A Detection Guided Normalized Least-Mean-Squares Adaptive Partial Crosstalk Canceller for Multi-User DSL Environments

Mandar L. Gujrathi, John Homer, I. Vaughan L. Clarkson, Raphael Cendrillon, and Marc Moonen

***Abstract*—Block crosstalk cancellation techniques in practical multi-user digital subscriber line (DSL) environments may involve a high computational complexity as the channel and noise statistics can vary over time. We follow an adaptive approach by designing a structurally consistent significance-test feature within the normalized least-mean-square (NLMS) adaptive crosstalk canceller, aimed to detect significant crosstalkers within a DSL binder. The proposed detection-guided NLMS adaptive partial crosstalk canceller for DSL targets the dominant crosstalkers across user lines and tones, has low run-time complexity, demonstrates significantly faster convergence, and requires smaller training sequences when compared via simulation to the equivalent standard NLMS adaptive crosstalk canceller.**

***Index Terms*—Adaptive canceller, column-wise diagonal dominant (CWDD), detection, normalized least mean squares (NLMS), partial crosstalk cancellation.**

I. Introduction

In a multi-user digital subscriber line (DSL) environment, performance degradation can be caused by near end crosstalk (NEXT) and far end crosstalk (FEXT) [1], [2] and by intercarrier crosstalk, which limits the data rate and service reach. Frequency division duplexing can be used to avoid NEXT, while various block processing techniques can be applied at the receivers for FEXT supression. This can be achieved by deploying crosstalk cancellers (for upstream) or crosstalk pre-compensators (for downstream) at the central office (CO) or DSL access multiplexers (DSLAMs), as shown in Fig. 1, depending on whether the channel state information (CSI) is available after reception or before transmission respectively. Dynamic spectrum management (DSM) in DSL aims to maximise performance by spectrum optimization and joint signal processing through co-ordination (via co-location) of modems in the DSLAM [3]. Further, synchronization of the DSLAMs to a common clock [3] eliminates intercarrier crosstalk.

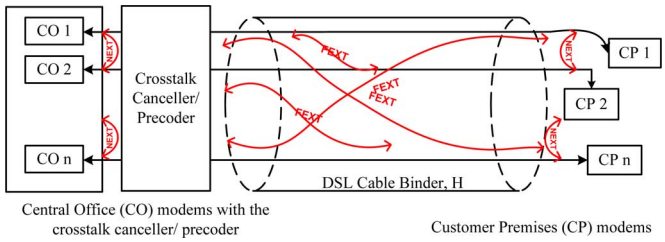

Fig. 1. Multi-user DSL binder with crosstalk canceller.

Block crosstalk cancellation techniques, e.g., zero-forcing, can involve high computational complexity if the intrinsic and extrinsic factors of the DSL channel [4] are considered, making it time varying. Adaptive cancellers address these variations using a gradient-descent rule to update (typically on per sample basis) the coefficients of the canceller [5]. Within a DSL binder, only those lines in close proximity produce appreciable mutual crosstalk [6]. In this paper, we assume that each line in a DSL binder has only a small number of nearby crosstalkers. The crosstalk contribution from other lines is considered zero [6], which makes the channel matrix sparse. Using this property, we propose an adaptive estimation of the dominant crosstalking users via an enhanced normalized least-mean-square (NLMS) approach. Classical (Standard) NLMS is known for its reduced complexity and ease of implementation [5]. However when applied to the DSL crosstalk problem, it treats all the crosstalkers as significant, delaying the convergence thereby requiring longer training sequences. With a view to faster convergence and low run-time complexity, we augment the NLMS algorithm with an activity criterion designed to detect and adapt only to the significant crosstalkers. During the training/initiation phase of the new detection-guided NLMS it is envisaged that, the co-located modems will co-operate to obtain direct and crosstalk CSI. Thereafter, the modems enter an operational phase ("Showtime") in which small variations are tracked using occasional pilot tones.

