ROBUSTLY INVARIANT POLYTOPES BASED ON THE L_1-NORM

ABSTRACT
Invariant sets are tools that are used in practice as terminal sets for model predictive control (MPC) in order to ensure stability. By enforcing the terminal state to lie inside an invariant set stability and feasibility with respect to the constraints can be ensured. In this work the main focus will go to sets based on the L_1-norm invariant for linear systems with polytopic uncertainty description subject to input constraints. The paper first proposes a method to determine an initial robustly feasible invariant set. This set is used as starting point for an optimization scheme that increases the volume of the set by solving a sequence of convex programs. A numerical example shows the efficiency of the proposed methods.

KEY WORDS
Polytopic invariant sets; Optimization; Constrained control

1 Introduction

The notion of invariant sets arises in many problems concerning analysis of dynamical systems, controller design and the construction of Lyapunov functions. An overview of the concept of set invariance can be found in the overview paper [4]. Typically invariant sets are ellipsoidal or polyhedral as both sets are convex and give raise to convex terminal constraints. Examples of algorithms capable of determining ellipsoidal invariant sets can be found in [3],[8] and [10]. Though ellipsoidal sets scale well with the state dimension they have the drawback they lead to the online solution of a SDP (semi-definite programming) when used as terminal sets for an MPC controller. Polytopic sets, however, consist of a set of linear constraints and therefore lead to the online solution of a QP (quadratic program) when used as terminal sets. Therefore, the main focus of this work will go to polytopic sets.

In [6] a method was proposed for constructing polyhedral invariant sets for linear time-invariant systems. The proposed method constructs an invariant set by iteratively adding additional constraints until invariance is obtained. In [2] these results were extended towards uncertain linear systems with polytopic model uncertainty. However, when dealing with higher dimensional systems, these methods could still lead to a large number of non-redundant constraints, making the algorithm impractical in these cases. Therefore, in [1] an algorithm was introduced able to make a trade-off between the number of constraints and the volume of the invariant set. This resulted to an algorithm that was capable to determine invariant sets for uncertain linear systems in high dimensions.

The calculation of low complexity polytopes is an approach that leads to sets that can be represented with a number of constraints that scale linearly with the state dimension. In [9] an algorithm to determine low-complexity invariant polytopes based on the L_∞-norm for nonlinear systems was proposed. Within an inclusion polytope the nonlinear system was described as an uncertain linear system for which a low-complexity polytope was derived based on a vertex-based optimization approach. However, vertex-based approaches are not suited for low-complexity polytopes based on the L_∞-norm because the number of vertices increases exponentially with the state dimension. Moreover, the proposed method consisted of an iterative procedure that needed an initial robustly feasible invariant set in order to start the optimization sequence. However, no method to determine such a starting set was proposed nor is such a method known in literature.

In this paper we consider the problem of constructing robustly feasible invariant low-complexity polytopes based on the L_1-norm. These polytopes have the advantage that the number of vertices scales linearly with the state dimension. This property makes vertex-based optimization schemes suited to determine invariant L_1-norm based sets for high dimensional systems. First a method is described to find an initial robustly feasible invariant L_1-norm based set by solving a number of LMI's. This set is than used as a starting point for an optimization scheme that at each iteration increases the volume of the resulting set by solving a convex program. A numerical example compares the methods proposed in this paper with the ones discussed in [9] and shows that the methods in this paper outperform those in [9].

This paper is organised as follows. In section 2 the problem statement is presented. In section 3 a method is described to determine an initial robustly feasible invariant L_1-norm based set for linear time-invariant systems as well as for uncertain linear systems with polytopic uncertainty. In the second part of section 3 a method is discussed that increases the initial robustly feasible invariant set by solving a sequence of convex programs. In section 4 the proposed algorithms are applied on a numerical example and compared with the methods in [9]. In section 5 some conclusions and future directions are discussed.

