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OBSERVER DESIGN FOR SINGULAR SYSTEMS USING CANONICAL FORMS

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Abstract.

The reduced order observer for singular systems is derived in this paper. Of particular interest is the presentation of the separation property.

1.Introduction.

In this paper, we consider the problem of designing an observer for the singular system

$$E\dot{x} = Ax + Bu \qquad y = Cx \tag{1}$$

Where E and A are nxn real matrices, B is an nxq matrix and C is a pxn matrix. Moreover, E is singular and $\det(Es-A)\not\equiv 0$ a.e. Such a problem has been examined by El-Tohami et al. [1]. Their construction is based on the generalized matrix inversion. Here, we shall take a different line of approach, and build up Luenberger-type observers by using the generalized observer canonical form developed in [2]. Our method is an easy generalization of the approach taken by Luenberger [3], thus it produces naturally reduced-order observers. Furthermore, if such an observer is used jointly with the controller, it is shown that the separation property, well-known in the regular theory (i.e. E=I), still holds.

2.(n-1)-order observer for singlevariable systems.

First recall that (1) is observable if rank $[E^T s-A^T \ -C^T]^T=n$ and rank $[E^T \ -C^T]^T=n$. Two singular systems are said to be restricted system equivalent (or r.s.e.) if they are connected by

$$\begin{bmatrix} M & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} Es-A & B \\ -C & 0 \end{bmatrix}\begin{bmatrix} N & 0 \\ 0 & I \end{bmatrix} = \begin{bmatrix} E_1 s-A_1 & B_1 \\ -C_1 & 0 \end{bmatrix} \tag{2}$$

Where M and N are nonsingular matrices (see Rosenbrock [4]). With the above definitions, the following theorem proven in [2] holds.

Theorem 1. An observable system (1) is transformable under r.s.e. operations, into the generalized observer canonical form

$$\begin{bmatrix} a_1 & -1 & 0 & \dots & 0 \\ a_2 & s & -1 & \dots & 0 \\ \cdot & \cdot & \cdot & & \cdot \\ a_{n-1} & 0 & 0 & \dots & -1 \\ a_n & 0 & 0 & \dots & s \end{bmatrix} x = \begin{bmatrix} b_1 \\ b_2 \\ \cdot \\ \cdot \\ b_n \end{bmatrix} u \qquad y = [1 \ 0 \ \dots \ 0]\, x \tag{3}$$

Now we are ready to construct an observer for (1). First apply r.s.e. transformations to bring the first equation set in (3) into the form

$$\begin{bmatrix} d_1 & -1 & 0 & \dots & 0 \\ d_2 & s+e_1 & -1 & \dots & 0 \\ \cdot & \cdot & \cdot & & \cdot \\ d_{n-1} & e_{n-2} & 0 & \dots & -1 \\ d_n & e_{n-1} & 0 & \dots & s \end{bmatrix} x = \begin{bmatrix} f_1 \\ f_2 \\ \cdot \\ \cdot \\ f_n \end{bmatrix} u \tag{4}$$

(Different Roman letters are used to indicate obvious changes resulting from the transformations. And we also use the same x to denote the transformed state vector, this is purely for convenience of notation). One should note that e_i (i=1,...,n-1) in (4) can be set up arbitrarily. Let x be the state which is to be observed. Since $y=x_1$, we only have to estimate $x_2, \dots, x_n$. This can then be achieved by applying the (n-1)-order observer shown as follows

$$\begin{bmatrix} s+e_1 & -1 & \dots & 0 \\ e_2 & s & \dots & 0 \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ e_{n-2} & 0 & \dots & -1 \\ e_{n-1} & 0 & \dots & s \end{bmatrix}\begin{bmatrix} \hat{x}_2 \\ \hat{x}_3 \\ \cdot \\ \cdot \\ \hat{x}_n \end{bmatrix} = -\begin{bmatrix} d_2 \\ d_3 \\ \cdot \\ \cdot \\ d_n \end{bmatrix} y + \begin{bmatrix} f_2 \\ f_3 \\ \cdot \\ \cdot \\ f_n \end{bmatrix} u \tag{5}$$

where $\hat{x}_i$ (i=2, ..., n) denote the estimates of x_i.

