot F \tag{20}$$

and the flow continuity equation

$$\sum f_i = 0 \longrightarrow +Q - A \cdot v = 0 \longrightarrow Q = A \cdot v \tag{21}$$

The reader who is not familiar with these two embedded constraint equations of a transducer is referred to Appendix A. Hence, Figure 6 (d) or (j) correspond with the bond graph.
Similarly the bond graph of Figure 6 (c) corresponds with the IPM shown in Figure 6 (e) or (k).

Such an IPM, augmented with reference power variable directions, is called an *augmented IPM*.

Observe that it should be clear that the reference direction of the effort and flow variables of the augmented IPM imposes constraints on the direction of the reference power of the bond graph of the transformer and the transformer modulus, and not on the sign of the instantaneous power exchange at the ports of the transformer. This last one depends solely

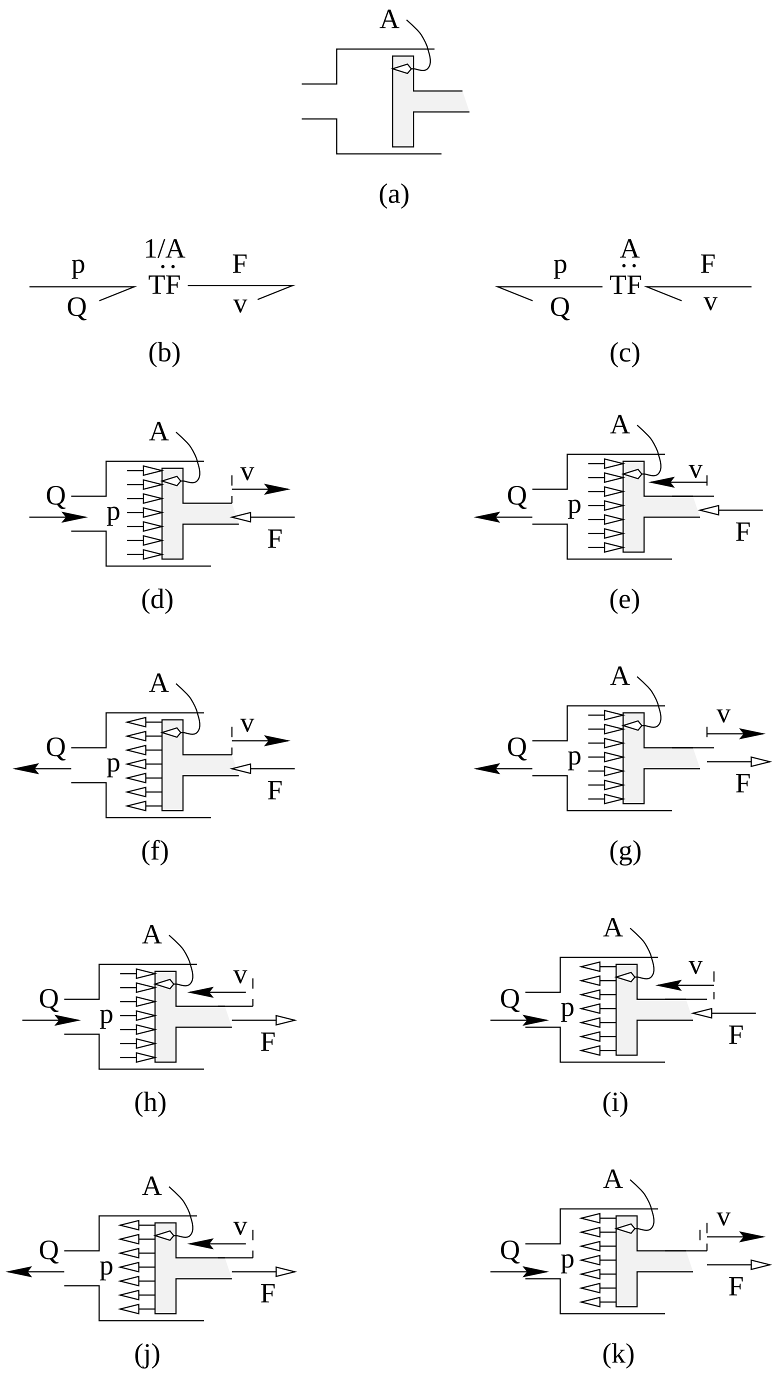

Figure 6: A hydraulic cylinder (a). In both the left and right column there are only two of the four physical models representing the bond graph on top