2 Problem Description

In this work the main focus will go to uncertain discrete-time systems of the following form:

$$x_{k+1} = \widetilde{A}x_k + \widetilde{B}u_k. \qquad (1)$$

with $\widetilde{A}$ and $\widetilde{B}$ uncertain, possible time-varying belonging to the polyhedral uncertainty class

$$(\widetilde{A}, \widetilde{B}) \in Co\left\{(A_1, B_1), \ldots, (A_{n_p}, B_{n_p})\right\}. \qquad (2)$$

Constraints on the inputs are assumed to be convex

$$u_k \in \{u| \, |u| \leq \bar{u}\} \qquad (3)$$

with $x \in \mathbb{R}^{n_x \times 1}$ and $u \in \mathbb{R}^{n_u \times 1}$. The extension to state constraints and/or non-symmetric constraints is straightforward and will not be discussed here.

The invariant sets are assumed to have the following form:

$$\varphi = \{x| \, \|Vx\|_1 \leq \gamma\} \qquad (4)$$

Invariance of the uncertain system (1) under a linear state feedback K requires $\forall x \in \varphi$ and $\forall (\widetilde{A}, \widetilde{B}) \in \Omega$

$$\left\| V\left(\widetilde{A} + \widetilde{B}K\right)x \right\|_1 \leq \|Vx\|_1 . \qquad (5)$$

It can be shown that it is sufficient to invoke condition (5) on the vertices (A_i, B_i) of the polytopic uncertainty description

$$\|V\left(A_i + B_iK\right)x\|_1 \leq \|Vx\|_1 , i = 1, \ldots, n_p. \qquad (6)$$

However, the invariance conditions (6) only imply Lyapunov stability. To improve the performance the invariance conditions can be strengthened into

$$\|V\left(A_i + B_iK\right)x\|_1 \leq \alpha \|Vx\|_1 , i = 1, \ldots, n_p. \qquad (7)$$

with $\alpha < 1$. Condition (7) ensures assymptotic convergence of the state as it can easily been shown that it leads to:

$$\forall x \in \varphi, i \in \{1, \ldots, n_p\} : \lim_{k \to \infty} \left(A_i + B_iK\right)^k x = 0. \qquad (8)$$

Note that for $n_p = 1$ the system (1) describes a linear time-invariant system, so the methods discussed in the remainder of the paper are also applicable on this type of systems.

3 Results

In this section the main algorithm is discussed. In the first part the notion β-contractive set is introduced and used in order to determine an initial feasible invariant set solving a sequence of LMI's. In the second part this set is used as a starting point for an algorithm that iteratively increases the volume of the set ensuring feasibility and invariance of the resulting set by solving a sequence of convex programs.

3.1 Initialization

In subsection 3.2 an iterative procedure is outlined in order to determine an invariant set with maximum volume. In order for the procedure to guarantee a feasible solution at each iteration an initial feasible invariant set has to be determined that can be used as a feasible starting point for the procedure. In the sequel a method will be discussed to find such a set. For a time-invariant linear system an initial invariant set can be found by using the following theorem:

Theorem 3.1. *For each linear feedback matrix K that stabilizes the time-invariant system $x_{k+1} = Ax_k + Bu_k$ in such a way that the eigenvalues λ_i of $A + BK$ are real the vertices of an initial invariant set induced by the 1-norm can be found as the n eigenvectors of $A + BK$.*

Proof. The set $\varphi = \{x| \, \|Vx\|_1 \leq 1\}$ is invariant for the time-invariant system if the corresponding vertices v_i satisfy $\|V(A + BK)v_i\|_1 \leq \|Vv_i\|_1$. Suppose v_i is a real eigenvector of $A + BK$ corresponding to a real eigenvalue λ_i, than $\|V(A + BK)v_i\|_1 = |\lambda_i| \, \|Vv_i\|_1$. Since K is stabilizing and the eigenvalues are assumed real this means $|\lambda_i| \leq 1$. Therefore $\|V(A + BK)v_i\|_1 \leq \|Vv_i\|_1$ and the set φ is invariant. □

Remark that if the linear time-invariant system is stabilizable, it is always possible to find such a feedback matrix K by choosing n stable λ_i and using the pole placement method [5]. This procedure, however, cannot be used for the time-variant system (1). In order to find an initial invariant set for uncertain linear systems the notion of robustly β-contractive set is introduced.

