Face Recognition as a Kronecker Product Equation

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***Abstract*—Various parameters influence face recognition such as expression, pose, and illumination. In contrast to matrices, tensors can be used to naturally accommodate for the different modes of variation. The multilinear singular value decomposition (MLSVD) then allows one to describe each mode with a factor matrix and the interaction between the modes with a coefficient tensor. In this paper, we show that each image in the tensor satisfying an MLSVD model can be expressed as a structured linear system called a Kronecker Product Equation (KPE). By solving a similar KPE for a new image, we can extract a feature vector that allows us to recognize the person with high performance. Additionally, more robust results can be obtained by using multiple images of the same person under different conditions, leading to a coupled KPE. Finally, our method can be used to update the database with an unknown person using only a few images instead of an image for each combination of conditions. We illustrate our method for the extended Yale Face Database B, achieving better performance than conventional methods such as Eigenfaces and other tensor-based techniques.**

I. Introduction

Face recognition is an important problem in computer vision with many applications within domains such as information security, surveillance, and biometric identification [1]. Although many recognition systems use matrix-based methods, face recognition is inherently a multidimensional problem due to variations in facial expression, pose, illumination conditions, etc [2]. Linear algebra is of limited use because it only captures a single variation using a mode of a matrix. For example, the well-known Eigenfaces method stacks vectorized images in the second mode, obtaining a matrix with modes pixels $\times$ persons [3]. Although some methods have tried to accommodate for different conditions [4], [5], the multidimensional structure remains a challenging problem for matrix-based methods.

Recently, tensor tools have gained increasing popularity in signal processing and machine learning applications [6], [7]. Tensors are higher-order generalizations of vectors (first order) and matrices (second order). The higher-order structure allows one to explicitly accommodate for the multidimensional structure of facial images: each mode of a tensor can represent a single variation of the image [2]. For example, a set of (vectorized) images of several persons under different illumination conditions can be represented by a third-order tensor with modes pixels $\times$ illuminations $\times$ persons. An important tensor tool is the multilinear singular value decomposition (MLSVD) of a higher-order tensor which is a generalization of the well-known singular value decomposition (SVD) [8]. The MLSVD allows one to approximately represent the tensor by a set of factor matrices that are each related to a single mode and a core tensor that explains the interaction between the different modes. This type of representation is also used in TensorFaces, enabling improved accuracy in face recognition in comparison with conventional techniques such as Eigenfaces [9]. Several other tensor-based methods have been proposed, see [10], [11].

In this paper, we explain that tensor-based face recognition using the MLSVD model can be expressed as a Kronecker Product Equation (KPE). A KPE is a linear system of equations with a Kronecker product constrained solution for which the authors have developed a generic framework in [12]. We show that by solving a KPE for a new unlabeled image, one can obtain a feature vector that enables better recognition rates than conventional methods. In practice, the robustness can be improved by coupling multiple images of the same person under different conditions, leading to a set of KPEs that are coupled. Additionally, our method allows one to add a new unknown person using only a few images instead of an image for each combination of conditions. We illustrate our method for the extended Yale Face Database B which contains facial images of multiple persons under different illuminations [13]. Our KPE-based method achieves higher performance than conventional Eigenfaces and the tensor-based approach in [9].

We conclude this section with an overview of the notation and basic definitions. We also define the MLSVD and KPEs. Next, we reformulate face recognition as a KPE in Section II. We apply our approach to a real-life dataset in Section III.

A. Notations and basic definitions

We denote vectors, matrices, and tensors by bold lowercase, bold uppercase, and calligraphic letters, respectively. A natural extension of the rows and columns of a matrix, is a mode-n vector of a tensor $\mathcal{A} \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_N}$, defined by fixing every index except the nth, e.g., $\mathbf{a}_{i_1 \cdots i_{n-1} : i_{n+1} \cdots i_N}$. A mode-$n$ unfolding of $\mathcal{A}$ is a matrix $\mathbf{A}_{(n)}$ with the mode-n vectors as its columns (following the ordering convention in [14]). The vectorization of $\mathcal{A}$, denoted as $\text{vec}(\mathcal{A})$, maps each element $a_{i_1 i_2 \cdots i_N}$ onto $\text{vec}(\mathcal{A})_j$ with $j = 1 + \sum_{k=1}^{N} (i_k - 1) J_k$ and $J_k = \prod_{m=1}^{k-1} I_m$. We indicate the nth element in a sequence by a superscript between parentheses, e.g., $\{\mathbf{A}^{(n)}\}_{n=1}^{N}$.

