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A Comparison of a Cellular Automaton and a Macroscopic Model

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Summary. In this paper we describe a relation between a microscopic stochastic traffic cellular automaton model (i.e., the STCA) and the macroscopic first-order continuum model (i.e., the LWR model). The innovative aspect is that we explicitly incorporate the STCA's stochasticity in the construction of the fundamental diagram used by the LWR model. We apply our methodology to a small case study, giving a comparison of both models, based on simulations, numerical, and analytical calculations of their tempo-spatial behavior.

1 Introduction

Dating back to the mid '50s, Lighthill, Whitham, and Richards introduced their macroscopic first-order continuum model (i.e., the LWR model) [1, 2]. It is based on a fluid dynamics analogy, in which the collective behaviour of infinitesimally small particles is described, using aggregate quantities such as flow q, density k and (space) mean speed $\overline{v}_s$. Models like this, can be solved using cell-based numerical schemes (e.g., using the Godunov scheme [3, 4]).

Later, microscopic traffic flow models have been developed that explicitly describe vehicle interactions at a high level of detail. During the early nineties, these models were reconsidered from an angle of particle physics: cellular automata models were applied to traffic flow theory, resulting in fast and efficient modelling techniques for microscopic traffic flow models [5]. These cellular automata models can be looked upon as a particle based discretisation scheme for macroscopic traffic flow models.

It is from this latter point of view that our paper addresses the common structure between the seminal STCA model and the first-order LWR model. Our main goal is to provide a means for explicitly incorporating the STCA's stochasticity in the LWR model. After explaining the methodology of our approach, we present an illustrative case study that allows us to compare the tempo-spatial behavioural results obtained with both modelling techniques.

2 Methodology

Already, relations between both types of models (i.e., STCA and LWR) have been investigated (e.g., [6]). Our approach is however different, in that it provides a practical methodology for specifying the fundamental diagram to the LWR model. Assuming that a stationarity condition holds on the STCA's rules, this allows us to to incorporate the STCA's stochasticity directly into the LWR's fundamental diagram.

We assume that we have the ruleset of the STCA available, as well as the maximum allowed speed $v_{\max}$ and the stochastic noise term p (i.e., the *slowdown probability*). Furthermore, a discretisation is given, expressed by the cell length $\Delta X = 7.5$ m, the time step $\Delta T = 1$ s, and its coupled speed increment $\Delta V = \Delta X \div \Delta T = 27$ km/h.

Relating both the STCA and the LWR models is done using a simple two-step approach, in which we first rewrite the STCA's rules (assuming a stationarity condition holds), and then convert these new rules into a space gap/speed diagram (which is equivalent to a stationary density/flow fundamental diagram).

2.1 Rewriting the STCA's rules

Starting from the ruleset of the STCA, we rewrite it using a min-max formulation. Instead of having several individual rules that give a discrete speed, we now have one rule that returns a continuous speed (for an individual vehicle):

$$\begin{aligned} v(t + \Delta T) = {}& p \cdot \min \left\{ v(t), g_s(t) - 1, v_{\max} - 1 \right\} \\ & + (1 - p) \cdot \min \left\{ v(t) - 1, g_s(t), v_{\max} \right\}, \end{aligned} \tag{1}$$

with $v(t + \Delta T) = \max\{0, v(t + \Delta T)\}$. The stationarity condition previously mentioned, asserts that the speed $v(t)$ of a vehicle at time t is the same as its speed at time $(t + \Delta T)$:

$$v(t + \Delta T) = v(t). \tag{2}$$

This allows us to reformulate equation (1) as a set of linear inequalities that express constraints on the relations between $v(t)$, $v_{\max}$, p and the space gap g_s.