We now prove that (5) is indeed an asymptotic observer. Define $\bar{x}_i = x_i - \hat{x}_i$ (i=2, ..., n). By substracting (5) from the last (n-1) equations in (4) we have

$$\begin{bmatrix} s+e_1 & -1 & \dots & 0 \\ e_2 & s & \dots & 0 \\ \cdot & \cdot & & \cdot \\ e_{n-2} & 0 & \dots & -1 \\ e_{n-1} & 0 & \dots & s \end{bmatrix}\begin{bmatrix} \bar{x}_2 \\ \bar{x}_3 \\ \cdot \\ \cdot \\ \bar{x}_n \end{bmatrix} = 0 \tag{6}$$

We have seen from the foregoing argument that if (1) is observable, $e_1, \dots, e_{n-1}$ can be chosen arbitrarily. This implies that the poles of (6) can be assigned to any desired locations. Hence the error $\bar{x}_i$ can be controlled to zero asymptotically in any exponential speed. Moreover, since the input does not play any role in the foregoing discussion, this observation remains valid for multi-input systems.

The preceding design method is obviously a generalization of the reduced order observer design method found in the regular system theory. The process shows quite naturally how the (n-1)-order observer operates on observable singular systems.

3.Reduced-order observer for multivariable systems.

Similar to Theorem 1, we find in [2] the following for the multivariable systems

Theorem 2. Every observable system (1) is r.s.e. to the following generalized observer canonical form. where X's denote possible nonzero elements, and a_i (i=1, ..., k) are either 1 or zero.

$$y = \left[\begin{array}{cccc|cccc|c|cccc} 1 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & \dots & 0 & 0 & \dots & 0 \\ X & 0 & \dots & 0 & 1 & 0 & \dots & 0 & \dots & 0 & 0 & \dots & 0 \\ X & 0 & \dots & 0 & X & 0 & \dots & 0 & \dots & 0 & 0 & \dots & 0 \\ \cdot & \cdot & & \cdot & \cdot & \cdot & & \cdot & & \cdot & \cdot & & \cdot \\ X & 0 & \dots & 0 & X & 0 & \dots & 0 & \dots & 1 & 0 & \dots & 0 \\ \hline X & X & \dots & X & X & X & \dots & X & \dots & X & X & \dots & X \\ \cdot & \cdot & & \cdot & \cdot & \cdot & & \cdot & & \cdot & \cdot & & \cdot \\ X & X & \dots & X & X & X & \dots & X & \dots & X & X & \dots & X \end{array}\right] x \tag{7}$$

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$$\begin{pmatrix} s+X & -1 & \cdots & 0 & X & & & X & & \\ X & s & \cdots & 0 & X & & & X & & \\ \vdots & & & & \vdots & & & \vdots & & \\ X & 0 & \cdots & -1 & X & & & X & & \\ X & 0 & \cdots & s & X & & & X & & \\ X & & & & a_2s+X & -1 & \cdots & 0 & X & \\ X & & & & X & s & \cdots & 0 & X & \\ \vdots & & & & \vdots & & & \vdots & & \\ X & & & & X & 0 & \cdots & -1 & X & \\ X & & & & X & 0 & \cdots & s & X & \\ \vdots & & & & & & & & \vdots & \\ X & & & & X & & & a_ks+X & -1 & \cdots & 0 \\ X & & & & X & & & X & s & \cdots & 0 \\ \vdots & & & & \vdots & & & \vdots & & & \\ X & & & & X & & & 0 & \cdots & -1 \\ X & & & & X & & & 0 & \cdots & s \end{pmatrix} \begin{pmatrix} x_{11} \\ x_{12} \\ \vdots \\ x_{1,r_1} \\ x_{21} \\ x_{22} \\ \vdots \\ x_{2,r_2} \\ \vdots \\ x_{k1} \\ x_{k2} \\ \vdots \\ x_{k,r_k} \end{pmatrix} = Bu$$

It is then easily seen from the structure of (7) that a transformation F can be found such that all X's in the first k rows of FC can be eliminated (this is equivalent to a coordinate change in the output space). Denoting by $\bar{y}$ the first k entries in the output vector Fy, $\bar{y}$ together with the first equation in (7) can be looked upon as consisting of "k" multi-input single-output systems of the form

$$\begin{pmatrix} a_is+X & -1 & \cdots & 0 \\ X & s & \cdots & 0 \\ \vdots & & & \\ X & 0 & \cdots & -1 \\ X & 0 & \cdots & s \end{pmatrix} \begin{pmatrix} x_{i1} \\ x_{i2} \\ \vdots \\ x_{i,r_i-1} \\ x_{i,r_i} \end{pmatrix} = H_i\bar{y} + B_iu \qquad (8)$$

$$\bar{y}_i = [1 \quad 0 \ \cdots \ 0]\, x_i$$

where H_i, B_i are appropriate matrices. Therefore, the result obtained in the previous section can be applied here. For equation (8), we can construct an asymptotic observer of dimension r_i-1 no matter what value a_i can take (zero or 1). Consequently, for the system (7) we need k observers with total dimension n-k (k is the number of generalized companion blocks in (7)). The observers will generate asymptotically n-k states, while the remaining k states can be found from $\bar{y}$. The eigenvalues of all observers can be arbitrarily chosen. To summarize, we state

Theorem 3. If system (1) is observable, an (n-k)-order asymptotic state observer with any desired eigenvalues can be constructed, where k is the number of the generalized companion blocks in its generalized observer canonical form.

