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Measurement of guitar string coupling[1]

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Abstract

This paper describes the measurement and extraction of plucked string waveguide model parameters. We determine the string coupling parameters for a simplified digital waveguide structure and a fully coupled model.

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Abstract

This paper describes the measurement and extraction of plucked string waveguide model parameters. We determine the string coupling parameters for a simplified digital waveguide structure and a fully coupled model.

1 Introduction

In this paper, we report on the experimental determination of the coupling parameters for two coupled string waveguide models. After a description of the experiment and a brief overview of the results, we determine the different transfer functions and reflection coefficients. Finally, we calculate the coupling coefficients for the simplified coupled string structure (Karjalainen et al. 1998) and the fully coupled structure (Nackaerts et al. 2001a).

2 Measurement setup

All measurements were done on a Taylor 514CE steel-string acoustic guitar. This guitar has a built-in Fishman Stereo Blender with a Fishman Acoustic Matrix undersaddle pickup and an internal electret microphone. We added a modified Roland GK-2A hexaphonic magnetic pickup at 1 cm of the saddle, not attached to the guitar but independently mounted on a rigid construction. The 8-channel sensor outputs were recorded on a PC equipped with 4 synchronised LynxOne soundcards, at 24-bit resolution and a 44.1 kHz sampling rate. The recorded signals were imported in Matlab (Mathworks Inc.) for further signal processing.

Figure 1 shows the outputs of the magnetic pickup, undersaddle pickup and built-in electret microphone for the high E string. The envelopes are quite similar. When looking at at the spectra of these signals (figure 2), the difference is quite clear. The output of the magnetic pickup clearly shows the comb-filter effect of the plucking point, while the internal microphone signal shows the influence of the instrument's body.

The magetic pickups are inherently non-linear. Their principle of operation is the modification of the magnetic flux through a coil by moving a ferromagnetic string in the field of a permanent magnet. A rough calculation yields that the output of the pickup $V \sim y^{-3}$, with y the pickup-string distance. This gives the trade-off (for the same magnet): larger distance, smaller distortion (relative to signal strength) but weaker signal (so worse signal-to noise ratio). At the bridge level, the string displacement is relatively small, so we disregard this non-linear distortion. The undersaddle pickup is an electret-film transducer, which could be seen as a capacitor with variable interplate distance. Its output is proportional to the differential movement of the saddle and the top plate of the guitar. Figure 3 shows the magnitude response of the magnetic pickup signal and the undersaddle pickup over a wider frequency range. The response of the undersaddle pickup shows the characteristic double peak of the body resonances. Finally, figure 4 shows the amplitude spectrum of the estimated transfer functions from magnetic pickup to undersaddle pickup to microphone. The transfer function from undersaddle pickup to microphone is essentially flat, while the transfer function from magnetic pickup to undersaddle

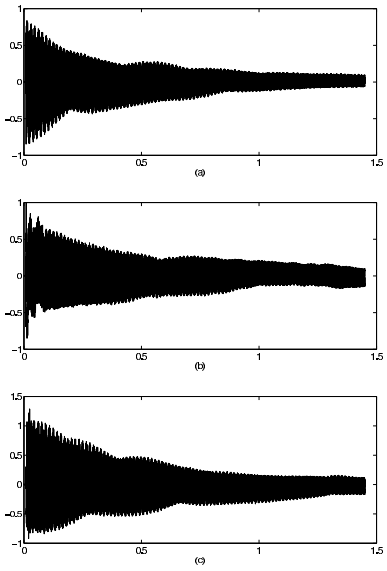

Figure 1: Amplitude vs time plots of the recording for the high E string (a) magnetic pickup, (b) undersaddle pickup, (c) electret microphone

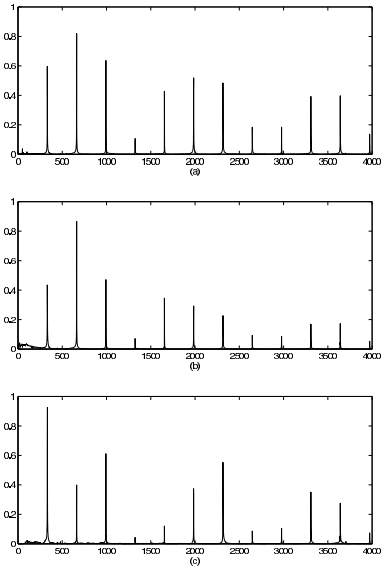

Figure 2: Spectrum of the recording for the high E string (a) magnetic pickup, (b) undersaddle pickup, (c) electret microphone

pickup closely resembles the body impulse response. The transfer functions were calculated using Welch's averaged periodogram method.

3 String coupling

Figure 5 shows a part of the recorded signals. The figure shows the outputs of the hexaphonic pickup, when the high E string is plucked. We see light coupling to the D string and stronger coupling to the A and low E string. Examination of the spectrum of the signals shows that coupling only occurs for matching modes. This confirms the near-linearity of the guitar body and string coupling mechanism. In table 1, we indicate for each string, which harmonic corresponds to a harmonic of the high E string. Based on this table, we expect strong coupling between the high E and the A and low E strings, and only slight coupling to the other strings. This is exactly as seen in figure 5.

