Flood Prevention of the Demer using Model Predictive Control

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Abstract: In order to prevent flooding of a river system the local water administration of the Demer provided the river system with hyraulical structures and water reservoirs. Though these actions have reduced the damage and frequency of flooding events, simulations have shown that a better usage of these structures and reservoirs could further decrease the flooding in the bassin. Therefore, in this work the main focus will be to ameliorate the usage of structures and reservoirs by controlling them with model predictive control. In this paper a conceptual model of the Demer will be derived. Afterwards a model predictive controller will be used to avoid flooding of the Demer bassin. A comparison will be made between the performances of the model predictive controller and the currently used three position controller.

1. INTRODUCTION

Flooding of rivers causes worldwide for a lot of trouble. In Belgium there is a river, the Demer, which caused for similar flooding problems in its bassin during periods of heavy rainfall . In order to prevent these flooding events the local water administration provided the river system with hydraulical structures (gates) in order to be able to control the discharges in the river system. Extra storage capacity for periods of heavy rainfall was provided through reservoirs. Structures to control the flow from and into the available reservoirs were also added to the system. Nowadays, the hydraulical structures are controlled by a three position controller. The three position controller determines the control actions of the gates based on some very simple rules. The main disadvantage of these rules is that they are only based on the current measurements of the water levels but don't take the future rain predictions into account. This causes the control actions to be far from optimal. The local water administration has verified that the damage of past flooding events could have been significantly reduced and even completely avoided if different control decisions would have been applied than the ones obtained by the three position controller. Therefore, the main purpose of this work is to come up with a better control strategy.

1.1 Model Predictive Control

In this work the control strategy to be investigated is model predictive control (MPC) ([Camacho],[Rossiter]). MPC is a control technique that in the past decennia has become more and more popular in the process industry because of some specific advantages. The biggest advantages of MPC are the possibility to take input and state constraints into account and the prediction horizon used in order to determine the optimal control input. An important disadvantage of MPC is the fact that because of the calculation time it is only applicable to systems with relatively slow dynamics. Because the dynamics of river systems are relatively slow, because to prevent flooding input and state constraints need to be considered and because future rain predictions need to be taking into account model predictive control is a suitable control strategy in order to solve the flooding problem.

In the literature several works can be found in which automatic control techniques are used in order to control a river system ([Brian et al.],[Burt et al.],[Litrico et al.]). A good overview of the different controllers can be found in [Malaterre(1998)].There are also several works available in which a MPC is used to control river systems([Rutz et al.],[Rodellar et al.]). These works however have as main goal to control the different waterlevels to some desired target value and not to prevent flooding. In these applications it is usually sufficient to linearize the system round the desired steady state value in order to obtain good results. But this simple linearization doesn't work when trying to avoid flooding. The main reason is that during periods of heavy rainfall the complete nonlinear dynamics of the system is excitated. So in this application it is really important to use a MPC that is capable of taking the nonlinear model behaviour into account. In the sequel of the paper such a MPC will be discussed.

Also remark that to the authors knowledge there are no works published in which MPC is used in order to avoid flooding, with exception from [Thai(2005)]. However, in [Thai(2005)] the nonlinear behaviour introduced by the

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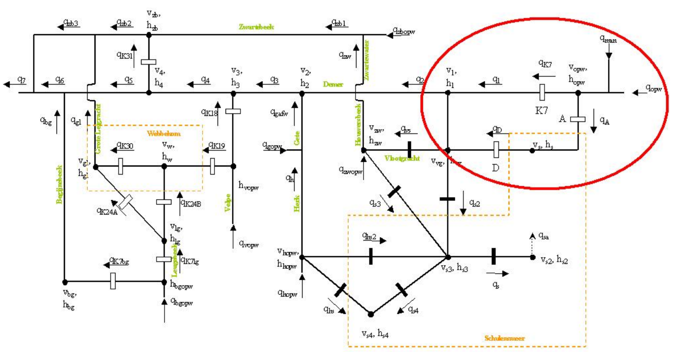