The outer and Kronecker product are denoted by $\otimes$ and $\otimes$, respectively. The mode-n product of a tensor $\mathcal{A} \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_N}$ and a matrix $\mathbf{B} \in \mathbb{R}^{J_n \times I_n}$ is a tensor $\mathcal{A} \cdot_n \mathbf{B} \in \mathbb{R}^{I_1 \times \cdots \times I_{n-1} \times J_n \times I_{n+1} \times \cdots I_N}$ and is defined element-wise as $(\mathcal{A} \cdot_n \mathbf{B})_{i_1 \cdots i_{n-1} j_n i_{n+1} \cdots i_N} = \sum_{i_n=1}^{I_n} a_{i_1 i_2 \cdots i_N} b_{j_n i_n}$. Hence, each mode-$n$ vector of the tensor $\mathcal{A}$ is multiplied with the matrix $\mathbf{B}$, i.e., $(\mathcal{A} \cdot_n \mathbf{B})_{(n)} = \mathbf{B}\mathbf{A}_{(n)}$.

B. Multilinear singular value decomposition

The multilinear singular value decomposition (MLSVD) of a higher-order tensor is a generalization of the singular value decomposition (SVD) of a matrix [6]–[8]. The MLSVD writes a tensor $\mathcal{A} \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_N}$ as the product

$$\mathcal{A} = \mathcal{S} \cdot_1 \mathbf{U}^{(1)} \cdot_2 \mathbf{U}^{(2)} \cdots \cdot_n \mathbf{U}^{(N)},$$

in which $\mathbf{U}^{(n)} \in \mathbb{R}^{I_n \times I_n}$ is a unitary matrix, $1 \leq n \leq N$, and the core tensor $\mathcal{S} \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_N}$ is ordered and all-orthogonal; see [8] for more details. The mode-n rank of an Nth-order tensor is equal to the rank of the mode-n unfolding. The multilinear rank of the tensor is equal to the tuple of mode-n rank values. The MLSVD is related to the low-multilinear rank approximation (LMLRA) and the Tucker decomposition (TD); see, e.g., [8], [14], [15]. The decomposition has been used successfully in applications such as compression and dimensionality reduction [14], [16].

C. (Coupled) Kronecker Product Equations

A KPE is a linear system of equations with a Kronecker product constrained solution. Here, we limit ourselves to problems with the following simple Kronecker product structure:

$$\mathbf{A}\mathbf{x} = \mathbf{b} \quad \text{with} \quad \mathbf{x} = \mathbf{v} \otimes \mathbf{u}, \tag{1}$$

in which $\mathbf{A} \in \mathbb{R}^{M \times K}$, $\mathbf{x} \in \mathbb{R}^K$, and $\mathbf{b} \in \mathbb{R}^M$. The solution $\mathbf{x}$ can be expressed as a Kronecker product $\mathbf{v} \otimes \mathbf{u}$ with $\mathbf{u} \in \mathbb{R}^I$ and $\mathbf{v} \in \mathbb{R}^J$ such that $K = IJ$. As a matter of fact, a KPE is a simple case of a linear system with a solution that can be represented as a matrix or tensor decomposition [12]. Expression (1) could be solved by first solving the system without structure and subsequently decomposing a matricized version of the solution. This approach works well if $\mathbf{A}$ has full column rank, but, in contrast to the methods in [12], fails when $\mathbf{A}$ is rank deficient or in the underdetermined case. The methods in [12] compute the least-squares (LS) solution of (1).

