Irregular Heartbeat Classification Using Kronecker Product Equations

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***Abstract*—Cardiac arrhythmia or irregular heartbeats are an important feature to assess the risk on sudden cardiac death and other cardiac disorders. Automatic classification of irregular heartbeats is therefore an important part of ECG analysis. We propose a tensor-based method for single- and multi-channel irregular heartbeat classification. The method tensorizes the ECG data matrix by segmenting each signal beat-by-beat and then stacking the result into a third-order tensor with dimensions channel $\times$ time $\times$ heartbeat. We use the multilinear singular value decomposition to model the obtained tensor. Next, we formulate the classification task as the computation of a Kronecker Product Equation. We apply our method on the INCART dataset, illustrating promising results.**

I. Introduction

Cardiac arrhythmia or irregular heartbeats are conditions where the behavior of the heart is abnormal. This is characterized by the heart beating either too slow, too fast, or irregularly. In many cases irregular heartbeats do not require medical attention. However, certain types of arrhythmia such as ventricular fibrillation are medical emergencies that may lead to sudden cardiac death. The presence of arrhythmia can also be an indication of cardiac disorders. It is therefore essential that irregular heartbeats can be detected in a reliable way. Also, the rise of online and long-term ECG monitoring has increased the need for automated heartbeat classification methods. When an ECG signal contains thousands of heartbeats, manual beat inspection becomes a time-consuming and tedious task which is prone to human errors. Automatic irregular heartbeat detection methods are therefore an important tool in the diagnosis of patients at risk for cardiac events.

Traditional heartbeat classification methods often use RR interval or ECG morphology features [1], [2]. These methods typically represent the ECG signal as a vector. Recently, there is a trend to represent the signals in multi-lead ECG as a tensor in order to preserve structural information [3], [4], [5]. A tensor is a higher-order generalization of a vector (first-order) and a matrix (second-order). In this paper we tensorize the ECG data matrix into a third-order tensor with dimensions channel $\times$ time $\times$ heartbeat by means of segmentation [3]. This tensorization technique segments the ECG signal of each channel beat-by-beat and stacks the results into a third-order tensor, enabling the use of tensor decompositions.

We propose a new tensor-based method for irregular heartbeat classification which can classify new heartbeats as regular or irregular using the ECG signal of a single channel. First, we model the obtained tensor using a multilinear singular value decomposition (MLSVD) [6]. We show that every heartbeat in the tensor can then be expressed as a Kronecker Product Equation (KPE). The latter is a linear system of equations with a Kronecker product structured solution [7]. In order to classify a new heartbeat signal with an unknown label, we solve a similar KPE which allows us to find the closest match with a labeled heartbeat in the tensor. In practice, the MLSVD model is only approximate and robustness can be improved by using several channels instead of just one, leading to a coupled KPE. We illustrate our method on the INCART dataset.

In the remainder of this section we introduce the notation and basic definitions as well as the MLSVD and (coupled) KPEs. We present our method in Section II and discuss experiments in Section III. We conclude the paper in Section IV.

A. Notation and definitions

We denote vectors, matrices, and tensors by bold lower (e.g., $\mathbf{a}$), bold uppercase (e.g., $\mathbf{A}$), and calligraphic letters (e.g., $\mathcal{A}$), respectively. The nth element in a sequence is indicated by a superscript between parentheses, e.g., $\{\mathbf{A}^{(n)}\}_{n=1}^{N}$. A mode-$n$ vector of a tensor $\mathcal{A} \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_N}$ is defined by fixing every index except the nth and is a natural extension of the rows and columns of a matrix. The mode-n unfolding of $\mathcal{A}$ is a matrix $\mathbf{A}_{(n)}$ with mode-n vectors as its columns (following the ordering convention in [8]). The vectorization of $\mathcal{A}$, denoted as $\text{vec}(\mathcal{A})$, maps each element $a_{i_1 i_2 \cdots i_N}$ onto $\text{vec}(\mathcal{A})_j$ with $j = 1 + \sum_{k=1}^{N}(i_k - 1)J_k$ and $J_k = \prod_{m=1}^{k-1} I_m$.