Fig. 1. The Complete Demer Model

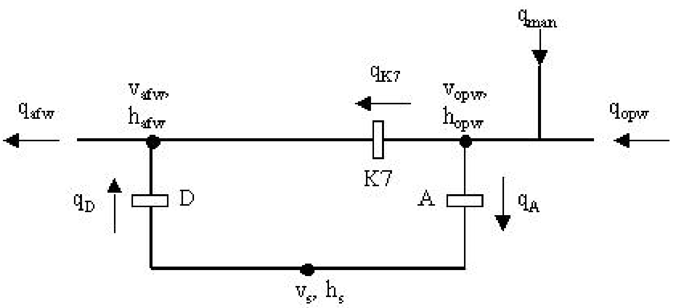

Fig. 2. Model Scheme

presence of the gates is not considered which is a serious simplification of the actual problem.

1.2 Modelling

MPC is a control paradigm that needs the model of the system in order to determine the optimal control inputs. So the first step in any application in which an MPC will be used, is to determine an appropriate model of the system to be controlled. In this work a discrete time conceptual model of the river system is determined rather than a finite element model. The tuning of the parameters of the conceptual model is done by taking hystorical data into account. Results of this model will be shown in the sequel.

2. MODELLING

The first step in this work is to make a model of the river system to be controlled. In figure 1 a schematic overview of the complete Demer model is shown. The local water administration has an accurate finite element model of this river system generated in the software package Infoworks [OBM-Demer(2003)]. Because it is not straightforward to make a software coupling with this software package, it was necessary to derive an own conceptual model based on simulation data generated in infoworks. Since this work is the first step towards the use of MPC for flooding the focus was limited to control only the part indicated on the figure by the (red) ellipse. A more detailed view of this part is depicted in figure 2. The river system considered in this work consists of 10 states (three water levels, four discharges and three volumes) and three inputs. The outputs of the system are the three water levels. The water

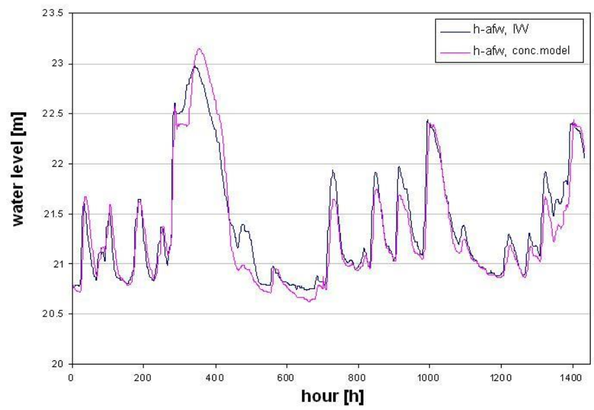

Fig. 3. Model Validation

level upstream is hopw , the water level downstream is hafw. There is one reservoir that can be used during heavy rainfall and its water level is hs. There are three gates that need to be controlled by the controller, namely A, D and K7. There are two disturbance inputs qman en qopw to model rainfall entering the river system.

The conceptual model derived here is from the reservoir type. The equations are determined according to the methodologies described in [Vaes et al.(2002)]. The model is calibrated based on the data of 1995 generated by Infoworks. The resulting model is a discrete time model with a simulation time step of 1 hour. Internally, however, the model uses a 5 minutes simulation time step. Remark the equations to describe the discharges over the gates make the model of the system hybrid. The model was validated by comparing its simulation results with data from 1998 and 2002 generated by the infoworks model. In figure 3 this validation is shown for water level hafw. The first 600 hours correspond to the year 1998 and the next hours to the year 2002. It can be seen that the conceptual model is a relatively good match of the data generated by infoworks.

3. CONTROLLER DESIGN

In literature a lot of control strategies can be found in order to control a river system. A good overview of all these control strategies can be found in [Malaterre(1998)]. In this work was opted for MPC. MPC has a typical structure that can cope with all issues related to controlling a river system. The main issues justifying the use for MPC in order to prevent flooding, are the following:

(1) The calculation time of a MPC controller limits its use to control systems with relatively slow dynamics. Because river systems have slow dynamics MPC can be applied to them.

(2) The gates in the water system have some important physical limitations that have to be taken into account. The gates have upper and lower limits that can never be violated in reality. There is also a restriction on the speed of the gate movement as the gates can't move infinitely fast in real time. MPC is perfectly capable of taking both constraints easily into account.