A coupled KPE (cKPE) is a set of KPEs that have a common coefficient vector. We limit ourselves to cKPEs of the form:

$$\mathbf{A}(\mathbf{v} \otimes \mathbf{u}^{(q)}) = \mathbf{b}^{(q)} \text{ for } 1 \leq q \leq Q$$

with $\mathbf{A} \in \mathbb{R}^{M \times K}$, $\mathbf{v} \in \mathbb{R}^I$, $\mathbf{u}^{(q)} \in \mathbb{R}^J$, and $\mathbf{b}^{(q)} \in \mathbb{R}^M$ such that $K = IJ$. By defining $\mathbf{U} \in \mathbb{R}^{J \times Q}$ with $\mathbf{u}_q = \mathbf{u}^{(q)}$ and $\mathbf{B} \in \mathbb{R}^{M \times Q}$ with $\mathbf{b}_q = \mathbf{b}^{(q)}$, we obtain $\mathbf{A}(\mathbf{v} \otimes \mathbf{U}) = \mathbf{B}$.

II. Face recognition using KPEs

A. Tensorization and MLSVD model

Higher-order tensors can explicitly accommodate for the multidimensional nature of facial images by representing each variation by a mode of the tensor [2], [9]. Although our method can be used for any combination of variations, we illustrate the strategy for the following particular case. Consider a set of facial images of J persons taken under I different illumination conditions. Each image is represented by a matrix of size $M_x \times M_y$ with M_x and M_y pixels in the x- and y-direction, respectively. All *vectorized* images of length $M = M_x M_y$ are stacked into a third-order tensor $\mathcal{D} \in \mathbb{R}^{M \times I \times J}$ with modes pixels (px) $\times$ illuminations (i) $\times$ persons (p).

Next, we compute a truncated MLSVD of the tensor $\mathcal{D}$:

$$\mathcal{D} \approx \mathcal{S} \cdot_1 \mathbf{U}_{\text{px}} \cdot_2 \mathbf{U}_{\text{i}} \cdot_3 \mathbf{U}_{\text{p}}, \tag{2}$$

in which $\mathbf{U}_{\text{px}} \in \mathbb{R}^{M \times P}$, $\mathbf{U}_{\text{i}} \in \mathbb{R}^{I \times R}$, and $\mathbf{U}_{\text{p}} \in \mathbb{R}^{J \times L}$ form an orthonormal basis for the pixel, illumination, and person mode, respectively. The interaction between the different modes is expressed by the core tensor $\mathcal{S} \in \mathbb{R}^{P \times R \times L}$. Each row of $\mathbf{U}_{\text{p}}$, denoted by $\mathbf{c}_p^{\text{T}}$, can be interpreted as the coefficients for person p and each row of $\mathbf{U}_{\text{i}}$, denoted by $\mathbf{c}_i^{\text{T}}$, can be interpreted as the coefficients for illumination i.

B. Kronecker Product Equation

Each mode-1 fiber of $\mathcal{D}$ in (2) corresponds to an image and can be modeled by a KPE as follows. Consider a vectorized image $\mathbf{d} \in \mathbb{R}^M$ for a *particular* person p and illumination i:

$$\mathbf{d} = (\mathcal{S} \cdot_1 \mathbf{U}_{\text{px}}) \cdot_2 \mathbf{c}_i^{\text{T}} \cdot_3 \mathbf{c}_p^{\text{T}}. \tag{3}$$

$$\mathbf{d} = \mathbf{U}_{\text{px}} \mathbf{S}_{(1)} (\mathbf{c}_p \otimes \mathbf{c}_i). \tag{4}$$

Expression (4) is the mode-1 unfolding of (3) and is a KPE: each $\mathbf{d}$ is a linear combination of the columns of $\mathbf{U}_{\text{px}}\mathbf{S}_{(1)}$ with Kronecker product constrained coefficients $(\mathbf{c}_p \otimes \mathbf{c}_i)$.