on the sign of the instantaneous power variables, since $P_i(t) = e_i(t) \cdot f_i(t)$, with i the port number, and t representing the time dependency. This last fact is sometimes used to show the perfect *two-way behavior* of a transformer, which is for some bond graphers a reason to avoid pre-causal definitions of transducer parameters. But in practice, the only thing one has to know if one uses such pre-causal definitions is an augmented IPM.

Note that this method of assigning in the bond graph the reference power direction and the corresponding field parameter by observing the reference power variables of the augmented IPM is valid for every 'power port' of any bond graph component. So two port storage components are for that aspect of modeling treated alike.

Summarizing, one can conclude that the reference power variable directions of the augmented IPM can define *uniquely* the reference power arrows and the field parameter. This fact is also a reason to use pre-causal definitions.

Observing the reference power variable directions, it is possible that a two port transducer has both reference power directed towards or outwards the transformer field. Because this representation of the transformer is not a normal one, a major problem occurs. The next subsection deals with that issue.

3.3 Abnormal transducer representations

3.3.1 The transformer case

Let's take again the ideal hydraulic cylinder, now augmented with reference power variables, shown in Figure 7 (a). Due to the pre-defined power variable reference directions in the mechanical and hydraulic domain the modeler seems to have a transformer with all the reference power arrows pointing towards the transformer, as shown in Figure 7 (b). Although this representation is not in contradiction with the second definition of a transformer, Eqn. (2), the unique transformer modulus m does not exist any more. Since the two power ports have their reference power directed towards the TF, Eqn. (1) cannot be used any more to define the modulus. Moreover, if one tries to define the transformer modulus with the embedded

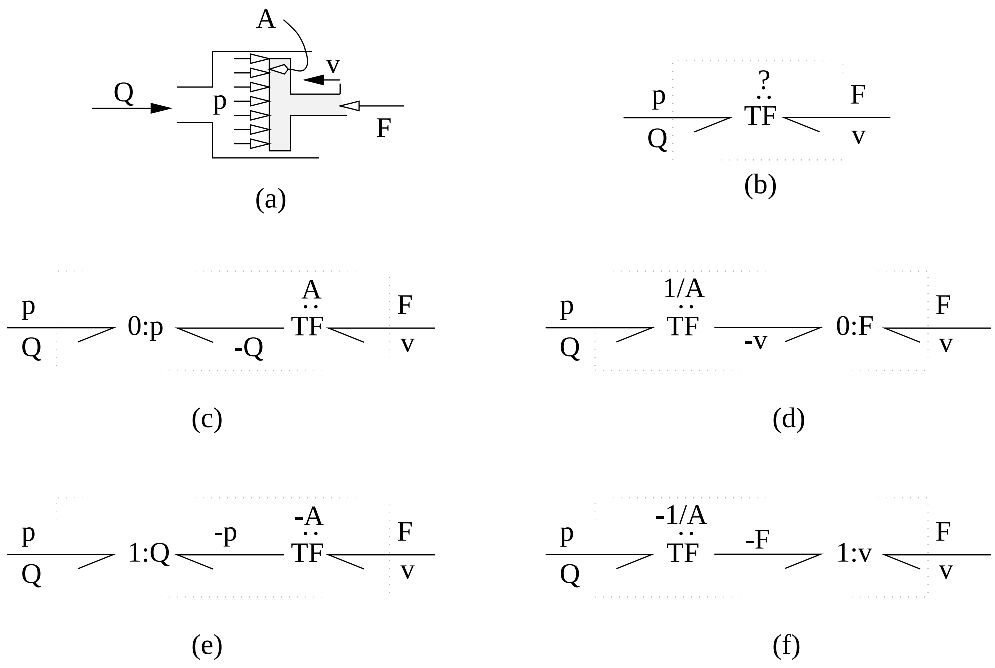

Figure 7: A hydraulic cylinder, revised. Since the IPM forces the TF to have the two power arrows directed towards the TF field, a unique modulus fails to describe the transformer behavior.