4. Joint controller-observer design configuration and the separation property.

In this section, we shall briefly look at another property related to the observer constructed above. For simplicity, we limit the discussion to the singlevariable systems. The generalization to multivariable case can easily be done.

Since the observer (5) is a regular system, its states (the state estimates) are directly accessible. These estimates plus the output of the system (3) enable us to set up a state feedback control scheme (Fig.1).Note that the notation used in Fig.1 is consistent with the following proper partitioning of (4)

$$\begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_r \end{pmatrix} = \begin{pmatrix} h_1 & p^T \\ q & A_r \end{pmatrix} \begin{pmatrix} x_1 \\ x_r \end{pmatrix} + \begin{pmatrix} f_1 \\ f_r \end{pmatrix} u \qquad (9)$$

where $x_r^T = [x_2 \ x_3 \ \cdots \ x_n]$, and all matrices in (9) are partitioned consistently. As usual, the state feedback takes the form

$$u = k \begin{pmatrix} x_1 \\ x_r \end{pmatrix} + v = k_1x_1 + k_rx_r + v \qquad (10)$$

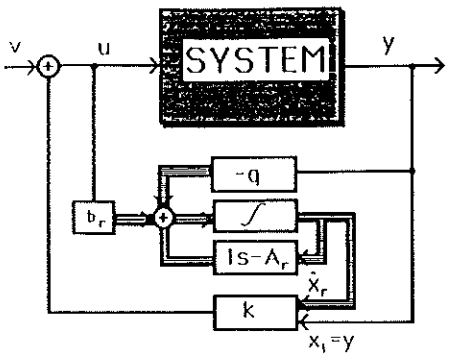

Fig.1 Observer and state feedback control scheme.

where $k=[k_1 \ k_r]$. By noting the structure of the (n-1)th order observer (5), we can easily set up the dynamic equation of the overall system in Fig.1.

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{pmatrix} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_r \\ \dot{\hat{x}}_r \end{pmatrix} = \begin{pmatrix} h_1+f_1k_1 & p^T & f_1k_r \\ q+f_rk_1 & A_r & f_rk_r \\ q+f_rk_1 & 0 & A_r+f_rk_r \end{pmatrix} \begin{pmatrix} x_1 \\ x_r \\ \hat{x}_r \end{pmatrix} + \begin{pmatrix} f_1 \\ f_r \\ f_r \end{pmatrix} v \qquad (11)$$

Apply r.s.e. transformations to (11), simple calculation gives

$$\begin{pmatrix} E & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} \dot{x} \\ \dot{\hat{x}}_r \end{pmatrix} = \begin{pmatrix} A+Bk & -Bk_r \\ 0 & A_r \end{pmatrix} \begin{pmatrix} x \\ \hat{x}_r \end{pmatrix} + \begin{pmatrix} B \\ 0 \end{pmatrix} v \qquad (12)$$

where

$$E = \begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix} \quad A = \begin{pmatrix} h_1 & p^T \\ q & A_r \end{pmatrix} \quad B = \begin{pmatrix} f_1 \\ f_r \end{pmatrix} \quad x = \begin{pmatrix} x_1 \\ x_r \end{pmatrix}$$

From (12) the immediate observation is that the characteristic polynomial of the overall system is the product of det(Es-A-Bk) and det($Is-A_r$). A simple interpretation is that as far as the eigenvalues are concerned, the feedback of the vector $[x_1 \ \hat{x}_r^T]^T$ is as perfect as the real state x. As a consequence, the design of the state feedback and the design of the observer can be carried out independently. This nice property, known in the regular theory, is also called the separation property.

5. Conclusions.

By applying the canonical forms developed in [2], we have successfully constructed the reduced order observers for singular systems. Compared with the construction of El-Tohami et al. in [1], our method is considerably simpler and serves as a natural extension of the well-known state observer found in the regular system theory. Following the same line of approach, other kinds of observers, for example the functional observer [3], can easily be constructed for singular systems.

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