The paper is organised as follows. Section II gives an overview of the signalling environment in a multi-user DSL system. In Section III we introduce our activity criterion to detect significant crosstalkers using the least squares approach and propose the new detection-guided NLMS adaptive partial cancellation algorithm. We then discuss the simulation environment and compare the convergence of the new algorithm with that of the standard NLMS adaptive cancellation algorithm in Section IV. Finally, we conclude the paper in Section V.

Manuscript received November 18, 2008; revised February 03, 2009. Current version published April 24, 2009. This paper was presented in part at the Proceedings of Asilomar Conference on Signals, Systems and Computers, November 2006. The associate editor coordinating the review of this manuscript and approving it for publication was Prof. Jen-Tzung Chien.

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Digital Object Identifier 10.1109/LSP.2009.2017480

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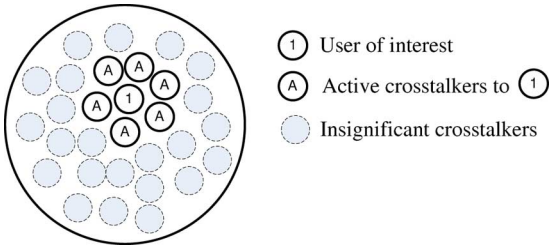

Fig. 2. Cross section of a DSL binder showing six crosstalking neighbors to a user of interest.

II. Multi-User DSL System Model

A typical very high speed DSL (VDSL2) configuration uses discrete multitone (DMT) modulation and has 4096 tones with a tone spacing of 4.3125 kHz [2]. For any particular tone k, at symbol period t the multi-user DSL system model is given as

$$\mathbf{y}(k,t) = \mathbf{H}(k,t)\mathbf{x}(k,t) + \mathbf{z}(k,t) \tag{1}$$

where the vectors $\mathbf{y}(k,t), \mathbf{x}(k,t)$ and $\mathbf{z}(k,t)$, represent the received, transmitted and noise signals respectively. The noise vector incorporates radio frequency interference, thermal noise and crosstalk from various other sources [4]. With N number of users, the $N \times N$ multi-user crosstalk channel matrix is defined by $\mathbf{H}(k,t) = [h_{n,m}(k,t)]$. The diagonal element $h_{m,m}(k,t)$, represents the (complex valued) gain of the direct channel, i.e., from transmitter modem m to receiver modem m and so forth. Typically, due to shielding on the twisted pairs, the crosstalk channel gains $h_{n,m}(k,t), (n \neq m)$ are significantly smaller than the direct channel gains, i.e., $|h_{m,m}(k,t)| \gg |h_{n,m}(k,t)|, \forall m \neq n$. The channel matrix is then said to be column-wise diagonally dominant (CWDD) [7]. Further, synchronization of DSLAMs to the same baud clock allows crosstalk on each tone to be treated independently [3]. We therefore simplify the notation by dropping the subscript k from the symbols in (1) and propose independent crosstalk cancellers on each tone.

III. Detection Guided Adaptive Partial Crosstalk Cancellation

The DSL channel environment is susceptible to changes over time. Major changes in the operating environment such as a modem in the binder powering up and down can cause a significant variation to crosstalk. Hence at different times, the conventional crosstalk canceller needs to scan the channel environment if a major initiation phase is necessary. Such scanning is not generally required with the NLMS adaptive canceller. The standard NLMS adaptive cancellation (standard NLMS-AC) algorithm, however, treats all the coefficients of the crosstalk canceller as significant. With many lines in the binder, the high dimensionality can slow convergence. To speed up the convergence, we incorporate an active or significant coefficient detector stage to estimate the number M of significant/active crosstalkers for any given line from a total of N lines, where $M \ll N$ as illustrated in Fig. 2. Typically in a 20-line binder, $M = 5$ [6]. The remaining crosstalkers, those not deemed significant, are treated as if their crosstalk contribution is nil. This reduces the number of crosstalk coefficients to be estimated which would otherwise slow the convergence [8]. We recognize significant crosstalkers by developing an activity criterion, based on the least-squares (LS) technique. We now present a mathematical derivation of the LS activity criteria and then incorporate it within a LS activity-detection-guided NLMS-based adaptive partial cancellation (NLMS-APC) algorithm for crosstalk cancellation in upstream DSL.