constraint equations (Eqns. (20) and (21)) applied on the augmented IPM, i.e.

$$Q = (-A) \cdot v \tag{22}$$

$$p = \left(\frac{1}{A}\right) \cdot F \tag{23}$$

it turns out that the effort and flow gain are different. To be able to draw the normal representation with one unique modulus this transformer representation has to have one of the power arrows pointing in the other direction. Several solutions are possible. For example, the reference scheme of the effort or flow variable in the mechanical or hydraulic domain of the IPM can be changed by reversing one of them. However this could influence other parts of the IPM and is therefore from a practical point of view not always recommended. Another method is introducing a zero junction in the bond graph of the transformer. This last solution, shown in Figure 7 (c), makes it possible to reverse *internally* the sign of the reference flow variable Q since, at that junction, it holds that $\sum f_i = 0$. This method can be called *tearing* *a junction out of the transducer.* Tearing makes it possible to preserve the one to one relation of the reference power variables in the IPM and in the bond graph, shown in the bond graph outside the dashed box. After this tearing has been done the transformer representation is in normal form, and hence the modulus m can be calculated from the definition of Eqn. (1), and written down into the (a-causal or causal) bond graph.

Remark that this tearing is not unique. The four possible teared bond graphs representing the same augmented IPM are shown in Figure 7 (c) to (f). Summarizing, in order to obtain an unambiguous representation of the transformer of Figure 7 (b) one can choose between four possibilities. It is a good habit to select the most natural one, which in this case is the one with positive modulus, Figure 7 (c) or (d).

3.3.2 The gyrator case

Analog remarks can be made for the gyrator transducer type, shown in normal form in Figure 8 (a). This bond graph gyrator component can be defined as

$$e_1 \stackrel{\text{def}}{=} r \cdot f_2 \tag{24}$$

with e_1 and f_2 the effort and flow variables at both power ports, and r the *gyrator ratio.* Note that in fact the ratio symbol r originates from some similarities of its definition with the constitutive relation of a resistor, Eqn. (13).

Since this is also a transducer it obeys power conservation, Eqn. (2). Hence the meaning of the *first* and *second* power port becomes redundant, i.e. both the power port with dummy index 1 and dummy index 2 can serve as *first* power port. This is illustrated in Figure 8.

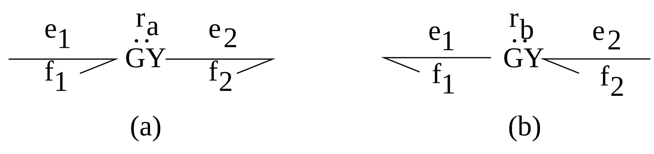

Figure 8: A gyrator representing effort to flow constraints

From this figure one can readily prove that $r_a = r_b$. However, the augmented IPM of both gyrators, i.e. including the reference power variable directions, differs. As a result, in contrast to transformers, different tearing of abnormal gyrator representations will not inverse the gyrator ratio r in the bond graph, it can only change the sign of the ratio. This is graphically shown in Figure 9. The IPM shows a current carrying conductor of length l in a magnetic