Definition 1. *An ellipsoidal set $\mathcal{E}$ is called robustly β-contractive for system (1) if and only if $\forall x \in \mathcal{E}$ and $\forall (\widetilde{A}, \widetilde{B}) \in Co\left\{(A_1, B_1), \ldots, (A_{n_p}, B_{n_p})\right\}$ it follows that $(\widetilde{A} + \widetilde{B}K)x \in \beta\mathcal{E}$.*

Remark 1. *From the definition it follows that a robustly β-contractive set is also invariant for the uncertain system (1).*

The following theorem uses this notion in order to determine robustly invariant L_1-norm based sets for linear systems with polytopic uncertainty:

Theorem 3.2. *Consider the robustly β-contractive ellipsoidal set $\mathcal{E} = \{x | \|Vx\|_2 \leq \gamma\}$. The set $\varphi = \{x | \|Vx\|_1 \leq \gamma\}$ is robustly invariant for system (1) w.r.t. the state feedback K if the following condition is satisfied:*

$$\beta\mathcal{E} \subseteq \varphi \subseteq \mathcal{E} \tag{9}$$

$$\forall(\widetilde{A}, \widetilde{B}) \in Co\left\{(A_1, B_1), \ldots, (A_{n_p}, B_{n_p})\right\}.$$

Proof. From the property $\|x\|_1 \geq \|x\|_2$ it follows that $\|Vx\|_1 \geq \|Vx\|_2$. Therefore, if $x \in \varphi \Rightarrow x \in \mathcal{E}$ meaning that $\varphi \subseteq \mathcal{E}$. Since $\mathcal{E}$ is β-contractive and $\varphi \subseteq \mathcal{E}$ it follows $\forall x \in \varphi$ that $(\widetilde{A} + \widetilde{B}K)x \in \beta\mathcal{E}$. If $\beta\mathcal{E} \subseteq \varphi$ than the set φ is invariant because $(\widetilde{A} + \widetilde{B}K)x \in \beta\mathcal{E} \subseteq \varphi$. □

Remark 2. *Theorem 9 provides sufficient (not necessary) conditions for invariance of the set φ.*

The following set of LMI's determines a robustly β-contractive set ([3],[8] and [10]):

Algorithm 1.

$$\min_{Q,X,Y} -log\ det(Q) \tag{10}$$

subject to

$$Q \succeq 0 \tag{11}$$

$$\begin{bmatrix} X & Y \\ Y^T & Q \end{bmatrix} \succeq 0, \quad X(j,j) \leqslant \bar{u}^2 \ , j = 1, \ldots, n_u \tag{12}$$

$$\begin{bmatrix} \beta^2 Q & (A_iQ + B_iY)^T \\ A_iQ + B_iY & Q \end{bmatrix} \succeq 0, j = 1, \ldots, n_p \tag{13}$$

By taking $P = Q^{-1}$ and $K = YQ^{-1}$ a feedback is obtained stabilizing the uncertain system (1) in such a way that the ellipsoidal set $\mathcal{E} = \left\{x | \left\|P^{0.5}x\right\|_2 \leq 1\right\}$ is β-contractive invariant and satisfies the input constraint (3). Note that the vertices of the corresponding L_1-norm set $\varphi = \left\{x | \left\|P^{0.5}x\right\|_1 \leq 1\right\}$ lie on the boundary of the ellipsoidal set $\mathcal{E}$. It is easy to see that for a sufficiently small contraction rate β the set φ satisfies the conditions of Theorem 3.2 and is robustly invariant. Therefore, in order to determine an initial robustly feasible invariant set for the uncertain linear system (1) the following strategy is proposed:

1. Initialize $\beta = \beta_0$ with $\beta_0 < 1$.

2. Solve the LMI of algorithm 1.

3. If the solution satisfies condition (9) a robustly feasible invariant set is obtained. If the condition is not satisfied decrease β and go back to step 2.

This iteration is needed because it is not clear how to determine the upper bound $\bar{\beta}$ that ensures for each $\beta \leq \bar{\beta}$ that the 1-norm set φ resulting from algorithm 1 is invariant.