To simplify things, we assume that only two strings interact. The procedure is however applicable to N coupled strings. First, we determine the parameters of the model show in figure 7. This is the simplified coupling structure with two strings and two polarizations per string. The input of the digital waveguides is characterized by

$$X = I + \begin{bmatrix} R_{11} & 0 & 0 & 0 \\ 0 & R_{12} & c_{21} & 0 \\ 0 & 0 & R_{21} & 0 \\ c_{12} & 0 & 0 & R_{22} \end{bmatrix} Y,$$

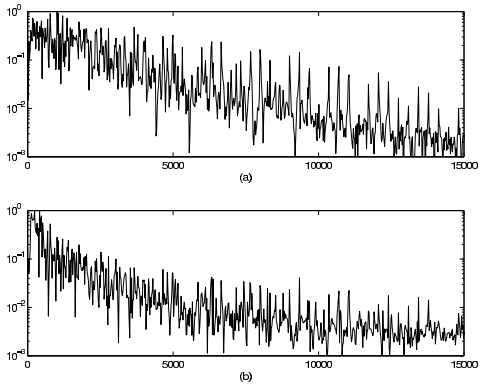

Figure 3: Amplitude vs Frequency plot of (a) the magnetic pickup signal and (b) the undersaddle pickup

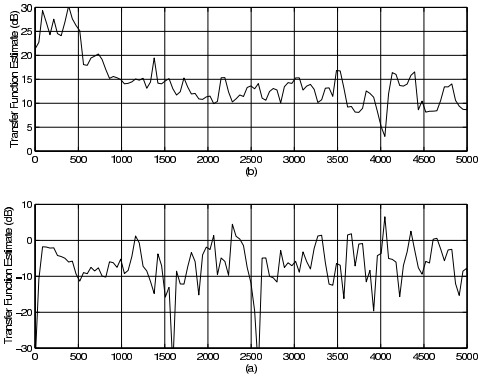

Figure 4: Amplitude response of the estimated transfer function from (a) the magnetic pickup to undersaddle pickup and (b) the undersaddle pickup to electret microphone

where Y denotes the output of the waveguides and I the excitation, R_{ij} are the reflection coefficients and c_{ij} the coupling parameters. This system consists of a pair of digital waveguides driving a second pair of waveguides. It is always stable if all $R_{ij} < 1$. Several ways are possible to determine the model parameters (Schroeder 1965), (Laroche 1989), (Nackaerts et al. 2001b). One could use the analytical solution for two coupled oscillators, and determine the pair-wise coupling parameters, or use a non-linear optimization algoritm (Nackaerts et al. 2002). We need two sets of measurements: one with the high E string excited and the low E string at rest, and one were the low E string is excited. The coupling parameters in both directions should be concurrently determined. Using non-linear optimisation, we obtain the following set of parameters (for delay line lengths of 533.23 for the low E and

high E	**329.6 Hz**	**fund**	**2nd**	**3rd**	**4th**
B	246.9 Hz			4th	
G	195.9 Hz			5th	
D	146.8 Hz				9th
A	110.0 Hz	3rd	4th	9th	12th
low E	82.4 Hz	4th	8th	12th	16th

Table 1: This table indicates which harmonics of the non-excited strings correspond to within a few Hz to the harmonics of the high E strings. Based on this table, we expect strong coupling to the A and low E strings, and weak coupling to the other strings

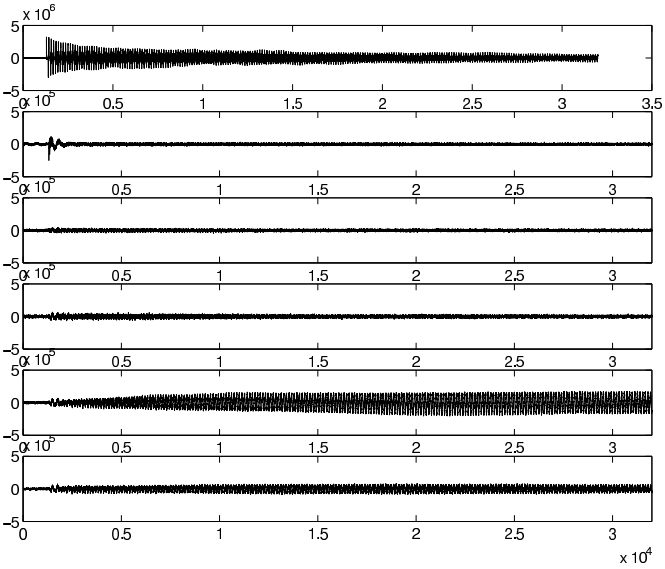

Figure 5: This figure shows the output of the hexaphonic magnetic pickup. From top to bottom: high E, B, G, D, A, low E. The high E string was plucked. Note that the scale for the high E string is ten times larger.