The outer and Kronecker product are denoted by $\otimes$ and $\otimes$, respectively, and are related by $\text{vec}\,(\mathbf{a} \otimes \mathbf{b}) = \mathbf{b} \otimes \mathbf{a}$. The mode-$n$ product of a tensor $\mathcal{A} \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_N}$ and a matrix $\mathbf{B} \in \mathbb{R}^{J_n \times I_n}$ is a tensor $\mathcal{A} \cdot_n \mathbf{B} \in \mathbb{R}^{I_1 \times \cdots \times I_{n-1} \times J_n \times I_{n+1} \times \cdots I_N}$ and is defined element-wise as $(\mathcal{A} \cdot_n \mathbf{B})_{i_1 \cdots i_{n-1} j_n i_{n+1} \cdots i_N} = \sum_{i_n=1}^{I_n} a_{i_1 i_2 \cdots i_N} b_{j_n i_n}$. Hence, each mode-$n$ vector of the tensor $\mathcal{A}$ is multiplied with $\mathbf{B}$, i.e., $(\mathcal{A} \cdot_n \mathbf{B})_{(n)} = \mathbf{B}\mathbf{A}_{(n)}$.

An Nth-order tensor of rank one is defined as the outer product of N nonzero vectors [9]. The rank of a tensor equals the minimal number of rank-1 tensors that generate it as their sum. The mode-n rank of a tensor is defined as the rank of the mode-n unfolding of the tensor. The multilinear rank of an Nth order tensor is equal to the N-tuple of mode-n ranks.

B. Multilinear singular value decomposition

The multilinear singular value decomposition (MLSVD) of a higher-order tensor is a multilinear generalization of the singular value decomposition (SVD) of a matrix [6], [9], [10].

Definition 1. A *multilinear singular value decomposition* (MLSVD) writes a tensor $\mathcal{A} \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_N}$ as the product

$$\mathcal{A} = \mathcal{S} \cdot_1 \mathbf{U}^{(1)} \cdot_2 \mathbf{U}^{(2)} \cdots \cdot_n \mathbf{U}^{(N)},$$

in which $\mathbf{U}^{(n)} \in \mathbb{R}^{I_n \times I_n}$ is a unitary matrix, $1 \leq n \leq N$, and the core $\mathcal{S} \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_N}$ is ordered and all-orthogonal.

The MLSVD is a powerful tensor tool in applications such as compression and dimensionality reduction [8], [11]. It is related to the low-multilinear rank approximation (LMLRA) and the Tucker model, see [6], [12] and references therein.

C. Kronecker Product Equations

A KPE is a linear system of equations with a solution that has a Kronecker product structure [7]. Consider a system $\mathbf{A}\mathbf{x} = \mathbf{b}$ with $\mathbf{A} \in \mathbb{R}^{M \times K}$, $\mathbf{x} \in \mathbb{R}^K$, and $\mathbf{b} \in \mathbb{R}^M$. Assume the solution $\mathbf{x}$ is constrained to the following simple Kronecker product structure: $\mathbf{x} = \mathbf{v} \otimes \mathbf{u}$ with $\mathbf{u} \in \mathbb{R}^I$ and $\mathbf{v} \in \mathbb{R}^J$ such that $K = IJ$. As such, we have that:

$$\mathbf{A}(\mathbf{v} \otimes \mathbf{u}) = \mathbf{b}. \tag{1}$$

The Kronecker product structure can be exploited in order to rewrite (1) as a multilinear system of equations [7]:

$$\mathcal{A} \cdot_2 \mathbf{u}^{\mathrm{T}} \cdot_3 \mathbf{v}^{\mathrm{T}} = \mathbf{b}$$

with the coefficient tensor $\mathcal{A} \in \mathbb{R}^{M \times I \times J}$ defined such that its mode-1 unfolding $\mathbf{A}_{(1)} \in \mathbb{R}^{M \times IJ}$ equals the coefficient matrix $\mathbf{A}$ in (1), i.e., we have that $\mathbf{A}_{(1)} = \mathbf{A}$.

