Online semi-supervised clustering regularized by Kalman filtering

Siamak Mehrkanoon, Oscar Mauricio Agudelo and Johan A. K. Suykens
KU Leuven, ESAT-STADIUS, Kasteelpark Arenberg 10, B-3001 Leuven (Heverlee), Belgium.
Email: Siamak.Mehrkanoon@esat.kuleuven.be

1 Introduction

An on-line semi-supervised learning algorithm formulated as a regularized kernel spectral clustering (KSC) approach is proposed. Given a few user-labeled data points the initial model is learned and then the class membership of the remaining data points in the current and subsequent time instants are estimated and propagated in an on-line fashion. Furthermore we show how the tracking capabilities of the Kalman filter can be used to provide the labels of objects in motion and thus regularizing the solution obtained by the MSS-KSC algorithm.

2 Formulation of the method

Consider training data points $\mathscr{D} = \{x_1, \ldots, x_{n_u}, x_{n_u+1}, .., x_n\}$, where $\{x_i\}_{i=1}^n \in \mathbb{R}^d$. The first n_u data points do not have labels whereas the last $n_L = n - n_u$ points have been labeled. Assume that there are Q classes, then the label indicator matrix $Y \in \mathbb{R}^{n_L \times Q}$ is defined as follows: $Y_{ij} = +1$, if the ith point belongs to the jth class and -1 otherwise. The formulation of Multi-class semi-supervised KSC (MSS-KSC) in primal is given as follows [1]:

$$\begin{aligned} \min_{w^{(\ell)}, b^{(\ell)}, e^{(\ell)}} \quad & \frac{1}{2}\sum_{\ell=1}^{Q} w^{(\ell)T} w^{(\ell)} - \frac{\gamma_1}{2}\sum_{\ell=1}^{Q} e^{(\ell)T} V e^{(\ell)} + \\ & \frac{\gamma_2}{2}\sum_{\ell=1}^{Q} (e^{(\ell)} - c^{(\ell)})^T \tilde{A} (e^{(\ell)} - c^{(\ell)}) \\ \text{subject to} \quad & e^{(\ell)} = \Phi w^{(\ell)} + b^{(\ell)} 1_n, \; \ell = 1, \ldots, Q, \end{aligned} \tag{1}$$

where c^{ℓ} is the ℓ-th column of the matrix C defined as

$$C = [c^{(1)}, \ldots, c^{(Q)}]_{n \times Q} = \left[\frac{0_{n_u \times Q}}{Y}\right]_{n \times Q}, \tag{2}$$

where $0_{n_u \times Q}$ is a zero matrix of size $n_u \times Q$ and Y is defined as previously. The matrix $\tilde{A}$ is defined as follows:

$$\tilde{A} = \left[\begin{array}{c|c} 0_{n_u \times n_u} & 0_{n_u \times n_L} \\ \hline 0_{n_L \times n_u} & I_{n_L \times n_L} \end{array}\right],$$

where $I_{n_L \times n_L}$ is the identity matrix of size $n_L \times n_L$. V is the inverse of the degree matrix. In the dual the solution is obtained by solving a system of linear equations [1]:

$$\gamma_2 \left(I_n - \frac{R 1_n 1_n^T}{1_n^T R 1_n}\right) c^{(\ell)} = \alpha^{(\ell)} - R\left(I_n - \frac{1_n 1_n^T R}{1_n^T R 1_n}\right) \Omega \alpha^{(\ell)}, \tag{3}$$

where $R = \gamma_1 V - \gamma_2 \tilde{A}$. The proposed on-line MSS-KSC consists of two stages [2]. In the first stage, one trains the MSS-KSC algorithm to obtain the initial solution vectors α_i and the cluster memberships. Assuming that N_c clusters are detected, the initial cluster representatives are then obtained. The aim of the second stage is to predict the membership of the new arriving data points using the updated solution vectors α_i. The score variables of the arrived batch of new data points is estimated by means of the out-of-sample extension property of the MSS-KSC method. Then the estimation of the projection of the points in the α-space and calculating the membership of the points are carried out. Finally the cluster representatives in both α and original spaces are updated. The interaction between Kalman filter and I-MSS-KSC algorithm is shown in Fig. 1.

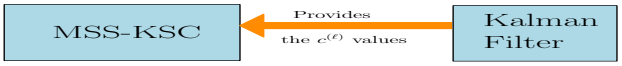

Figure 1: Kalman filter acts as a regularizer for the MSS-KSC algorithm

The applicability of the approach for online video segmentation are shown in the Fig. 2.

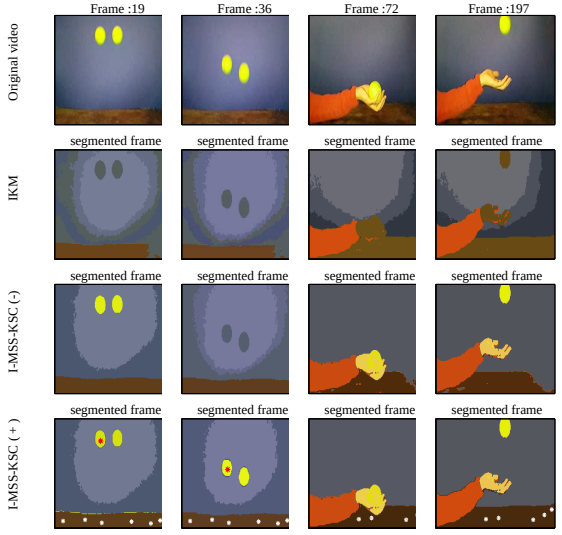

Figure 2: Bouncing balls and Siamak's hand video. **First row:** Original video, **Second row:** IKM, **Third row:** I-MSS-KSC(-), **Fourth row:** I-MSS-KSC(+)

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