(3) In order to prevent flooding it is necessary to be able to impose upper limits on the different water levels. In an MPC framework it is possible to impose them as hard constraints. In order the rainfall is to big to avoid flooding of some water levels, MPC can also impose them as soft constraints (see further).

(4) Taking the rainfall predictions into account is a very important issue when trying to prevent flooding. MPC is capable of taking this into account by modelling the rainfall as a known disturbance input into the system.

(5) The model of a river system is highly nonlinear. Because during flooding periods the complete nonlinear dynamics of this system is excitated, the control problem turns into a higly nonlinear problem. Most control strategies are based on linear models and therefore can't cope with this nonlinear behaviour. On the contrary, MPC is well suited to tackle this.

(6) A river system is a coupled system in which affecting the gate in one river channel also affects the other river channels in the river system. Traditional local control techniques do not work very well on such kind of systems. MPC, however, can cope with this coupling. Even more, if one of the system inputs, in this case one of the gates, doesn't work during operation, MPC is capable of use this coupling to still achieve a reasonable control action despite the failure of one input. Local control techniques are not capable to work properly when an input fails.

In the remainder of this section the principles of MPC will be explained as well as the implementation issues of it.

3.1 Principles of MPC

MPC is a control strategy that uses the model of the system in order to make future predictions on which an optimal input sequence is determined in order to minimize an objective function. The three basic components of MPC are the following:

(1) A process model is used to determine the future outputs within a time window with length N, the prediction horizon. In this step the relation between the unknown control inputs and the future ouputs is determined. This prediction strategy is shown in figure 4.

(2) An objective function is minimized. The objective function is typically a quadratic function that tries to minimize the water level errors and the gate movement by adjusting the unknown control inputs. Typically the objective function is also subject to constraints. There are always some constraints to take physical or desired limitations into account. When all the constraints are linear and the objective function is quadratic the optimization problem to be solved is called a quadratic program, which can be solved efficiently in practice.

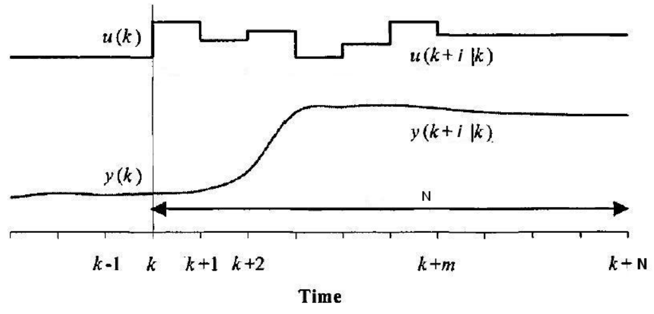

Fig. 4. MPC Strategy

(3) Once the sequence of future control actions that minimizes the desired objective function is determined, only the first set of control actions is implemented on the system. The system is then updated by measuring (estimating) the new state of the system and the process is repeated. The updating of the system and the repetition of the optimization can be seen as a feedback to compensate for measurement errors and model uncertainties.

3.2 Implementation of MPC

The best known MPC is the linear MPC in which the process model is a linear time (in)variant system. This may seem restrictive but since in most control applications the goal is to steer the output to some predefinied reference output and keep it there, linear MPC seems to work very well in practice. Furthermore, most nonlinear MPC strategies for nonlinear process models are based on linear MPC. Therefore, in the following a further outline of the linear MPC will be given and afterwards will be discussed how to extent this to come to the nonlinear MPC used in this work.

The linear time variant state space system of interest in this work has the following form:

$$x(k+1) = A_k x(k) + B_k u(k) + D_k d(k), \qquad (1)$$

$$y(k+1) = Cx(k+1). \qquad (2)$$

with x(k) the state vector of the system at time k, u(k) the input vector (gates) at time k, d(k) the disturbance vector (rainfall) at time k, y(k) the output vector (water levels) of the system at time k, A_k, B_k and D_k time variant system matrices and C a time invariant system matrix. With this process model it is possible to determine the output predictions as a function of the unknown control inputs. The output at the next time step can be written as follows:

$$y(k+1) = CA_k x(k) + CB_k u(k) + CD_k d(k) \qquad (3)$$

In a similar way the next predicted output can be written as:

$$y(k+2) = CA_{k+1}x(k+1) + CB_{k+1}u(k+1) + CD_{k+1}d(k+1)$$

$$y(k+2) = CA_{k+1}A_kx(k) + CA_{k+1}B_ku(k) + CA_{k+1}D_kd(k) + CB_{k+1}u(k+1) + CD_{k+1}d(k+1) \tag{4}$$