A coupled KPE (cKPE) is a set of KPEs that share a coefficient vector. We limit ourselves to cKPEs of the form:

$$\mathbf{A}(\mathbf{v}^{(q)} \otimes \mathbf{u}) = \mathbf{b}^{(q)} \text{ for } 1 \leq q \leq Q \tag{2}$$

with $\mathbf{A} \in \mathbb{R}^{M \times K}$, $\mathbf{v}^{(q)} \in \mathbb{R}^I$, $\mathbf{u} \in \mathbb{R}^J$, and $\mathbf{b}^{(q)} \in \mathbb{R}^M$ such that $K = IJ$. We can reformulate (2) as a multilinear system:

$$\mathcal{A} \cdot_2 \mathbf{u}^{\mathrm{T}} \cdot_3 \mathbf{V}^{\mathrm{T}} = \mathbf{B} \tag{3}$$

in which $\mathbf{V} \in \mathbb{R}^{I \times Q}$ with $\mathbf{v}_q = \mathbf{v}^{(q)}$, $\mathbf{B} \in \mathbb{R}^{M \times Q}$ with $\mathbf{b}_q = \mathbf{b}^{(q)}$, and $\mathbf{A}_{(1)} = \mathbf{A}$. Expression (3) is equivalent with:

$$\mathbf{A}(\mathbf{V} \otimes \mathbf{u}) = \mathbf{B}$$

which can be interpreted as a more general type of KPE.

II. Irregular heartbeat classification as a Kronecker Product Equation

A. Preprocessing and tensorization

The preprocessing step is necessary to remove noise from the ECG signal that may corrupt the final classification performance. Similarly as in [3], we consider baseline wander and high frequency noise from muscle artifacts as primary noise sources. They are removed channel-by-channel using quadratic variation reduction and wavelet-based filtering, respectively.

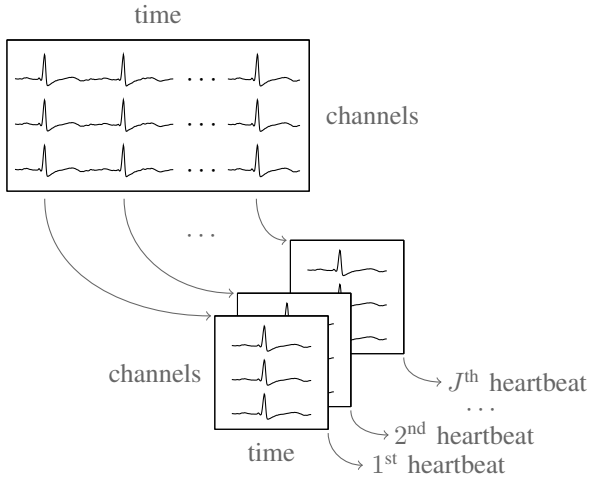


Fig. 1. Tensorization of an ECG data matrix into a third-order tensor with dimensions channel × time × heartbeat using segmentation.

Next, we transform the ECG data matrix into a third-order tensor as illustrated in Figure 1. First, we segment the signals into smaller segments of size I containing only a single heartbeat. As such, we obtain J heartbeats for all M channels. Next, we stack all heartbeats in the third mode, obtaining a third-order tensor $\mathcal{T} \in \mathbb{R}^{M \times I \times J}$ with dimensions channel × time × heartbeat. We use this particular tensorization because we are only interested in the differences between subsequent heartbeats but other techniques can be found in literature [13].

Segmentation in individual heartbeats is done here by taking a fixed-size window of 500 ms around each R peak, starting 200 ms before the peak. The R peak location can easily be detected using standard techniques such as Pan-Tompkins. Note that when the heart rate changes a lot throughout the signal (for example in long term ambulatory signals), resampling the heartbeats might be required to align the different ECG waves.