Consider a set of facial images of the same person under Q different illuminations, leading to a set of coupled KPEs that share the coefficient vector in the illumination mode:

$$\mathbf{d}^{(q)} = \mathbf{U}_{\text{px}} \mathbf{S}_{(1)} (\mathbf{c}_p \otimes \mathbf{c}_i^{(q)}) \text{ for } 1 \leq q \leq Q. \tag{5}$$

By stacking all vectorized images $\mathbf{d}^{(q)}$ and illumination coefficient vectors $\mathbf{c}_i^{(q)}$ into a matrix $\mathbf{D} \in \mathbb{R}^{M \times Q}$ and $\mathbf{C}_i \in \mathbb{R}^{R \times Q}$, respectively, we obtain $\mathbf{D} = \mathbf{U}_{\text{px}} \mathbf{S}_{(1)} (\mathbf{c}_p \otimes \mathbf{C}_i)$.

C. Face recognition

We explain how to recognize a person in a (set of) facial image(s) under a new illumination condition using (c)KPEs. First, we construct a tensor $\mathcal{D}$ by stacking a set of facial images of different persons under different illuminations in the way explained in Subsection II-A. Second, we compute the truncated MLSVD of $\mathcal{D}$, obtaining factor matrices $\mathbf{U}_{\text{px}}$, $\mathbf{U}_{\text{i}}$, and $\mathbf{U}_{\text{p}}$, and core tensor $\mathcal{S}$. Every (vectorized) image of $\mathcal{D}$ can then be expressed as a KPE as explained in Subsection II-B. Next, consider a new, unlabeled facial image $\mathbf{d}^{(\text{new})}$ of a known person. In order to recognize the person in the image, we solve the following KPE using the algorithms from [12]:

$$\mathbf{d}^{(\text{new})} = \mathbf{U}_{\text{px}} \mathbf{S}_{(1)} \left(\mathbf{c}_p^{(\text{new})} \otimes \mathbf{c}_i^{(\text{new})} \right), \tag{6}$$

obtaining estimates $\hat{\mathbf{c}}_p^{(\text{new})}$ and $\hat{\mathbf{c}}_i^{(\text{new})}$ for the coefficient vectors. We compare $\hat{\mathbf{c}}_p^{(\text{new})}$ with the rows of $\mathbf{U}_{\text{p}}$ using the Frobenius norm of the difference (after fixing scaling and sign invariance). One can then recognize the person in the image by assigning the label corresponding to the closest match. In other words, the estimated coefficient vector for the person mode $\hat{\mathbf{c}}_p^{(\text{new})}$ acts as a feature vector and $\mathbf{U}_{\text{p}}$ acts as a database. More robust results can be obtained by using images under multiple illumination conditions and coupling the KPEs as in (5).

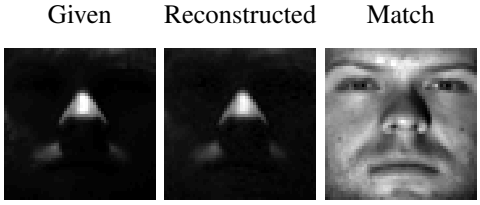


Fig. 1. Classification of a person that is included in the dataset. Note that we can identify the person even though the picture is almost completely dark.

In contrast to our method, the tensor-based approach in [9] solves (6) by fixing the illumination coefficients to a particular illumination. More specifically, the approach solves (6) by taking $\mathbf{c}_i^{(\text{new})}$ equal to a row of $\mathbf{U}_\text{i}$, reducing (6) to a linear system of equations for *each* illumination condition. Every estimate is then compared with $\mathbf{U}_\text{p}$ in a similar way as explained above. This approach is especially tedious when considering many modes because a linear system has to be solved for every possible combination. Our method, on the other hand, computes the LS solution of (6) by explicitly exploiting the Kronecker product structure of the coefficients.

III. Numerical experiments

We illustrate our KPE-based method for the extended Yale Face Database B[1] which contains cropped facial images of $J = 37$ persons under 64 illumination conditions. Some of the illuminations are missing for several persons and are therefore removed entirely from the dataset, obtaining $I = 57$ conditions. Each image of size 51×58 pixels is vectorized into a vector of length $M = 2958$. Hence, the resulting tensor $\mathcal{D}$ has size $2958 \times 37 \times 57$.