3.2 Maximal Robustly Feasible Invariant Set

The low-complexity polytope described in (4) consists of $2n$ vertices. The set φ is completely determined by the n linearly independent primary vertices $v_1, \ldots, v_n$. All other vertices can be expressed in terms of $\{v_1, \ldots, v_n\}$ as follows:

$$v_i = -v_{i-n}, i = n+1, \ldots, 2n. \tag{14}$$

At each vertex the following relation holds

$$Vv_i = e_i, \qquad i = 1, \ldots, n \tag{15}$$

$$Vv_i = -e_i, \qquad i = n+1, \ldots, 2n \tag{16}$$

with e_i the i-th column of the identity matrix. Therefore following relation holds between V and the primary vertices

$$V = \begin{bmatrix} v_1 & \ldots & v_n \end{bmatrix}^{-1}. \tag{17}$$

From [7] it can easily been shown that the volume of the set (4) is proportional to $|det(V)|$. Note that the number of vertices scales linearly with the state dimension, in contrary to the low-complexity polytopes described in [9] which have an amount of vertices that increase exponentially with the state dimension. Therefore, combining vertex-based approaches with sets of the form (4) lead to algorithms that scale linearly with the state dimension. The following theorem concerns a vertex-based optimization scheme in order to determine maximal volume feasible invariant sets of the form (4).

Theorem 3.3. *The maximum feasible invariant low-complexity polytope is the solution of the nonlinear program:*

$$\min_{v_1,\ldots,v_n,w_1,\ldots,w_n} -\log\left(\left|\det\left(\begin{bmatrix} v_1 & \ldots & v_n \end{bmatrix}\right)\right|\right) \tag{18}$$

subject to the following equations for $j = 1, \ldots, n_p$:

$$\begin{aligned} &A_j \begin{bmatrix} v_1 & \ldots & v_n \end{bmatrix} + B_j \begin{bmatrix} w_1 & \ldots & w_n \end{bmatrix} = \\ &\begin{bmatrix} v_1 & \ldots & v_n & -v_1 & \ldots & -v_n \end{bmatrix} Q_j \end{aligned} \tag{19}$$

$$|w_i| \leq \bar{u} \tag{20}$$

$$\left[\mathbf{1}^T \mathbf{1}^T\right] Q_j = \mathbf{1}^T \tag{21}$$

$$Q_j \geq 0 \tag{22}$$

with $\mathbf{1} = \left[\begin{array}{ccc} 1 & \dots & 1 \end{array}\right]^T \in \mathbb{R}^n$*. The corresponding stabilizing feedback K can be obtained as*

$$K = \left[\begin{array}{ccc} w_1 & \dots & w_n \end{array}\right] \left[\begin{array}{ccc} v_1 & \dots & v_n \end{array}\right]^{-1}. \quad (23)$$

Proof

Condition (19) together with (21)-(22) ensure the resulting set is invariant. To see this note that column p of constraint (19) can be re-written as $Av_p + Bw_p = \sum_{i=1}^{n} Q_j(i,p)v_i + \sum_{i=1}^{n} Q_j(n+i,p)(-v_i)$. Because (22) ensures $Q_j(i,p) \geq 0$ and (21) ensures $\sum_{i=1}^{2n} Q_j(i,p) = 1$ this means that $A_j v_p + B_j w_p \in$ Co$\{v_1,\dots,v_n,-v_1,\dots,-v_n\}$ and the set is invariant. Condition (20) ensures the input constraints are satisfied. Expression (23) follows trivially from $\left[\begin{array}{ccc} w_1 & \dots & w_n \end{array}\right] = K \left[\begin{array}{ccc} v_1 & \dots & v_n \end{array}\right]$. □

The program of Theorem 3.3 is nonlinear due to the nonlinear cost function (18) and the bilinear invariance constraints (19). However, by introducing some conservativeness it is possible to recast this problem into a convex programming problem as stated below.

Theorem 3.4. *The nonlinear program of Theorem 3.3 can be computed by solving the following convex program iteratively:*

$$\min_{\substack{v_1,\dots,v_n,w_1,\dots,w_n \\ Q_1,\dots,Q_{n_p} \\ a_1,\dots,a_n}} \sum_{i=1}^{n} -\log a_i \quad (24)$$

subject to the following linear constraints for $i = 1,\dots,n, j = 1,\dots,n_p$

$$A_j \left[\begin{array}{ccc} v_1 & \dots & v_n \end{array}\right] + B_j \left[\begin{array}{ccc} w_1 & \dots & w_n \end{array}\right] = \quad (25)$$

$$\left[\begin{array}{cccccc} v_1^0 & \dots & v_n^0 & -v_1^0 & \dots & -v_n^0 \end{array}\right] Q_j \quad (26)$$