B. Kronecker product equation

The (truncated) MLSVD of the tensor $\mathcal{T}$ is given by:

$$\mathcal{T} = \mathcal{S} \cdot_1 \mathbf{U}_{\mathrm{c}} \cdot_2 \mathbf{U}_{\mathrm{t}} \cdot_3 \mathbf{U}_{\mathrm{h}} \tag{4}$$

with $\mathbf{U}_{\mathrm{c}} \in \mathbb{R}^{M \times P}$, $\mathbf{U}_{\mathrm{t}} \in \mathbb{R}^{I \times R}$, and $\mathbf{U}_{\mathrm{h}} \in \mathbb{R}^{J \times L}$ forming an orthonormal basis for the spatial, temporal, and shape component, respectively. The coefficient tensor $\mathcal{S} \in \mathbb{R}^{P \times R \times L}$ explains the interaction between the different modes. *Every* heartbeat $\mathbf{t} \in \mathbb{R}^I$ of a *particular* channel, i.e., every mode-2 vector of the tensor $\mathcal{T}$, satisfies the following model:

$$\mathbf{t}^{\mathrm{T}} = \mathcal{S} \cdot_1 \mathbf{c}_{\mathrm{c}}^{\mathrm{T}} \cdot_2 \mathbf{U}_{\mathrm{t}} \cdot_3 \mathbf{c}_{\mathrm{h}}^{\mathrm{T}}. \tag{5}$$

Vectors $\mathbf{c}_{\mathrm{c}}^{\mathrm{T}}$ and $\mathbf{c}_{\mathrm{h}}^{\mathrm{T}}$ are rows of $\mathbf{U}_{\mathrm{c}}$ and $\mathbf{U}_{\mathrm{h}}$, respectively, corresponding to the coefficients of heartbeat h and channel c.

Clearly, the mode-2 unfolding of (5) is a KPE:

$$\mathbf{t} = \mathbf{U}_{\mathrm{t}} \mathbf{S}_{(2)} (\mathbf{c}_{\mathrm{h}} \otimes \mathbf{c}_{\mathrm{c}}). \tag{6}$$

Equation (6) expresses $\mathbf{t}$ in the column space $\mathbf{U}_{\mathrm{t}}$ and (the mode-2 unfolding of) an additional interaction tensor $\mathcal{S}$ that

links the different modes. The coefficients can then be written as a Kronecker product of the coefficient vectors $\mathbf{c}_\text{h}$ and $\mathbf{c}_\text{c}$.

We can also consider a set of K channels instead of just one. In that case we have a set of K KPEs that are coupled via the coefficient vector for the heartbeat dimension:

$$\mathbf{t}^{(q)} = \mathbf{U}_\text{t}\mathbf{S}_{(2)}(\mathbf{c}_\text{h} \otimes \mathbf{c}_\text{c}^{(q)}) \text{ for } 1 \leq q \leq Q.$$

We collect all heartbeat signals $\mathbf{t}^{(q)}$ in $\mathbf{T} \in \mathbb{R}^{I \times Q}$ and all channel coefficients $\mathbf{c}_\text{c}^{(q)}$ in $\mathbf{C}_\text{c} \in \mathbb{R}^{M \times K}$. As such, we obtain:

$$\mathbf{T}^\text{T} = \mathcal{S} \cdot_1 \mathbf{C}_\text{c}^\text{T} \cdot_2 \mathbf{U}_\text{t} \cdot_3 \mathbf{c}_\text{h}^\text{T}$$

which is equivalent with the following cKPE:

$$\mathbf{T} = \mathbf{U}_\text{t}\mathbf{S}_{(2)}(\mathbf{c}_\text{h} \otimes \mathbf{C}_\text{c}). \tag{7}$$

C. Irregular heartbeat classification

We explain how to classify a *new* heartbeat measured on a *single* channel as regular or irregular using KPEs. Consider an ECG data matrix with known heartbeat labels. First, we perform preprocessing and tensorization as explained in Subsection II-A, obtaining a tensor $\mathcal{T}$. Next, we compute a MLSVD of $\mathcal{T}$ as in (4), obtaining factor matrices $\mathbf{U}_\text{c}$, $\mathbf{U}_\text{t}$, and $\mathbf{U}_\text{h}$ and core tensor $\mathcal{S}$. Recall that every heartbeat in $\mathcal{T}$ can be expressed as a KPE as in (6). Consider now a *new* heartbeat $\mathbf{t}^{(\text{new})}$ with unknown label, i.e., a heartbeat that is not included in $\mathcal{T}$. In order to classify the new heartbeat, we solve a KPE:

$$\mathbf{U}_\text{t}\mathbf{S}_{(2)}(\mathbf{c}_\text{h}^{(\text{new})} \otimes \mathbf{c}_\text{c}^{(\text{new})}) = \mathbf{t}^{(\text{new})},$$