The standard LS technique chooses a crosstalk canceller coefficient matrix $\mathbf{W}(t)$ via

$$\mathbf{W}(t) = \arg \min_{\mathbf{W}(t) \in \mathbb{C}^{N \times N}} \mathbf{V}(\mathbf{W}(t)) \tag{2}$$

$$\text{where} \quad \mathbf{V}(\mathbf{W}(t)) = \frac{1}{t}\sum_{i=1}^{t} ||\mathbf{x}(i) - \mathbf{W}(i)\mathbf{y}(i)||^2 \tag{3}$$

and $\mathbf{W}(t)\mathbf{y}(t) = \tilde{\mathbf{x}}(t)$ is the LS-based estimate of the transmitted vector $\mathbf{x}(t)$ after crosstalk cancellation. The LS technique estimates $\mathbf{W}(t)$ at each symbol period such that the accumulated square of the error between the transmitted and the estimated symbols is minimum.

Using the LS-based procedures developed in [8], we propose the following 'significance' or 'activity' measure for the jth crosstalker of the mth user as

$$a_{m,j}(t) = \frac{\left\{\sum_{i=1}^{t} x_m(i)y_j(i)\right\}^2}{\sum_{i=1}^{t} y_j^2(i)} \tag{4}$$

where $x_m(i)$ and $y_j(i)$ are the mth and jth elements of the transmitted and received $\mathbf{x}(i)$ and $\mathbf{y}(i)$ at the ith symbol period. It can be seen that this activity measure is related to a normalized estimate of the correlation between $x_m(i)$ and $y_j(i)$. This activity measure is derived from the LS cost function of (3), under the assumption that the received signals $y_j(i), j = 0, 1, \ldots, N-1$ are spatially uncorrelated [8]. Hence, it can be shown that as $t \to \infty$, the LS estimate of $\mathbf{W}(t) = w_{m,j}(t), m, j = 1 \ldots N$ converges to

$$w_{m,j}(t) = \frac{\sum_{i=1}^{t} x_m(i)y_j(i)}{\sum_{i=1}^{t} y_j^2(i)} \tag{5}$$

and then the cost function of (3) converges towards

$$\mathbf{V}_m(t) = \frac{1}{t}\left\{\sum_{i=1}^{t} x_m^2(i) - \sum_{j=1}^{N} a_{m,j}(t)\right\}. \tag{6}$$

A limitation of the activity measure of (4) is that it fails if the received signals $y_{i,j}$ are spatially correlated/coloured. Based on the work in [9] for correlated sequences, we propose the following modified activity measure for DSL significant crosstalker identification:

$$a_{m,j}(t) = \frac{\left\{\sum_{i=1}^{t} [(x_m(i) - \tilde{x}_m(i)) + w_{m,j}(i)y_j(i)]y_j(i)\right\}^2}{\sum_{i=1}^{t} y_j^2(i).} \tag{7}$$

The term $-\tilde{x}_m(i) + w_{m,j}(i)y_j(i)$ attempts to decorrelate jth significant crosstalker of user m from all other $(f \neq j)$ crosstalkers to user m.

To classify a neighbor j as a significant crosstalker to a user m involves application of a 'significance' threshold to $a_{m,j}(t)$. This requires the LS cost function to be structurally consistent, i.e., it should be capable of accurate identification of active crosstalkers along with their position in the DSL binder. A standard approach is to penalise the estimator's order, i.e., the number of estimated coefficients. Following [8], we consider the Akaike's B-Information Criteria (ABIC) based cost function to

test the crosstalk activity of each of the crosstalkers to user m at a symbol period t

$$\mathbf{V}_m(t) = \mathbf{V}_m(\mathbf{W}_m(t)) \left(1 + \frac{KM\log(t)}{t \cdot \mathbf{V}_m(\mathbf{W}_m(t))}\right) \quad (8)$$

where $\mathbf{W}_m(t)$ is the mth row of the crosstalk canceller matrix $\mathbf{W}(t)$ and M is the unknown number of 'significant' elements within $\mathbf{W}_m(t)$. K is a constant invariant of t and M. From (6) and (8) we can rewrite $\mathbf{V}_m(t)$ as

$$\mathbf{V}_m(t) = \frac{1}{t}\left\{\sum_{i=1}^{t} x_m^2(i) - \sum_{a=1}^{M}[a_{m,j_a}(t) - K\log(t)]\right\}; \quad 1 \le j_a \le N, a = 1, 2, \ldots, M.$$

