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CONTRIBUTION

TITLE: Optimal Spectrum Management

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ABSTRACT

This document contains a detailed description of the *Optimal Spectrum Management* (OSM) algorithm for inclusion in the DSM report.

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Optimal Spectrum Management

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ABSTRACT

This document contains a detailed description of the *Optimal Spectrum Management* (OSM) algorithm for inclusion in the DSM report.

1. Optimal Spectrum Management

The OSM algorithm calculates the theoretically optimal spectra for a network of communication systems. This is done through the application of *optimisation theory* and a technique known as the *dual decomposition*. For ease of explanation we will describe the algorithm as applied to a system with 2 users. Extensions to more than 2 users will follow in Section 4.

The OSM algorithm was presented at the previous T1E1.4 meeting [2] and a specific text was requested for inclusion in the DSM report. This text is given below.

2. The Spectrum Management Problem – 2 user case

The basic problem of spectrum management is to maximize the rate of a user (in this case user 2), subject to minimum service rates for the other users within the network (in this case user 1). Mathematically we need to maximize the rate of user 2 over all of the possible transmit PSDs for user 1 and user 2

$$\max_{\mathbf{s}_1,\mathbf{s}_2} R_2(\mathbf{s}_1,\mathbf{s}_2)$$

$$\text{s.t. } R_1(\mathbf{s}_1,\mathbf{s}_2) \geq R_1^{\text{target}}$$

$$\sum_k s_k^1 \leq P_{\max}$$

$$\sum_k s_k^2 \leq P_{\max}$$

Target rate constraint for user 1

Power constraints for users 1 and 2

Here s_k^n is the transmit PSD of user n on tone k, $\mathbf{s}_n = [s_1^n \cdots s_K^n]$ is a vector containing the transmit PSD of user n on all tones, and $P_{\max}$ is the maximum transmit power supported by a modem. $R_n(\mathbf{s}_1,\mathbf{s}_2)$ is the data-rate achieved by user n when transmit spectra $\mathbf{s}_1$ and $\mathbf{s}_2$ are used by user 1 and user 2 respectively. R_1^{target} is the target service data-rate for user 1.

Unfortunately (1) is a non-convex optimisation and requires complexity $O(e^{KN})$ to solve where K is the number of tones in the system and N the number of users. With K=256 in ADSL and K=4096 in VDSL this leads to a computationally intractable problem.

Using a technique from optimization theory known as the *dual decomposition* allows us to solve the spectrum management problem (1) with a linear complexity in *K*. This leads to the OSM algorithm which is computationally tractable and can be solved on a standard PC in a number of minutes.

3. The OSM Algorithm - 2 user case

The OSM algorithm is based on maximising the so-called *Lagrangian* on each tone. We start with a 2 user case for ease of explanation. The Lagrangian on tone *k* is then defined

$$L_k = wb_k^1(s_k^1, s_k^2) + (1-w)b_k^2(s_k^1, s_k^2) - \lambda_1 s_k^1 - \lambda_2 s_k^2$$

where $b_k^n(s_k^1, s_k^2)$ denotes the bitloading achieved by user *n* on tone *k* when user 1 and user 2 adopt transmit PSDs s_k^1 and s_k^2 respectively. The optimal transmit spectra on tone *k* are found by maximising L_k

$$s_k^{1,\text{opt}}, s_k^{2,\text{opt}} = \arg\max_{s_k^1, s_k^2} L_k$$

The weight *w* determines the desired trade-off of data-rates between user 1 and user 2. Setting $w = 1$ gives full priority to user 1 and user 2 is switched off. Setting $w = 0$ gives full priority to user 2 and user 1 is switched off. The terms λ_1 and λ_2 are the *Lagrangian multipliers* and enforce the power constraints on modems 1 and 2 respectively.

During operation the OSM algorithm adjusts *w* such that the target data-rate of user 1 is just achieved. The algorithm does not give more priority to user 1 than is necessary to achieve their target data-rate, thereby maximising the data-rate of user 2. In a similar fashion λ_1 and λ_2 are adjusted such that the power constraints on both modems are enforced.