$$\left[\begin{array}{ccc} v_1 & \dots & v_n \end{array}\right] = \quad (27)$$

$$\left[\begin{array}{ccc} v_1^0 & \dots & v_n^0 \end{array}\right] \left[\begin{array}{ccc} a_1 & \mathbf{0}^T & 0 \\ \mathbf{0} & \ddots & \mathbf{0} \\ 0 & \mathbf{0}^T & a_n \end{array}\right] \quad (28)$$

$$|w_i| \leqslant u \quad (29)$$

$$\left[\begin{array}{cc} \mathbf{1}^T & \mathbf{1}^T \end{array}\right] Q_j = \mathbf{1}^T \quad (30)$$

$$Q_j \geq 0 \quad (31)$$

$$a_1,\dots,a_n \geq 1 \quad (32)$$

with $\mathbf{0} = \left[\begin{array}{ccc} 0 & \dots & 0 \end{array}\right]^T \in \mathbb{R}^{n-2}$ *and the robustly invariant feasible set* $\left\{v_1^0,\dots,v_n^0,-v_1^0,\dots,-v_n^0\right\}$ *computed at the previous iteration.*

Proof.

Similar to the proof of Theorem 3.3 conditions (26), (30)-(31) ensure for each $p = 1,\dots,2n$:

$$A_j v_p + B_j w_p \in Co\left\{v_1^0,\dots,v_n^0,-v_1^0,\dots,-v_n^0\right\}. \quad (33)$$

In order for condition (33) to guarantee invariance it is necessary that $Co\left\{v_1^0,\dots,v_n^0,-v_1^0,\dots,-v_n^0\right\} \subseteq Co\{v_1,\dots,v_n,-v_1,\dots,-v_n\}$. This is ensured by condition (28). To see this note that (28) is equivalent to $\frac{1}{a_p}v_p = v_p^0$. Condition (32) ensures $1 \geq \frac{1}{a_p} \geq 0$. It can easily be shown that for each $p = 1,\dots,n$ there exist $\lambda_1^p,\lambda_2^p \geq 0$ such that $v_p^0 = \lambda_1^p v_p + \lambda_2^p(-v_p)$ with $\lambda_1^p - \lambda_2^p = \frac{1}{a_p}$ and $\lambda_1^p + \lambda_2^p = 1$. Therefore, it is ensured that $v_p^0 \in Co\{v_1,\dots,v_n,-v_1,\dots,-v_n\}$. Because this holds for all p it is ensured that $Co\left\{v_1^0,\dots,v_n^0,-v_1^0,\dots,-v_n^0\right\} \subseteq Co\{v_1,\dots,v_n,-v_1,\dots,-v_n\}$. Condition (29) ensures the input constraints are satisfied. From condition (28) follows that $\left|\det\left(\left[\begin{array}{ccc} v_1 & \dots & v_n \end{array}\right]\right)\right| \sim \prod_{i=1}^{n} a_i$. Therefore the cost function (24) ensures the volume of the set $Co\{v_1,\dots,v_n,-v_1,\dots,-v_n\}$ is maximized. □

Remark 3. *The feedback gain* K *is recoverable from the solution for* $\{v_i, w_i, i = 1,\dots,n\}$ *via (23).*

Remark 4. *The initialization procedure of subsection 3.1 provides a feasible starting point for the procedure of Theorem 3.4. Therefore, Theorem 3.4 generates a sequence of feasible robustly invariant sets with nondecreasing volume.*

4 Numerical Example

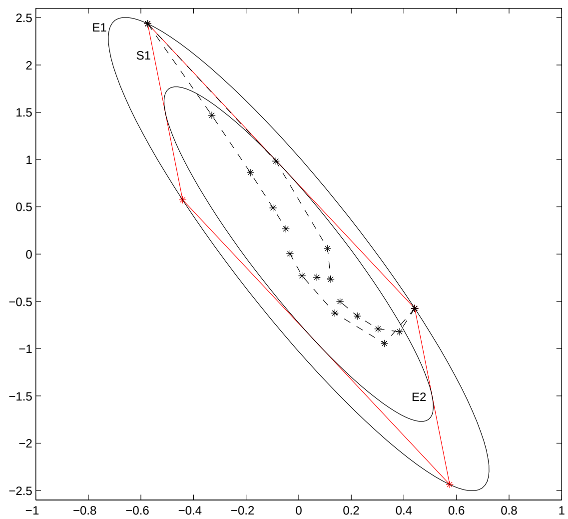
In this section a numerical example is used to compare the efficiency of the algorithm described in Theorem 3.4 with the algorithm discussed in [9]. In [9] an iterative sequence of LP's is proposed in order to find the maximal volume robustly feasible L_∞-norm based invariant set. However, with some minor modifications these results can easily be extended to L_1-norm based sets. The uncertain model in this example is the same as the one discussed in [2]. The model and constraints are given by:

$$A_1 = \left[\begin{array}{cc} 1 & 0.1 \\ 0 & 1 \end{array}\right], \quad B_1 = \left[\begin{array}{c} 0 \\ 1 \end{array}\right], \qquad A_2 = \left[\begin{array}{cc} 1 & 0.2 \\ 0 & 1 \end{array}\right], \quad B_2 = \left[\begin{array}{c} 0 \\ 1.5 \end{array}\right], \quad (34)$$

$$\overline{u} = 1. \quad (35)$$

Using the results of subsection 3.1 an initial robustly feasible invariant polytopic set is obtained that is depicted in figure 1, together with the corresponding ellipsoids. The trajectories of the vertices of the polytopic set $S1$ are also depicted and clearly show the set is robustly invariant. Ellipsoid $E1 = \left\{x | x^T P x \leqslant 1\right\}$ contracts in the next time

Figure 1. Initial robustly feasible invariant polytope.



step into ellipsoid $E2 = \left\{x|x^T(\frac{1}{\beta^2}P)x \leqslant 1\right\}$. From the figure it can be seen clearly that $E2 \subseteq S1 \subseteq E1$ which corresponds to the invariance condition (9). The primary vertices of the set S_1 are:

$$v_1 = \begin{bmatrix} 0.44 \\ -0.57 \end{bmatrix} \quad v_2 = \begin{bmatrix} -0.57 \\ 2.44 \end{bmatrix} \tag{36}$$

and the corresponding feedback K for which the system stays inside $S1$ is

$$K = \begin{bmatrix} -1.555 & -0.764 \end{bmatrix}. \tag{37}$$

Note that the input constraints are satisfied. The determined contraction rate β is equal to 0.7.

The matrix P determining the ellipsoids E_1 and E_2 is

$$P = \begin{bmatrix} 11.25 & 2.97 \\ 2.97 & 0.94 \end{bmatrix}. \tag{38}$$

Figure 2 depicts the resulting set after applying the volume maximization of Theorem 3.4 on the initial feasible set. The obtained optimal solution is the set $S2$. The paths of the vertices are also depicted in the figure and show the set is robustly invariant. The primary vertices of the set $S2$ are

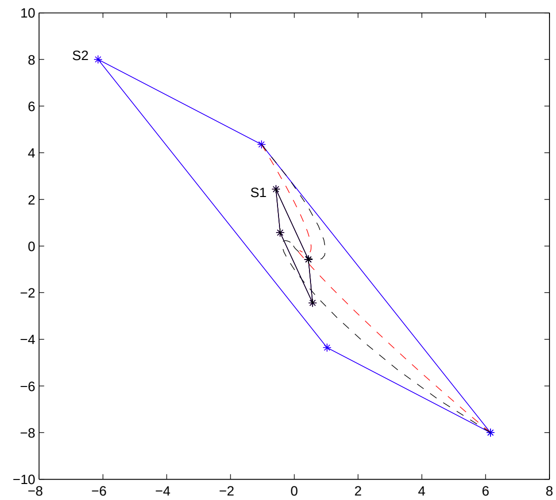
$$v_1 = \begin{bmatrix} 6.2 \\ -8 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1.1 \\ 4.4 \end{bmatrix} \tag{39}$$

and the corresponding feedback is

$$K = \begin{bmatrix} -0.213 & -0.280 \end{bmatrix}. \tag{40}$$

Therefore the inputs of the system are equal to

Figure 2. Maximal robustly feasible invariant polytope obtained with the algorithm described in Theorem 3.4.



$$w_1 = 0.9, \quad w_2 = -1.0 \tag{41}$$

and satisfy the input constraints. Also note that extending the iterative LP sequence specified in [9] to L_1-norm based sets does not increase the volume of the initial invariant set $S1$ showing that the algorithm in Theorem 3.4 outperforms that of [9].