We use a non-linear LS algorithm with random initialization in order to solve (c)KPEs [12]. All computations are done with Tensorlab [17]. We compute the MLSVD with a randomized algorithm called `mlsvd_rsi` [18]. We use $R = 15$, $L = J$, and $P = 1000 \ll M$ which we determined via cross validation. We project the given image onto the column space of $\mathbf{U}_\text{px}$ in order to reduce computation time. In order to accommodate for scaling and sign invariance, the rows of $\mathbf{U}_\text{p}$ and the estimated coefficient vectors are normalized as $\text{sign}(c_1)\frac{\mathbf{c}}{\|\mathbf{c}\|}$. As explained in Subsection II-C, the normalized rows of $\mathbf{U}_\text{p}$ act as database, denoted by $\mathbf{U}_\text{db}$, and the normalized coefficient vector for the person mode acts as a feature vector.

A. Proof-of-Concept

Although the facial image in Figure 1 is almost completely dark, our method correctly recognizes the person in the image. In this case, we constructed the MLSVD model in (2) using all facial images of all persons under every illumination condition. Hence, the coefficient vectors $\mathbf{c}_p$ and $\mathbf{c}_i$ are perfectly reconstructed and a correct match is found. The reconstructed image in Figure 1 can be obtained by recomputing the vectorized image using the estimated coefficient vectors.

[1]The extended Yale Face Database B can be downloaded from http://vision.ucsd.edu/~leekc/ExtYaleDatabase/ExtYaleB.html.

TABLE I
By reformulating face recognition as a Kronecker Product Equation, higher performance (%) can be obtained in comparison to conventional techniques such as Eigenfaces as well as the tensor-based approach in [9]. Lower recognition time (s) is achieved in comparison to the method of [9].

	Eigenfaces [3]	Vasilescu [9]	KPE
Accuracy	93.3	93.5	**95.7**
Precision	90.6	94.4	**96.6**
Recall	88.4	90.9	**95.8**
Time of PCA/MLSVD	**2.97**	3.29	3.29
Time of recognition	**0.004**	0.135	0.097

TABLE II
Higher performance (%) can be achieved by using multiple images under different illuminations. Our cKPE-based method outperforms Eigenfaces using majority voting.

	Eigenfaces [3]			cKPE-based method		
# illuminations	1	2	3	1	2	3
Accuracy	92.7	93.3	96.3	**95.8**	**97.1**	**97.3**
Precision	89.8	91.2	97.9	**97.0**	**99.3**	**99.9**
Recall	87.7	87.8	97.5	**96.2**	**99.2**	**99.9**

B. Performance

Our method obtains higher performance than conventional techniques as we effectively exploit the multilinear structure of facial images by reformulating the recognition task as a KPE. Although our method is slower than the matrix-based method, it is slightly faster than the tensor-based method from [9] for this dataset. In Table I we report the median performance and time across 50 trials for our method, Eigenfaces, and the tensor-based method from [9]. In particular, we report the accuracy as well as the precision and recall using macro-averaging as explained in [19]. In each trial, 75% of the illuminations conditions are selected randomly as training set and 25% as test set for each person. For Eigenfaces, we unfold the data tensor and apply principal component analysis (PCA): we have $\mathbf{D}_{(1)} = \mathbf{B}\mathbf{C}^\text{T}$ with $\mathbf{B} \in \mathbb{R}^{M \times T}$ and $\mathbf{C} \in \mathbb{R}^{IJ \times T}$ using $T = J = 37$.

C. Improving performance through coupling

More robust recognition can be achieved by using multiple images of the same person under different illuminations. In Table II we report the median performance across 15 trials when using $Q = \{1, 2, 3\}$ randomly chosen illuminations. We compare our cKPE-based method to Eigenfaces using majority voting. For the latter, we assign the label of the person with the lowest index in the case of a tie. We use the same experiment settings as in Subsection III-B and solve a cKPE with Q randomly chosen illuminations which we repeat 25 times for each trial and each person. Clearly, higher accuracy can be achieved by using multiple images for both approaches. Our cKPE-based method achieves higher accuracy than Eigenfaces.



Fig. 2. The MLSVD model captures the new person reasonably well.