2.2 Deriving the fundamental diagram

The linear inequalities derived in section 2.1 together form a set of boundaries that can be plotted in a diagram that shows the space gap g_s of a vehicle versus its speed v. Knowing that the space headway h_s equals the vehicle's length L plus its space gap g_s, we can plot a stationary (h_s,v) diagram as can be seen in the left part of figure 1. Because the space headway h_s is inversely proportional to the density k, we can derive an equivalent *triangular* (k,q) fundamental diagram, corresponding to the right part of figure 1.
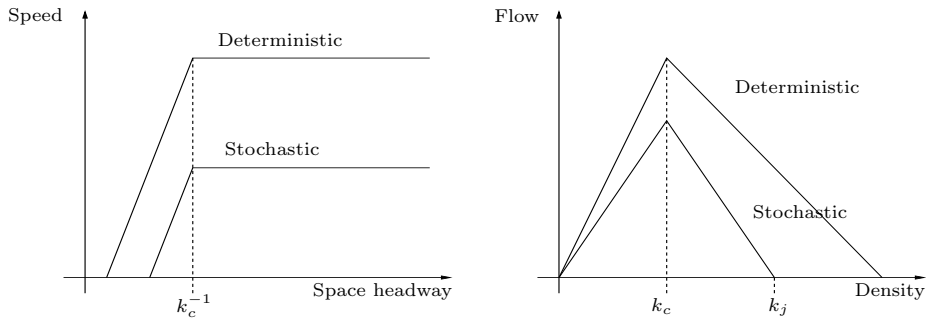

Fig. 1. Deriving stationary (h_s,v) (left) and (k,q) (right) fundamental diagrams for the LWR model, after incorporation of the STCA's stochastic noise.

Considering the previously derived constraints and the diagrams in figure 1, the following important observations can be made:

- The stochastic effects from the STCA, are now incorporated in a stationary fundamental diagram, which can then be specified as a parameter to the LWR model.
- The stochastic diagrams lie lower than their deterministic counterparts, so the capacity flow is lower for the stochastic variants.
- The jam density for stochastic systems is different from that for deterministic systems, but the critical density remains unchanged.

3 An illustrative case study

As a toy example, we apply our methodology to a case study, in which we model a single lane road that has a middle part with a reduced maximum speed (e.g., an elevation, or a speed limit, . . .). The road consists of three consecutive segments A, B, and C. The first segment A consists of 1500 cells (11.25 km), while the second and third segments B and C each consist of 750 cells (i.e., each approximately 5.6 km long). We consider a time horizon of 2000 s. The maximum speed for segments A and C is 5 cells/s, whereas it's 2 cells/s for segment B. The stochastic noise p was set to 0.1 for all three segments. Vehicles enter the road at segment A, travel through segment B, and exit at the end of segment C.

   This road is simulated using both the STCA and the LWR model. As for the boundary conditions, we assume an overall inflow of $q_c^B \div 2$ (q_c is the capacity flow), except from $t = 200$ s to $t = 600$ s, where we create a short traffic burst with an inflow of $(q_c^{A,C} + q_c^B) \div 2$. Figure 2 shows the individual vehicle trajectories in a time/space diagram: heavy congestion sets in and flows upstream into segment A, where it starts to dissolve at the end of the traffic burst.
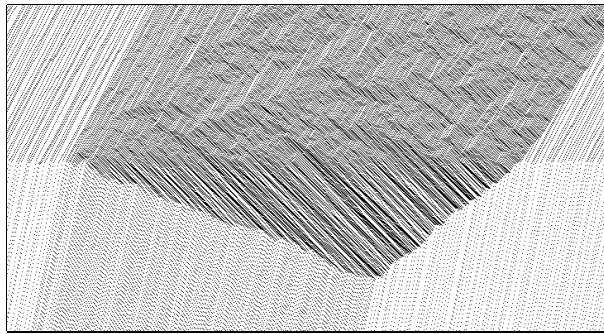

Fig. 2. A time/space diagram after simulation of the STCA: each vehicle is represented by a single dot (the time and space axes are oriented horizontally, respectively vertically). At the end of segment A, we can see the formation and dissolution of an upstream growing congested region, related to the short traffic burst.