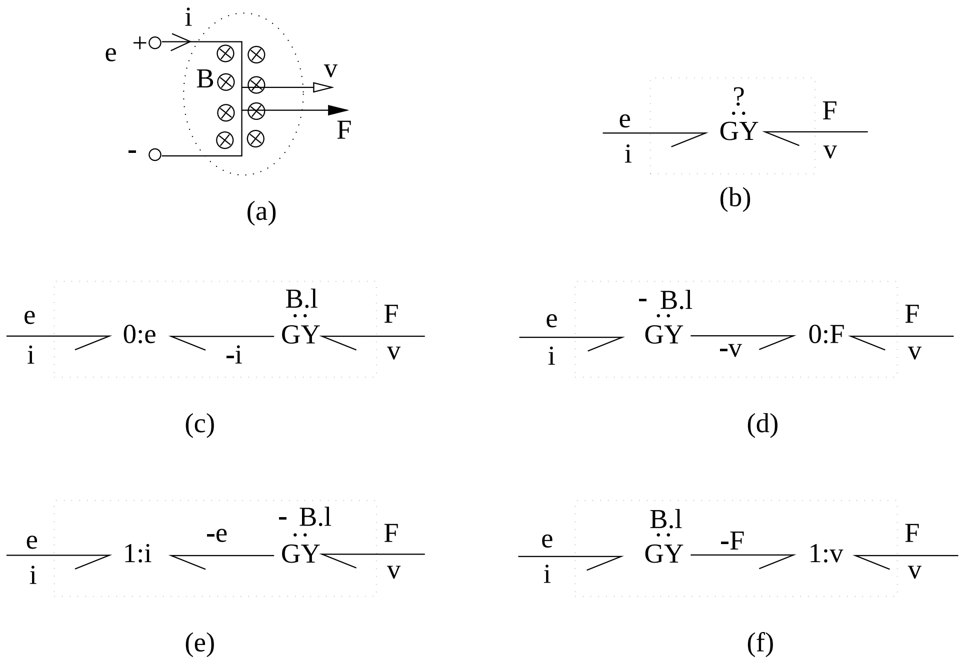

Figure 9: A gyrator representing the power continuous electro-mechanical energy conversion (a current in a magnetic field). The IPM forces to have the GY pointing its two power arrows towards the GY field, hence a unique ratio can not be determined

field B ($\mathbf{B}$ is pointing into the paper). In this figure the reference power variables acting on the conductor are determined by the environment and hence, cannot be inverted. The original bond graph is shown in figure 9 (b). The embedded constraint equations (Eqns. (20) and (21)) can be derived from the augmented IPM and the well known equations of Faraday-Lenz and Lorentz,

$$e = (B \cdot l) \cdot v \tag{25}$$

$$F = (-B \cdot l) \cdot i \tag{26}$$

revealing a non unique ratio. To conclude, in Figure 9 (c) to (f) the four possible non ambiguous bond graphs of this conductor are shown.

3.4 Some remarks

1. Only when one uses a dual bond graph of the original one, one can argue the existence of the reciprocal parameters G $(= \frac{1}{R})$, n $(= \frac{1}{m})$, and k $(= \frac{1}{r})$. In any case it is not good to mix the parameter symbols and their inverse. So let's forget G (as conductance), and n and k (as modulus and ratio), and keep it simple.

2. In a lot of standard textbook and papers of bond graphs no such definitions as Eqns. (1), (18), (13) or (24) are made. As long as no subsequent errors result from this, one can argue that what is said here, is a purely *academic discussion.* However the benefits of such pre-causal definitions are manifold:

   (a) The bond graph gets more informative and remains compact, resulting in a graph that is *denser.*

   (b) Nothing has to be *changed* on the a-causal bond graph during causal augmentation. Everything that should be known after imposing reference directions and before causal analysis can be written down into the bond graph. This information remains in the bond graph with the same original meaning. It is excluded that somewhere in the modeling and design process of a physical system the bond graph parameters become ambiguous. This could be called some kind of *knowledge conservation* (similar to other more physical conservation laws like energy, momentum, ...) representation in the bond graph. As will be shown in the next section, knowledge conservation makes bond graph modeling even more structured.

   (c) The bond graph gets *more portable* to other bond graph people. No extra (redundant) equations are needed to explain a typical parameter. All the parameters are well defined and hence, well understood.

3. Multi-ports transducers have to be decomposed into scalar two ports. This could be a drawback, but not for most practical engineering problems.