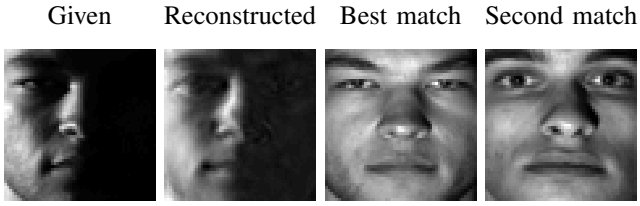


Fig. 3. Although we update the database with a new person using only one illumination condition, the KPE-based method recognizes that person in a new image under a different illumination condition.

D. Updating the database with a new person

Given an MLSVD model, the KPE-based method allows one to update the database $\mathbf{U}_{\text{db}}$ with a new person using *only a few* images instead of an image for each illumination. For example, consider an MLSVD model as in (2) which we have constructed using the facial images of all but one person under every illumination. The retained person is initially not included in the database $\mathbf{U}_{\text{db}}$, but can be recognized as follows. By solving (6) for a particular image, we obtain a feature vector $\hat{\mathbf{c}}_p^{(\text{new})}$ which we can add as a new row to the extended database $\mathbf{U}_{\text{db}}$ (with known label). In order to recognize the person in a new image under a different illumination, we can proceed as before, i.e., we solve a KPE and compare the obtained feature vector with the extended database. This strategy works well if the given image can be well approximated by the original MLSVD model. In practice, one can improve the recognition by extending the database using multiple illuminations and solving a cKPE to obtain a new row for the database.

We illustrate the approach for the person depicted in Figure 2 (left). In this example, we choose a neutral illumination to update the database. The current MLSVD captures the new person reasonably well as can be seen from the reconstructed image on the right. The person is correctly recognized in a new image with a different illumination as illustrated in Figure 3.

By using multiple illuminations to update the database, one can again further improve the performance. In Table III we report the performance when updating the database with the first, third, or 31th person in the extended Yale Face Database B using all other persons to construct the model. In other words, the data is divided into a training set of 36 persons and a test set of 1 person. In this experiment, we use $P = 1000 \ll M$. When using one, two, or three images, we use illumination setting 1, $\{1, 10\}$, and $\{1, 10, 55\}$, respectively. Illumination 1, 10, and 55 correspond to the neutral illumination and a left and right illumination of the face, respectively. Clearly, the accuracy improves by updating the model using multiple illuminations as can be seen for several persons in Table III. Also, one can see that the median performance over all persons in the dataset improves by using additional illuminations.

TABLE III
WHEN UPDATING THE DATABASE WITH A NEW PERSON, OUR METHOD CAN ACHIEVE HIGHER ACCURACY (%) BY FUSING MULTIPLE IMAGES UNDER DIFFERENT ILLUMINATION CONDITIONS INSTEAD OF USING ONLY ONE IMAGE OF THE NEW PERSON.

Person	One illumination	Two illuminations	Three illuminations
16	51.8	56.4	**59.3**
25	64.3	72.7	**75.9**
28	58.9	63.6	**70.4**
All	61.8	66.2	**68.1**

IV. Conclusion

In this paper, we propose a new tensor-based technique for face recognition that exploits the multidimensional nature of a collection of facial images under different conditions such as illumination, pose, and expression. First, we construct a tensor by stacking the images along several modes that each relate to a variation in the image. Our method models the obtained tensor by a multilinear SVD, describing each of the modes with a factor matrix and the interaction between the modes with a tensor. By reformulating the recognition task as the computation of a KPE, we can explicitly exploit the multilinear structure of the problem, obtaining a feature vector that enables higher performance than conventional methods. We illustrated our method for the extended Yale Face Database B, obtaining better performance than Eigenfaces and an other tensor-based technique. Our method performs well when using only a single image and the performance can be improved further by coupling a few images with different illuminations. Remarkably, our method also allows one to update the database with a new person using only a few images instead of an image for each combination of conditions. In future work, one can probably improve the performance by using neural networks or support vector machines in combination with KPE-computed feature vectors instead of using Euclidian distance-based comparisons. Additionally, one can take into account the non-negative nature of the data.

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