With the aim of minimising $\mathbf{V}_m(t)$ then $a_{m,j}(t)$ is considered significant if $a_{m,j}(t) \ge K\log(t)$. Furthermore, following [8], [9], via application of Donoho's thresholding criterion [10], we select K as per

$$K = \frac{c}{t}\sum_{i=1}^{t}\{x_m(i) - \tilde{x}_m(i)\}^2 \quad (9)$$

where $c < 1$ is a user-selectable constant. A smaller value of c improves the detection of the smaller significant coefficients, at the risk of slower convergence. To better cope with time variations within the system, a suitable alternative to the cost function of (3) is the corresponding exponentially-weighted LS cost function

$$\mathbf{V}(\mathbf{W}(t)) = \frac{1}{t}\sum_{i=1}^{t}\gamma^{t-i}\|\mathbf{x}(i) - \mathbf{W}(i)\mathbf{y}(i)\|^2, \quad 0 \ll \gamma < 1.$$

This modification leads to the inclusion of the term γ^{t-i} in (7) and (9) within all their summations. The LS-activity-detection-guided NLMS-based adaptive partial cancellation (NLMS-APC) algorithm, which can be implemented at the co-located CO modems in upstream DSL communication, is now described in full.

Step 0: Initialisation

Initialise the crosstalk canceller matrix, $\mathbf{W}(0) = [\mathbf{I}]_{N\times N}$ and the activity criterion parameters $\mathbf{P}(0) = \mathbf{D}(0) = \mathbf{G}(0) = l(0) = 0$.

Step 1: Apply crosstalk cancellation and calculate the error

At a particular symbol period t, the processed vector, output from the crosstalk canceller is

$$\tilde{\mathbf{x}}(t) = \mathbf{W}(t)\mathbf{y}(t)$$

The error is given by $\mathbf{e}(t) = \mathbf{x}(t) - \tilde{\mathbf{x}}(t)$, where $\mathbf{x}(t)$ is the transmitted training or pilot signal vector as per (1).

Step 2: Update the Crosstalk Canceller

We consider that, for each symbol period t, within tone k, the mth row of the crosstalk cancellation matrix, $\mathbf{W}(t)$, is estimated with the modified NLMS equation

$$\mathbf{W}_m(t+1) = \mathbf{B}_m(t) \circ \mathbf{W}_m(t) + \frac{\mu\mathbf{B}_m(t)e_m(t)\mathbf{y}(t)}{\mathbf{y}(t)^T \circ \mathbf{B}_m(t)\mathbf{y}(t) + \varepsilon}$$

where $\mathbf{W}_m(t)$ and $\mathbf{B}_m(t)$ represents mth row of $\mathbf{W}(t)$ and $\mathbf{B}(t)$ respectively, $\circ$ represents the Hadamard or Schur product between two vectors, μ is the step size, ε is a small positive constant. Here, the standard NLMS learning rule is modified with a pre-multiplication by a sparse matrix, $\mathbf{B}(t)$, such that $b_{m,j}(t) = 1$, if $w_{m,j}(t)$ is detected to be significant (using Step 3), otherwise $b_{m,j}(t) = 0$. The pre-multiplication helps retain and track only the significant coefficients (dominant crosstalkers) and ignores the rest. This assists in reducing the length of the training sequence and improving (minimising) the run-time complexity of the detection guided NLMS-APC algorithm.