Remark that the predicted outputs only depend on the current state $x(k)$, the disturbance inputs $d(k+i)$ which are known in this work and the current and future control inputs $u(k+i)$ that have to be determined. Doing this for all the other outputs, the predicted outputs can be written as follows:

$$\begin{bmatrix} y_{k+1} \\ y_{k+2} \\ y_{k+3} \\ \dots \\ y_{k+N} \end{bmatrix} = \begin{bmatrix} CP_0 \\ CP_1 \\ CP_2 \\ \dots \\ CP_{N-1} \end{bmatrix} x_k +$$

$$\begin{bmatrix} CB_0 & 0 & 0 & \dots & 0 \\ CP_0B_0 & CB_1 & 0 & \dots & 0 \\ CP_1B_0 & CP_0B_1 & CB_2 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ CP_{N-1}B_0 & CP_{N-2}B_1 & CP_{N-3}B_2 & \dots & CB_{N-1} \end{bmatrix} \begin{bmatrix} u_k \\ u_{k+1} \\ u_{k+2} \\ \dots \\ u_{k+N-1} \end{bmatrix} +$$

$$\begin{bmatrix} CD_0 & 0 & 0 & \dots & 0 \\ CP_0D_0 & CD_1 & 0 & \dots & 0 \\ CP_1D_0 & CP_0D_1 & CD_2 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ CP_{N-1}D_0 & CP_{N-2}D_1 & CP_{N-3}D_2 & \dots & CD_{N-1} \end{bmatrix} \begin{bmatrix} d_k \\ d_{k+1} \\ d_{k+2} \\ \dots \\ d_{k+N-1} \end{bmatrix}$$

with $P_i = \prod_{n=0}^{i} A_{k+i-n}$, $B_i = B_{k+i}$ and $D_i = D_{k+i}$.

This can be rewritten into the following shortened vector notation:

$$Y_p = Gx_k + H\mathbf{u} + J\mathbf{d} \tag{5}$$

The second component of MPC is the objective function to be minimized. The objective function typically has the following form:

$$\min_{\mathbf{u}} \|Y_p(\mathbf{u}) - Y_r\|_Q + \|\mathbf{u} - u_r\|_R$$

with

$$\|\mathbf{x}\|_Q = \mathbf{x}'Q\mathbf{x}$$

, Y_r the desired output references, u_r the desired input references and Q and R positive definite symmetrical cost matrices.

By taking (5) into account the cost function can be written as only a function of the unknown input vector $\mathbf{u}$, this leads to a quadratic objective function which together with the constraints imposed to the system leads to the following (constrained) quadratic program (QP) that has to be solved at each time instant:

$$\min_{\mathbf{u}} \mathbf{u}'(H'QH + R)\mathbf{u} + 2(x_k'G'QH + d'J'QH - Y_r'QH - u_r'R)\mathbf{u}$$
$$F\mathbf{u} \leq b \tag{6}$$

In this work the process model is not a linear time variant but a highly nonlinear one. However, the results of the linear time variant system can be used in order to solve the control problem with the nonlinear process model by means of the following steps:

(1) Simulation of the nonlinear model within the prediction horizon N with the inputs obtained by solving the QP in the previous time instant. This leads to a trajectory of future states. At initialization of the MPC there are no inputs from a previous time instant available so an arbitrary input sequence is chosen in order to do the simulation.

(2) At each time instant within the prediction horizon a linearization around the simulated future states is done. The linearization in this work was done iteratively by use of forward differences. The linearization gives rise to different linear systems at each sampling time which are the characteristics of a linear time variant systems.

(3) The QP (6) related to this linear time variant system is solved and a sequence of optimal inputs is obtained.

(4) The previous steps are repeated with the recently computed optimal input sequence untill convergence or untill time runs out. After convergence the first input is applied to the system, the systems gets an update and the MPC strategy is repeated.