133.22 for the high E string respectively).

$$\begin{aligned} R_{11} &= 0.9969 \\ R_{12} &= 0.9142 \\ R_{21} &= 0.9896 \\ R_{22} &= 0.9423 \\ c_{12} &= 0.0188 \\ c_{21} &= 0.0204 \end{aligned}$$

We obtain one lightly and one strongly damped polarization, and quite similar coupling coefficients c_{12} and c_{21} between high E and low E strings. Note that $R_{11} + c_{12} \approx R_{21} + c_{21} \approx 1$. Figure 9 shows the recorded and simulated amplitude envelopes for a common harmonic of the high and low E strings. The fully coupled model is both easier and more difficult to

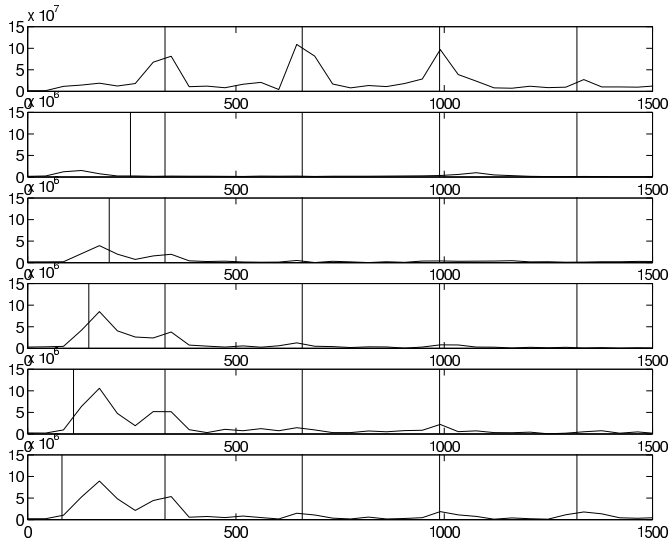

Figure 6: This figure shows a part of the spectrum of the signals shown in Figure 5. The full lines are the fundamental and the first three harmonics of the high E string. The dashed lines indicate the fundamentals of the other strings. We see coupling to matching modes only. Note that the scale for the high E string is different

calibrate. In this model, we have twelve string models and one body model, all coupled using a 13×13 coupling matrix. As there is feedback, care must be taken to ensure that the global model is stable. The heuristic rules as described in (Nackaerts et al. 2002) give a good starting point for further optimisation. When using these rules, one only determines the decay rate of the fundamentals for the two polarizations of each string. All other model parameters follow from basic principles. Exactly calibrating the parameters for this model requires non-linear optimisation, as the system stability has to be ensured. Slightly altering the feedback from the body to the strings yields the output shown in figure 10. This is a close match to the recording. Note that the two-stage decay now results from true coupling, as the two polarizations have comparable reflection coefficients.

4 Conclusion

This paper desribes a practical measurement setup that allows the determination of all the digital waveguide model parameters. We determine the coupling parameters for the simplified and the fully coupled structures. Both models are capable of closely simulating the behavior of the real instrument, where the fully coupled model holds the advantage that two-stage decay and beating are included in the model.

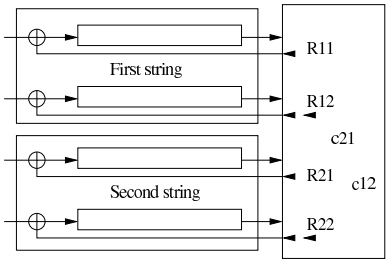

Figure 7: This simplified coupled string structure has two polarizations per string. The first polarization of the first string drives the second polarization of the second string, with no feedback.

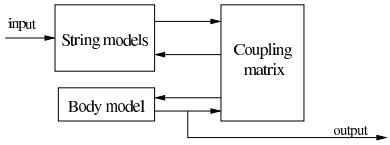

Figure 8: The fully coupled string model has twelve separate string models (two for each string) and a body model. All the outputs are mixed using a 13×13 coupling matrix and used as inputs.

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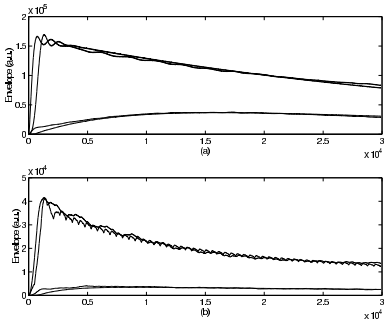

Figure 9: This figure shows the amplitude envelope of (full line) the recording of magnetic pickup, and (dotted line) the actual model output. Shown are (a) high E string excited (top pair), coupling with low E string (bottom pair); and (b) low E excited, coupling with high E

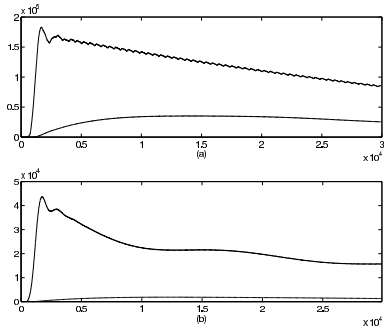

Figure 10: Amplitude envelopes for the same two cases as figure 9, when using the fully coupled model and after having slightly altered the parameters determined using the heuristic rules.

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