obtaining estimates $\hat{\mathbf{c}}_\text{h}^{(\text{new})}$ and $\hat{\mathbf{c}}_\text{c}^{(\text{new})}$ for the unknown coefficient vectors $\mathbf{c}_\text{h}^{(\text{new})}$ and $\mathbf{c}_\text{c}^{(\text{new})}$, respectively. We compare $\hat{\mathbf{c}}_\text{h}^{(\text{new})}$ with the rows of $\mathbf{U}_\text{h}$ using the norm of the difference (after fixing scaling and sign invariance). Finally, we classify the new heartbeat with the label corresponding to the closest match.

We use the data of all channels to compute the MLSVD but classify using the signal from a single channel. In practice, however, the MLSVD model holds only approximately and incorrect classification can possibly occur. We can make the classification more robust by using heartbeats from *multiple* channels which can be solved using a coupled KPE as in (7).

III. Results and Discussion

We illustrate the proposed method with two experiments using the first ten subjects of the St.-Petersburg Institute of Cardiological Technics 12-lead Arrhythmia (INCART) Database available on Physionet [14]. The dataset consists of 75 ECG recordings from 32 subjects. All signals are 30 minutes long and contain 12 standard leads. The sampling frequency is 257 Hz. The signals are collected during tests for coronary artery diseases. The dataset contains all ECG signals together with patient diagnoses, R peak locations and beat annotations. The beat annotations were first automatically determined and later corrected manually. We apply preprocessing and segmentation as explained in Subsection II-A and obtain heartbeats of length $I = 131$. The number of heartbeats J is different for each subject. The number of channels is $M = 12$.

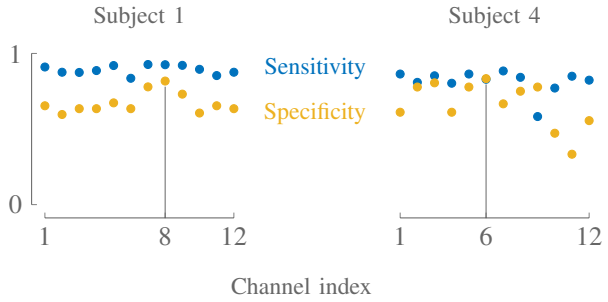


Fig. 2. Overall our method achieves good performance while better results can be obtained by using a suitable channel for a given subject, e.g., channels 8 and 6 achieve the highest specificity for subjects 1 and 4, respectively.

We developed nonlinear least-squares (NLS) algorithms for solving KPEs and cKPEs, called `kpe_nls` and `ckpe_nls`, respectively, which are available upon request [7]. All computations are done with Tensorlab [15]. We compute the MLSVD with a randomized algorithm called `mlsvd_rsi` which is faster but achieves similar accuracy than non-randomized MLSVD algorithms [16]. We use $P = M = 12$ and $R = I = 131$. Strongly truncating the third mode, i.e., taking $L \ll J$, decreases computation time and improves classification performance. The optimal value for L is subject dependent and can be determined via validation data with $2 \leq L \leq 10$. We use random initialization in all experiments. Each row of $\mathbf{U}_\text{h}$ and the estimated coefficient vectors are normalized to accommodate for scaling and sign invariance as follows: a vector $\mathbf{c}$ is normalized to $\bar{\mathbf{c}}$ as $\bar{\mathbf{c}} = \text{sign}(c_1)\frac{\mathbf{c}}{\|\mathbf{c}\|}$.

In a first experiment, we show that our method achieves high classification performance provided we choose a suitable channel for classification. This is illustrated in Figure 2 where we report the median across 30 trials of the sensitivity and specificity for subjects one and four and all channels. The data for subject one and four consist of 2411 and 2301 regular and 344 and 121 irregular heartbeats, respectively [14]. For each subject, we randomly divided the data in training (85%) and (15%) test set in each trial. We used $L = 4$ and $L = 8$ in the MLSVD model, respectively. Clearly, the performance depends on the choice of the channel and the choice is subject dependent. For example, the highest specificity for subject one (0.8173) and four (0.8173) is achieved if one uses channel eight (V2) and six (AVF), respectively. However, the overall performance is also good: the median sensitivity and specificity across all subjects, using, e.g., channel eight (V2), is 0.9083 and 0.7353, respectively. Moreover, in that case the F_1 score is 94.2% which is better than the best performance (92%) of traditional techniques as in [2] that use all channels. It is remarkable that our method can achieve high performance while using only a single channel for classification.