   Applying our previously discussed methodology, we construct a stationary (k,q) fundamental diagram, and numerically solve the LWR model. The result can be seen in the left part of figure 3. Comparing this spatio-temporal behaviour of the LWR model with the microscopic system dynamics from the STCA model (i.e., the right part of figure 3), we find a qualitatively good agreement between the two approaches.

   Even more interesting, is the fact that the STCA model reveals a higher-order effect that is not visible in the LWR model: there exists a fan of forward
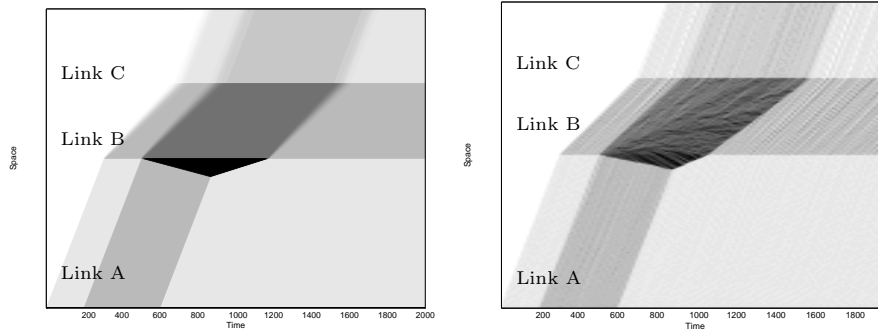
Fig. 3. Time/space diagrams showing the propagation of densities during 2000 s for the road in the case study. The left part shows the results for the LWR model, while the right part shows the microscopic system dynamics from the STCA model (note that darker regions correspond to congested traffic).

propagating density waves in segment B. Furthermore, in its tempo-spatial diagram, the STCA seems to be able to visualise the characteristics that constitute the solution of the LWR model.

4 Conclusions

The novel approach taken in our research, allows us to incorporate the STCA's stochasticity directly in the first-order LWR model. This is accomplished by means of a stationarity condition that converts the STCA's rules into a set of linear inequalities. In turn, these constraints define the shape of the fundamental diagram that is specified to the LWR model.

Our methodology sees the STCA complementary to the LWR model and vice versa, so the results can be of great assistance when interpreting the traffic dynamics in both models. Nevertheless, because the LWR model is only a coarse representation of reality, there are still some mismatches between the two approaches. One of the main concerns the authors discovered, is the fact that using a stationary fundamental diagram (i.e., an equilibrium relation between density and flow), always overestimates the practical capacity of a cellular automaton model (see e.g., figure 4, where the true capacity for $v_{\max} = 5$ cells/s lies somewhere near 2400 vehicles/h, which is a rather low value).

Further research will focus on the dynamics of multi-lane traffic, on the heterogeneity of the traffic stream (using a heterogeneous LWR model), and on the relation between the capacity of a cellular automaton and the level of stochastic noise in the system.
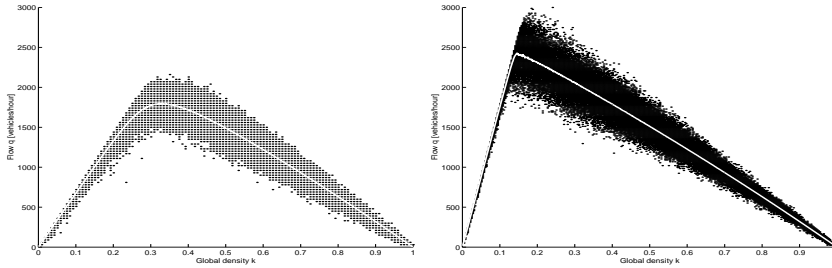
Fig. 4. The (k,q) phase space diagrams of the STCA for $v_{\max} = 2$ cells/s (left) and $v_{\max} = 5$ cells/s (right). The stochastic noise was $p = 0.1$ in both diagrams. The small points denote individual measurements, whereas the white curves represent long-time averages.

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