Step 3: Detection of significant coefficients—Satisfying the activity criterion

In order to classify the significant coefficients of $\mathbf{W}_m(t)$ with regards to crosstalk degradation to a particular user m, we employ the ABIC threshold of (9). In particular, the jth coefficient of $\mathbf{W}_m(t)$ is deemed significant if

$$\frac{p_{m,j}^2(t)}{d_m(t)g_j(t)} > \frac{c\log(l(t))}{l(t)} \quad (10)$$

where, $p_{m,j}(t) = \gamma p_{m,j}(t-1) + [x_m(t) - \tilde{x}_m(t) + w_{m,j}(t)y_j(t)]y_j(t)$, $d_m(t) = \gamma d_m(t-1) + e_m^2(t)$, $g_j(t) = \gamma g_j(t-1) + y_j^2(t)$, $l(t) = \gamma l(t-1) + 1$ and γ is a forgetting factor. Typically, γ is set between 0.99 and 0.999 (similar to that employed in the exponentially-weighted recursive least squares (RLS) algorithm).

Step 4: Iterate from Step 1 until $t = t_{\text{end}}$, *viz.*, t_{end} is length of the training or pilot sequence

A. Computational Complexity

We now compare the computational complexity of the NLMS-APC algorithm with the standard NLMS-AC algorithm. The proposed NLMS-APC algorithm requires (per user and per tone) $5N + 3M + 6$ multiplications per symbol period (MPSP), where N is the total number of users and M is the number of active/significant crosstalk neighbors to each user. In comparison, the standard NLMS-AC algorithm (which does not include activity detection and treats all the coefficients to be significant) requires $2N + 2 + (1/N)$ MPSP. Typically, for $N \gg M > 1$ the proposed algorithm involves essentially 2.5 times the computational complexity (MPSP) of standard NLMS-AC algorithm.

IV. Simulated Performance

In line with the model assumed in [6], we assume only the nearest crosstalking lines to a particular user are significant. We therefore simulate a DSL system of $N = 19$ users with $M = 6$ dominant/significant crosstalkers per user. 4-ary quadrature amplitude modulation (4-QAM) symbols are transmitted for each user with -60 dBm/Hz [11], [7] transmitter power (T_p) on each DSL tone. Additive temporal and spatial white Gaussian noise is assumed, such that the noise power $N_p = -140$ dBm/Hz [11] and the received signal power is $T_r = -64$ dBm/Hz. The NLMS adaptive step size parameter μ controls the convergence speed, as well as the steady-state error (SSE). A larger μ leads to faster convergence, but higher SSE. Keeping this in mind, we choose $\mu = 0.1$. The parameter ε in the denominator of the canceller update step 2, is set to $\varepsilon = 0.01$, to avoid numerical issues when the received power is low. The forgetting factor is set to $\gamma = 0.999$. The user-selectable constant in (10) is set to $c = 0.1$.

Fig. 3 shows the progressive detection of active coefficients within $\mathbf{W}_m(t)$, for a particular user m, in the presence of white noise. The activity test is performed as shown in Step 3 of the

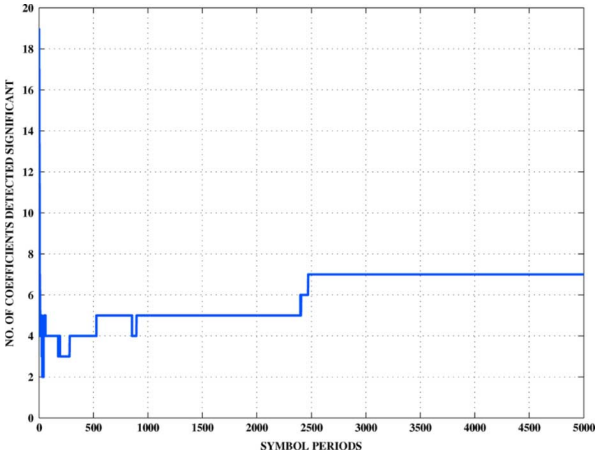

Fig. 3. Active coefficient detection.