4. Tearing can be done in several ways, some of which are not very appropriate with respect to the physical interpretation of the transducer parameter.

A drawback of this method is that it fails to determine uniquely the field parameter of components having more than two power ports, except for 1- and 0-junctions of which the field parameter is the common flow or common effort. In that case it seems unavoidable to number every power port in order to define properly the field parameter. In design objectives however, numbering is error prone since the power port number of the scalar power port can change during design, and hence influence the field parameter. If, in such cases the designer wants to proceed with the more topological determination of the field parameters, one has to stick with at most two-port components (except for 1- and 0-junctions). Therefore a scalar realization of a multi-port field is the only solution to proceed with that method. Such a realization can be based on a canonical decomposition [1]. It should be noted that such a realization can introduce unwanted algebraic loops [9], which in turn influences the ease of deriving a mathematical model. However, since multi-port components with more than two power ports do not occur frequently in practical engineering design problems this drawback is not really a problem.

4 A last example

Many bond graph books and papers are confusing in their parameter interpretation. The bond graph bible [5, pp.48] shows a common situation. In Figure 10 (a) and (b) the augmented IPM and the proposed bond graph are redrawn.
This bond graph seems all right since we all know that such a transducer is a transformer. The question can be raised, however, why we do not take full advantage of this knowledge before causal assignment? This can be done as follows: From the IPM one can clearly see that such a system can be represented as a transducer TD, graphically shown in Figure 10 (c). In this figure the reference power arrows are directed according to the reference power variables in the augmented IPM. Consequently a possible candidate for the transducer modulus t can

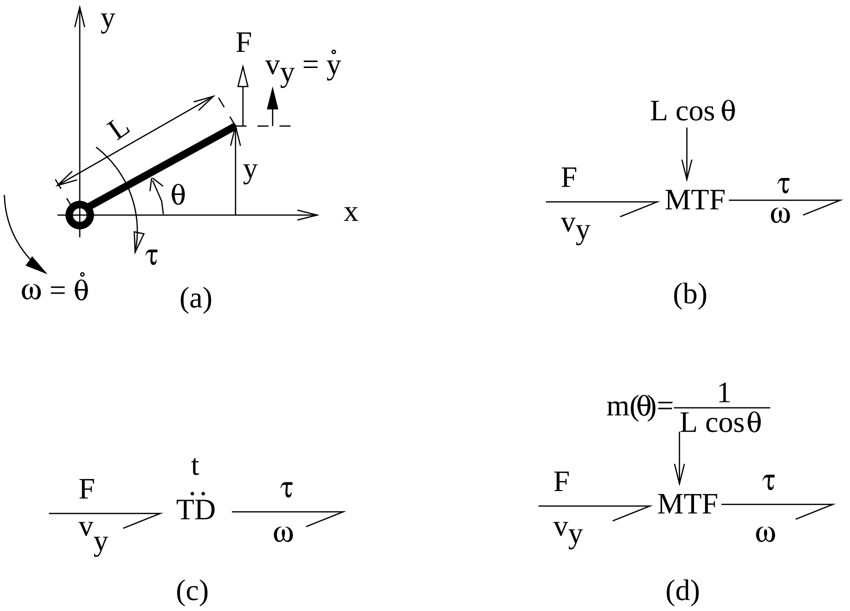

Figure 10: A nonlinear mechanical transformer

be found by taking the time derivative of the position equation $L \cdot \sin(\theta) = y$, or

$$L \cdot \cos(\theta) \cdot \dot{\theta} = \dot{y} \longrightarrow t = \frac{1}{L \cdot \cos\theta} \tag{27}$$

This in fact is the flow continuity equation of the transducer. Since this is a constraint equation of the flow variables of the power ports of the transducer, it is a well known fact that the TD is a TF. Since the modulus is dependent on the angular position θ it is a modulated one. Since the proposed MTF is a two port and since just one reference power arrow is pointing towards it (i.e. the normal representation), tearing can be skipped and one can take full advantage of the pre-causal definition, Eqn. (1), and hence

$$m(\theta) = \frac{1}{L \cdot \cos(\theta)} \tag{28}$$

This result is graphically represented in Figure 10 (d). Notice again that, in order to be consistent with the two basic definitions of a transformer, i.e. Eqns. (1) and (2), the modulus of the effort variables and the flow variables is the same. From this point the structure of the bond graph and the imposed parameters aren't changing any more when one would now impose some kind of causal assignment or different bond numbering.