In literature [Allgöwer et al.] it has been shown that this procedure converges to a local minimum of the nonlinear control problem. In this work this procedure was used in order to obtain the results presented in section 4.

3.3 Constraint and Cost Function Strategy

In order to obtain a satisfying control strategy it is important the constraints and the cost functions are chosen in a 'good' way. During normal periods there are some specific desires from the water administration that should be achieved as close as possible. But during heavy rainfalls some of those desires lose importance because the main desire is to avoid flooding. This means that it is not possible to keep the objective function constant during operation.

In this work the constraints and control goals change according to some predefined strategy based on the experience of the local water administration. The main idea behind the strategy is to avoid as long as possible to fill the reservoirs of the system. If some water levels reach a critical level the constraints and cost function are adjusted in order to be able to store the excessive water volume in the reservoirs. The critical levels are determined by experience of the water administration and are chosen in such a way that there is still enough time to avoid flooding of the water levels by starting to fill the reservoirs.

Another important issue is the fact some water levels are more important than others. At some point when the rainfall is extremely big it is impossible to keep all water levels under their flood level. A possible strategy could be to introduce slack variables to the violated constraints and minimize the value of those variables. A disadvantage of this, however, is that the number of optimization variables increases which leads to a bigger computation time. Instead of introducing slack variables in this work was opted to classify the constraints into sets of different importance. If it is impossible to ensure feasibility of all the constraints, the constraints belonging to the least important set are removed from the optimization problem. In the cost function the weight of the water levels corresponding to the removed constraints is increased and the optimization is redone. If the problem is still infeasible the least important constraint set of the remaining sets is removed. This is repeated untill a feasible solution is obtained. This strategy ensures the biggest effort goes to ensure the most important water levels stay beneath their flood level.

4. EXPERIMENT

The main objective of this experiment is to compare the current three position controller with a MPC controller. A three position controller is a controller that consists of some very simple logical rules based on the current water levels of the system in order to decide the control action to be implemented. A MPC is the controller that is described in more detail in section 3. In this section both controllers are compared by a simulation based on the rainfall data of 1998. In the following some important details of the simulation are discussed:

(1) In practice the gates of the river system change each 15 minutes. This is a very important issue because this is an upper limit to the computation time of the QP optimization.

(2) Another very important remark is the fact that in this experiment it is assumed the nonlinear model is perfectly known, the rain predictions are perfectly known and the current state of the system is exactly known at each time step. In practice this is never the case. But as pointed out in section 5 future work will focus in taking these uncertainties into account during the experiments.

(3) The flood levels of the 3 water levels are:

- hopw $\leq$ 23.2 m
- hs $\leq$ 23.2 m
- hafw $\leq$ 22.75 m

(4) Two hard constraints to take into account are related to the gates and are the consequence of physical limitations. The first gate constraint limits the speed of all the gate movements to 0.1 m/hour. The second gate constraint defines the physical upper and lower limits of the gates. The three valves k7, A and D (see figure 2) have 20 m as lower limit and 23 m as upper limit.

(5) The control objectives in this experiment are as follows:

- During normal operation the objectives are to steer hopw to 21.5 m and to avoid hs increases.
- When hopw reaches the critical water level of 23 m or hafw reaches the critical water level of 22.55 m, it is allowed to fill the reservoir in order to avoid both water levels reach their flood level.
- Under normal circumstances it is desired to steer the water level hs of the reservoir to 20.4 m.
- Another important objective is to always try to empty the reservoir as fast as possible. This is important in order to handle 2 successive extreme rainfall events.
- Concerning the water level of the reservoir it is preferred to avoid its water level to exceed 23 m. Though exceeding 23 m doesn't mean the reservoir is flooded (flood level hs is 23.2 m), in practice the local water administration tries to avoid to exceed this water level because from that point on part of the reservoir consists of farm land. When this land is used in order to avoid flooding, the water administration is obliged to give the farmers a financial recompensation for it.

(6) The rainfall data used in this experiment is based on data of the Demer bassin from 1998, a year where a serious flooding event caused for a lot of damage in the bassin. In order to be able to have a better assessment of the advantages of MPC the rainfall data was modified. In the original rainfall data of 1998 only one extreme rainfall peak was present. In this experiment the original data was modified by extending it with a second peak. At the end of the second peak a period with very little rainfall was assumed.