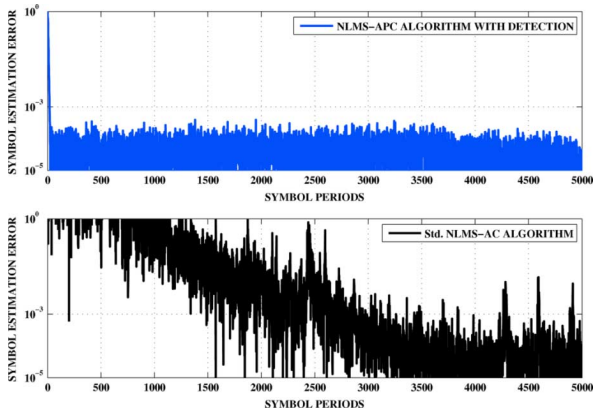

Fig. 4. Convergence and SSE performance of proposed NLMS-APC and standard NLMS-AC for white noise, $N_p = -140$ dBm/Hz.

proposed algorithm in Section III. Initiating from 0, the activity criterion terms $l(t), p_{m,j}(t), d_j(t)$ and $g_j(t)$ are updated as per (10) in each iteration. Progressive updating leads to the number of detected coefficients converging towards the correct value, 7.

Fig. 4 compares the convergence and SSE performance of detection-guided NLMS-APC and the standard NLMS-AC algorithms for one user, on one tone. The results plotted show the squared symbol estimation error $\mathbf{e}_m^2(t)$ against the number of symbol periods t. It is seen that the NLMS-APC converges considerably faster than the standard NLMS-AC thereby requiring shorter training sequences and shifts more rapidly into operational phase. Better data efficiency results. Also, the faster convergence comes with similar SSE performance. We add that, the earlier convergence of the proposed detection-guided NLMS-APC ensures that in general, for a given training length sequence, the proposed NLMS-APC will have converged to a lower SSE. This results in lower run-time bit error rates.

To further examine the achieved speed in the convergence, we make a comparison of the proposed algorithm with the RLS algorithm (which is known for its fast convergence but relatively high complexity [5]) under conditions in which their SSE performances are similar. We simulate an exponentially-weighted RLS algorithm with its forgetting factor set to 0.99. Fig. 5 shows that the detection-guided NLMS-APC algorithm converges faster than RLS, eventually reaching a similar SSE. Thus, by augmenting the activity criterion within the

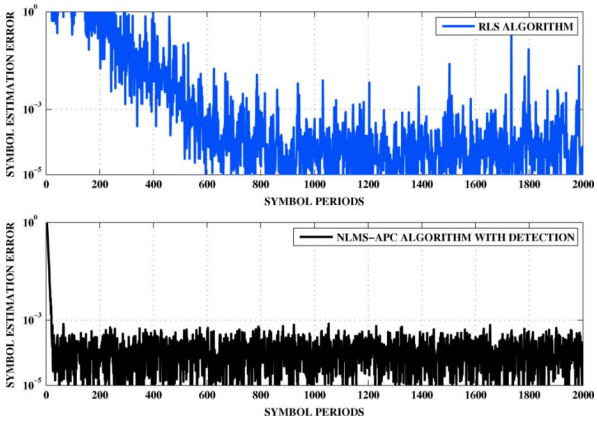

Fig. 5. Convergence and tracking performance of proposed NLMS-APC and RLS algorithm for white noise, $N_p = -140$ dBm/Hz.

NLMS-AC, the new detection-guided NLMS-APC is capable of achieving faster convergence than the RLS algorithm but with low complexity.

V. Conclusion

In this paper, we proposed an efficient detection-guided NLMS based adaptive partial crosstalk cancellation algorithm for multi-user DSL environments. The proposed NLMS-APC algorithm employs a LS criterion to detect and subsequently guide the NLMS estimation of the significant crosstalk coefficients within the unknown 'desired' crosstalk canceller matrix. The basic aim is to enhance the convergence speed and subsequently keep the length of the canceller training sequences to a minimum, improving reliability of services in changing crosstalk environments. Simulations confirmed that in comparison to standard NLMS-AC the proposed algorithm shows faster convergence while maintaining similar SSE performance.

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