In general, this method can be called *condensed bond graph modeling*, which is outlined formally in Figure 11.

This figure has to be interpreted as follows. During design, several modeling constraints, shown in the left column, have to be taken into account. These constraints are said to work *effectively* if all their consequences are exploited fully in the actual IPM or bond graph, shown at the same horizontal level as the respective constraints. The second column shows the different modeling steps; the reality as starting point, the specific desired mathematical representation as final point, and the different stages of the modeling process in between. The most important difference with other *so-called* structured modeling methods (even with use of bond graphs) is the fact that, at every stage of the modeling process, the known information of the system is represented *as much as possible* in the graph itself. In order to obtain the model (i.e. the graph) of the next stage, only the constraint of the next stage and the model (i.e. the graph) of the actual stage have to be considered. Thus the modeler should not rely on earlier graphs or assumptions and hence, no backtracking traces are encountered during modeling because the necessary information is included in the actual graph. Therefore it is called *condensed bond graph modeling* [9]. This modeling method exploits the graphical capabilities of a bond graph as much as possible. Note that this way of modeling is not possible without using pre-causal definitions of bond graph parameters and normal representations of bond graph components.

5 Closing

Although it is not very well defined where the effort and flow variable is put on the power bond, it seems that most of the bond graph people agree that the effort variable has to stand above a horizontal or inclined bond, and left of a vertical bond (*above-left rule*), as proposed in [5]. The opposite holds for the corresponding flow variable. However, there are situations where these rules are superfluous. Therefore another rule has more or less been accepted by other people: *put the flow variable on the side of the harpoon arrow* (*harpoon rule*) [11]. In the U.S. however, one sometimes tends to do the opposite since there the harpoon arrow