The complete algorithm is listed below:

Algorithm 1: Optimal Spectrum Management – 2 Users

initialise w, λ_1, λ_2
while $R_1 \neq R_1^{\text{target}}$
  while ($\sum_k s_k^1 \neq P_{\max}$) and ($\lambda_1 > 0$)
    while ($\sum_k s_k^2 \neq P_{\max}$) and ($\lambda_2 > 0$)
      for each tone *k*: find PSD pair (s_k^1, s_k^2) which maximises $L_k(w,\lambda_1,\lambda_2,s_k^1,s_k^2)$
      if $\sum_k s_k^2 > P_{\max}$ increase λ_2, else decrease λ_2
    end
    if $\sum_k s_k^1 > P_{\max}$ increase λ_1, else decrease λ_1
  end
  if $R_1 < R_1^{\text{target}}$ increase *w*, else decrease *w*
end

4. The OSM Algorithm - *N* user case

We now describe the OSM algorithm in the general *N* user case. With *N* users we desire to maximize the rate of a user (in this case user *N*), subject to minimum service rates for the other users within the network (in this case users 1...*N*-1). Mathematically we need to maximize the rate of user *N* over all of the possible transmit PSDs for users 1...*N*

$$\max_{\mathbf{s}_1,\ldots,\mathbf{s}_N} R_N(\mathbf{s}_1,\ldots,\mathbf{s}_N)$$

$$\text{s.t. } R_n(\mathbf{s}_1,\ldots,\mathbf{s}_N) \geq R_n^{\text{target}} \ \forall n < N$$

$$\sum_k s_k^n \leq P_{\max} \forall n$$

Target rate constraints for users 1...*N*-1

Power constraints for users 1...*N*

With *N* users, *N*-1 weights $w_1 \ldots w_{N-1}$ are required. These enforce the target rates on users 1...*N*-1. We define the weight for user *N* arbitrarily as

$$w_N = 1 - \sum_{n=1}^{N-1} w_n$$

A Lagrangian multiplier λ_n is required for each user to enforce the total power constraint. The Lagrangian on tone *k* is then defined

$$L_k = \sum_{n=1}^{N} \left(w_n b_k^n(s_k^1,\ldots,s_k^N) - \lambda_n s_k^n \right)$$

where $b_k^n(s_k^1,\ldots,s_k^n)$ denotes the bitloading achieved by user *n* on tone *k* when the users adopt transmit PSDs $s_k^1,\ldots,s_k^N$. The optimal transmit spectra on tone *k* are found by maximising L_k

$$(s_k^{1,\text{opt}},\ldots,s_k^{N,\text{opt}}) = \arg \max_{s_k^1,\ldots,s_k^N} L_k$$

During operation the OSM algorithm adjusts $w_1 \ldots w_{N-1}$ such that the target data-rates of users 1...*N*-1 are just achieved. The algorithm does not give more priority to users 1...*N*-1 than is necessary to achieve their target data-rates, thereby maximising the data-rate of user *N*. In a similar fashion $\lambda_1 \ldots \lambda_N$ are adjusted such that the power constraints are enforced on each modem.

The complete algorithm is listed on the following page. For more details see [1].

Algorithm 2: Optimal Spectrum Management – *N* Users

initialise $w_1,\ldots, w_{N-1}$, $\lambda_1,\ldots,\lambda_N$
while $R_1 \neq R_1^{\text{target}}$
⋮
while $R_{N-1} \neq R_{N-1}^{\text{target}}$
while ($\sum_k s_k^1 \neq P_{\max}$) and ($\lambda_1 > 0$)
⋮
while ($\sum_k s_k^N \neq P_{\max}$) and ($\lambda_N > 0$)
$w_N = 1 - \sum_n w_n$
for each tone k:
find PSD tuple ($s_k^1,\ldots,s_k^N$) which maximises $L_k = \sum_n w_n b_k^n(s_k^1,\ldots,s_k^N) - \lambda_n s_k^n$
if $\sum_k s_k^N > P_{\max}$ increase λ_N, else decrease λ_N
end
⋮
if $\sum_k s_k^1 > P_{\max}$ increase λ_1, else decrease λ_1
end
if $R_{N-1} < R_{N-1}^{\text{target}}$ increase w_{N-1}, else decrease w_{N-1}
end
⋮
if $R_1 < R_1^{\text{target}}$ increase w_1, else decrease w_1
end

5. Proposal

We propose that the OSM algorithm be included (informative) in Appendix A of the DSM report for centralised (Level 2) operation.

References

[1] R. Cendrillon, W. Yu, M. Moonen, J. Verlinden, T. Bostoen, "Optimal Multiuser Spectrum Management for Digital Subscriber Lines," *submitted to IEEE Transactions on Communications, accepted for IEEE Intl. Conf. on Communications (ICC) 2004.*
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[2] R. Cendrillon, W. Yu, M. Moonen, J. Verlinden, T. Bostoen, "On the Optimality of Iterative Waterfilling in DSL," ANSI T1E1.4 Working Group (DSL Access) Meeting, contrib. 2003-325, San Diego, December 2003.

[3] W. Yu, G. Ginis, J. Cioffi, "Distributed Multiuser Power Control for Digital Subscriber Lines," in *IEEE Journal on Selected Areas in Communications*, vol. 20, no. 5, pp. 1105 – 1115, June 2002.