The results of the experiment are depicted in figures 5 and 6. In figure 5 the three position controller was used to control the system and in figure 6 the MPC was used. Comparing both results it is obvious the MPC outperforms the three position controller. During the first 250 hours and the last 800 hours the MPC steers hopw much closer to the desired 21.5 m. During the two extreme rainfall peaks the MPC almost completely avoids flooding. During the second peak only hafw violates its flood level for a short time. The three position controller, however, is not capable to avoid flooding during the two peaks. Especially during the second peak the flooding is dramatic. The main reason for this extreme flooding is the fact that the three position controller isn't capable of emptying the reservoir sufficiently between the two rain peaks, while the MPC almost completely empties the reservoir. A closer look to the results during the period between the two peaks reveals the MPC steers hopw to his critical water level without exceeding it. By doing this hafw will be lower which on his turn offers the possibility to empty the reservoir sufficiently fast without making hopw flood. The MPC is capable of

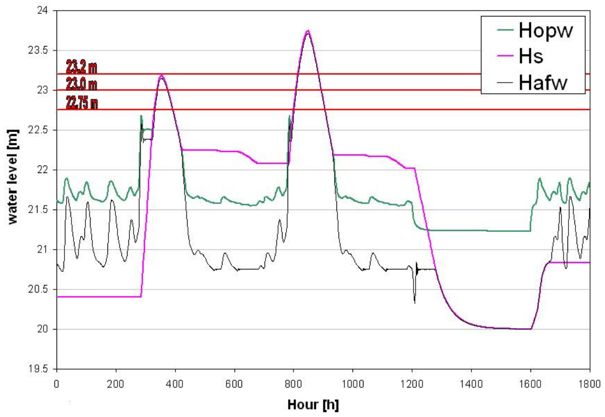

Fig. 5. Result Three Position Controller

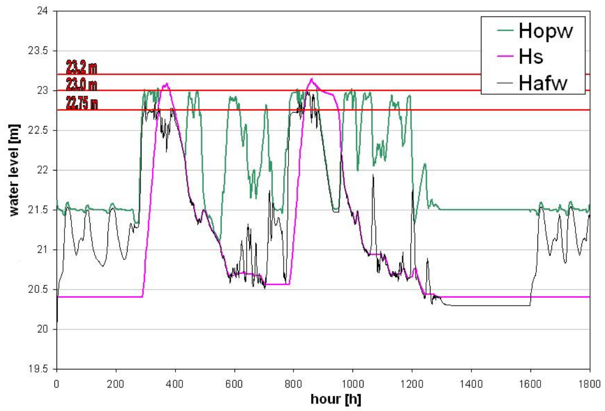

Fig. 6. Result MPC

reaching this limit because it uses a process model and the future rain predictions in order to determine the optimal control actions, which makes the controller less conservative than the three position controller.

5. CONCLUSION AND FUTURE WORKS

In this work a conceptual model of the Demer was determined. The resulting model was of the reservoir type. It was calibrated by historical data of 1995 generated by a finite element model of the Demer made with the Infoworks software package. This model was used in order to compare the current three position controller with a model predictive controller. In order to make the comparison the rainfall data of 1998 was extended with a second peak of extreme rainfall. The simulations with this rainfall data showed the MPC outperformed the three position controller. With MPC flooding was almost completely avoided despite two successive peaks of extreme rainfall. With the three position controller the experiment showed drastic flooding was inevitable during the second peak. Also during normal rainfall periods it was shown MPC steered the water levels much closer to the desired levels.

Future works will focus on controlling the complete model of the Demer instead of a small part of it. The complete model has much more states and is more nonlinear which will raise challenges concerning computational speed as well as stability of the QP's to be solved. As stated before in this work the assumptions were made that the model contains no model uncertainties, the rain predictions are exactly known and the states are completely known. In practice however, much of these assumptions are not the case. So a next step will be to take those uncertainties into account during the experiments and add a state estimator (e.g. Kalman filter) into the system. In a further step a robust MPC scheme could be designed to take all the uncertainties explicitly into account.

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