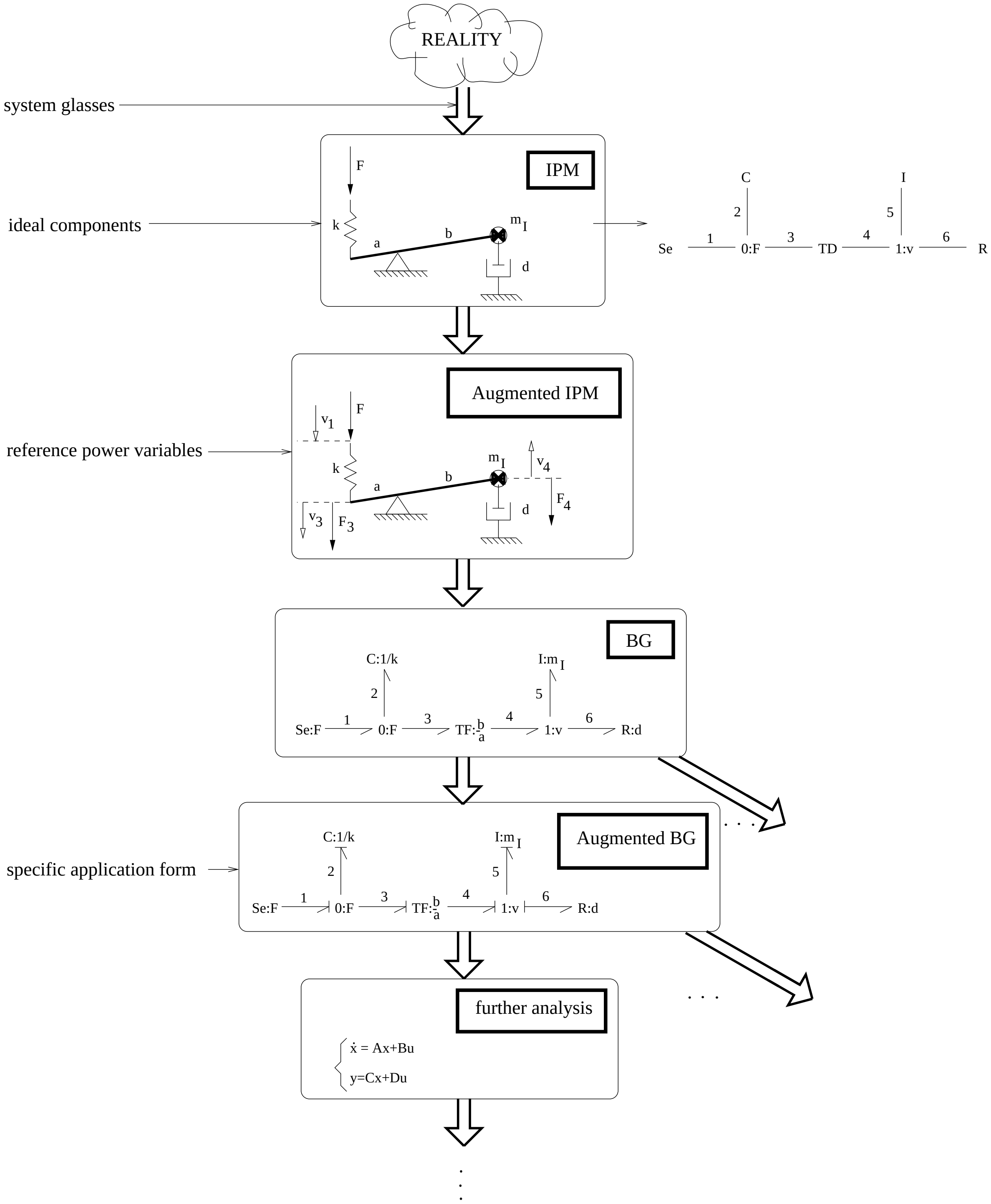

Figure 11: Flow diagram of *condensed bond graph modeling*. The key-step in this method is the explicit representation of a minimum number of reference power variables in the IPM, which is denoted as the *Augmented IPM*.

sometimes is drawn above a horizontal power connection. Such ambiguities have to be avoided since they make the shown power variables on the bond graph ambiguous. Therefore it is better to use the *harpoon rule*, rather than the *above-left rule*, since the last one is ambiguous for inclined bonds. Note that it is true that the side where the harpoon arrow is placed on the bond differs in Europe (commonly placed below and right) and the US (commonly placed above and left), even within one paper or book. However, using the harpoon rule, everybody can continue this habit, without making the power variables ambiguous for other people.
Maintaining these small differences result in a misunderstanding of what has been written down in the bond graph. In the opinion of the authors it is wrong to state that the real meaning of a certain parameter goes to the context in which the bond graph is used, and that the position of that symbol (either an effort, flow or a field parameter) in the bond graph has no relation to its meaning.

6 Conclusions

From the experience with students and research people from industry, it is the belief of the authors that some ambiguities on bond graph parameters are to be avoided, since they render the bond graph superfluous and hence hamper the use and propagation of bond graphs throughout the technological world. This can be prevented, (1) by using pre-causal definitions of parameter symbols and their use and, (2) by using *normal* representations of bond graph symbols. A solution procedure, tearing, is proposed if bond graph elements are encountered in an abnormal form. The resulting bond graphs contain more information, both qualitatively and quantitatively, and are more portable, without using complicated or unnatural conventions. Finally the *condensed bond graph modeling* method is presented as an example to illustrate the benefits of the propositions made.

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A About embedded constraint equations

Observe the bond graph of Figure 12 (a).