5 Conclusion

5.1 Conclusion

A strategy is proposed to find an initial robustly feasible invariant low-complexity polytope based on the L_1-norm for linear time-invariant systems as well as for uncertain linear systems with polytopic uncertainty subject to input constraints. This set is used as starting point in an iterative procedure that increases the volume of the set by solving a sequence of convex programs. The main advantage of the proposed method is that the number of vertices of the proposed low-complexity polytope scales linearly with the state dimension. Therefore, the vertex-based optimization approach of this paper makes it possible to calculate invariant low-complexity polytopes for high dimensional systems. The efficiency of the methods were shown with a numerical example. The example showed that the methods in this work outperform that of [9].

5.2 Future Works

The strategy proposed in this paper in order to determine an initial robustly feasible invariant 1-norm set consists in solving an iterative sequence of LMI's trying to find a $\beta \leq \bar{\beta}$ such that invariance of the 1-norm set is guaranteed. However, if $\bar{\beta}$ could be determined in advance an initial invariant set could be found solving just 1 LMI for $\beta = \bar{\beta}$ and therefore speeding up this initialization step. Also conditions must be found for which it is guaranteed the LMI of algorithm 1 returns a feasible solution for $\beta = \bar{\beta}$. In a next step the volume maximization algorithm could be modified. By chosing a different type of conservatism a new sequence of convex programs might be developed leading to possibly bigger invariant sets. Also extending these results to systems with bounded disturbances could be a future research direction.

Acknowledgements: Research supported by: Research Council KUL: GOA AMBioRICS, CoE EF/05/006 OPTEC, IOF-SCORES4CHEM, several PhD/postdoc and fellow grants;Flemish Government:FWO (PhD/postdoc grants, G.0452.04, G.0499.04, G.0211.05, G.0226.06, G.0321.06, G.0302.07, G.0320.08, G.0558.08, G.0557.08, research communities (ICCoS, ANMMM, MLDM)); IWT (PhD Grants, McKnow-E, Eureka-Flite+);Helmholtz: viCERP; Belgian Federal Science Policy Office: IUAP P6/04 (DYSCO, Dynamical systems, control and optimization, 2007-2011) ;EU: ERNSI; Contract Research: AMINAL

References

[1] Pluymers B. *Robust Model Based Predictive Control - An Invariant Set Approach*. PhD thesis, Faculty of Engineering, K.U.Leuven (Leuven, Belgium), May 2006.

[2] J. Suykens B. De Moor B. Pluymers, J.A. Rossiter. The efficient computation of polyhedral invariant sets for linear systems with polytopic uncertainty. *Proceedings of the American Control Conference*, 2:804–809, 2005.

[3] El Ghaoui L. Feron E. Boyd, S. and V. Balakrishnan. *Linear matrix inequalities in system and control theory.* Philadelphia: SIAM, 1994.

[4] F.Blanchini. Set invariance in control. *Automatica*, 35:1747–1767, 1999.

[5] J . D. Powell G. F. Franklin and A. Emami-Naeini. *Feedback Control of Dynamic Systems*. Addison-Wesley Publishing Company, Reading, 1986.

[6] E. G. Gilbert and Tan. K. T. Linear systems with state and control constraints : The theory and application of maximal output admissable sets. *IEEE Transactions on Automatic Control*, 36(9):1008–1020, 1991.

[7] Klee V. Gritzmann, P. *On the complexity of some problems in computational convexityII: Volume and mixed volumes.* In Polytopes: abstract, convex and computational, Vol. 440 of NATO ASI Series C. Dordrecht: Kluwer Academic., 1994.

[8] V.Balakrishnan M.Kothare and M. Morari. Robust constrained model predictive control using linear matrix inequalities. *Automatica*, 32:1361–1379, 1996.

[9] B. Kouvaritakis V. Deshmukh, M. Cannon. Nonlinear model predictive control with polytopic invariant sets. *Automatica*, 39:1487–1494, 2003.

[10] M. Cannon Y.I. Lee and B. Kouvaritakis. Extended invariance and its use in model predictive control. *Automatica*, 41:2163–2169, 2005.