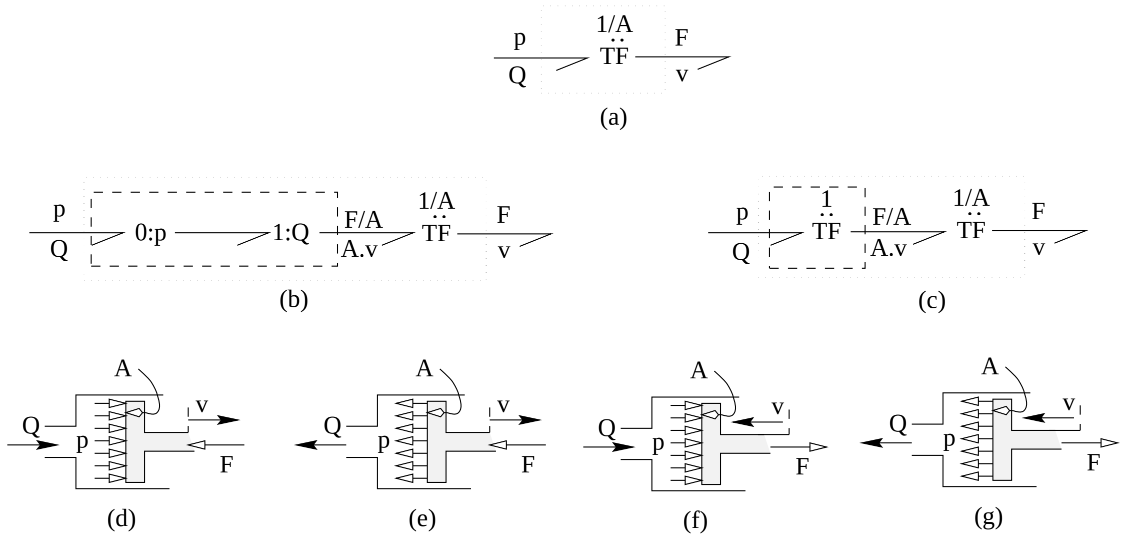

Figure 12: Constraint equations of a TF

The question we want to solve is: Which IPM represents this bond graph? Observing the transformer node, one knows that the reference power variable arrows of the hydraulic side have the same direction, and that the reference power variable arrows of the mechanical side have opposite directions. Hence four possible candidates are shown in Figure 12 (d) to (g), of which two of them are right. This can be found by means of observing the relationship between the four reference power variables. This relationship can be visualized graphically by introducing a series connection of an artificial zero and one junction at one side of the transformer, as shown in Figure 12 (b). Then, at the one junction it holds that

$$\sum e_i = 0 \tag{29}$$

and at the zero junction it holds that

$$\sum f_i = 0 \tag{30}$$

These two equations are called the *embedded constraint equations* of the transformer. Eqn. (29) is called the *effort equilibrium equation*, and Eqn. (30) is called the *flow continuity equation* of the transducer. In this example the effort equilibrium equation is $+p - \frac{F}{A} = 0$, or

$$p = \frac{1}{A}F \tag{31}$$

and the flow continuity equation is $Q - A \cdot v = 0$, or

$$Q = A \cdot v \tag{32}$$

These two equations then can be used in finding the right IPM, which in this case is Figure 12 (d) or (g).

Remarks:

1. The names *effort equilibrium equation* and *flow continuity equation* originates form the physical interpretation of the describing reference power variables in the augmented IPM.

2. The reference power variables shown in the bond graph or in the IPM of the transformer are assumed to act from the outside to the transformer field, as with all other bond graph components.

3. For transformers in normal form this method may look trivial, since these two equations can be derived directly from the effort and flow equations, Eqns. (1) and (3). However, if both power arrows are directed towards or in the other direction of the transformer, the effort and flow equations alone cannot solve this problem. Therefore the embedded constraint equations are more appropriate.

4. Embedded constraint equations (Eqns. (29) and (30)) and constituent relations (e.g. Eqns. (1) and (3)) can differ.

5. Another bond graph representation explicitly showing these embedded equations is shown in Figure 12 (c). There the transformer with unity modulus replaces the series connection of the one and zero junction. The same embedded equations can be used to derive the flow and effort gain equation of this unity transformer.

6. Similar things hold for two-port gyrators.