Fusing the ECG signals from multiple channels with our method improves classification performance. In Figure 3 we report the median across 10 trials of the sensitivity and specificity for subject one (using $L = 4$). In each trial we also randomly divided the data in training (85%) and test (15%) set.

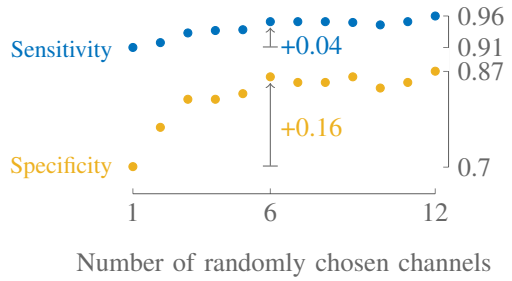


Fig. 3. Fusing signals from multiple channels leads to a better performance.

The number of channels that is used for classification is varied from one to twelve and in each trial the channels are chosen randomly. For example, coupling six random channels greatly improves the specificity for subject one. However, only a small improvement is obtained for the sensitivity. Also, coupling more than six channels does not seem to increase the overall performance significantly for this subject.

IV. Conclusion

We presented a new tensor-based method for single- and multi-channel irregular heartbeat classification. The proposed method tensorizes the ECG data matrix using segmentation. The obtained tensor is modeled by a MLSVD which allows us to express every heartbeat in the tensor as a KPE. We have shown that the classification task can then be formulated as the computation of a KPE. While the method performs well for only a single channel, the performance can be improved by coupling the ECG signals from multiple channels by means of a cKPE. We illustrated our method on the INCART dataset. The proposed method can achieve high performance by choosing a suitable channel for classification. Coupling multiple channels, improved the overall classification performance. In future work, the method can be extended to multi-class classification. Also, one can possibly improve the performance by using more intricate schemes to determine the best match in the database. Finally, further research is necessary to determine the best channel(s) to use for classification in both the single- and multi-channel case.

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References

[1] M. G. Tsipouras, D. I. Fotiadis, and D. Sideris, "An arrhythmia classification system based on the RR-interval signal," *Artificial Intelligence in Medicine*, vol. 33, no. 3, pp. 237–250, 2005.

[2] M. Llamedo, A. Khawaja, and J. Martínez, "Analysis of 12-lead classification models for ECG classification," in *2010 Computing in Cardiology*, Sept. 2010, pp. 673–676.

[3] G. Goovaerts, O. De Wel, B. Vandenberk, R. Willems, and S. Van Huffel, "Detection of irregular heartbeats using tensors," in *Proceedings of the 42nd annual Conference of Computing in Cardiology (CinC 2015), Nice, France*, Sept. 2015, pp. 573–576.

[4] K. Huang and L. Zhang, "Cardiology knowledge free ECG feature extraction using generalized tensor rank one discriminant analysis," *EURASIP Journal on Advances in Signal Processing*, vol. 2014, no. 1, p. 2, 2014.

[5] Q. Li and D. Schonfeld, "Multilinear discriminant analysis for higher-order tensor data classification," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 36, no. 12, pp. 2524–2537, Dec. 2014.

[6] L. De Lathauwer, B. De Moor, and J. Vandewalle, "A multilinear singular value decomposition," *SIAM Journal on Matrix Analysis and Applications*, vol. 21, no. 4, pp. 1253–1278, Apr. 2000.

[7] M. Boussé, N. Vervliet, I. Domanov, O. Debals, and L. De Lathauwer, "Linear systems with a canonical polyadic decomposition constrained solution: Algorithms and applications," *Technical Report 17-01, ESAT-STADIUS, KU Leuven, Leuven, Belgium*, 2017.

[8] T. G. Kolda and B. W. Bader, "Tensor decompositions and applications," *SIAM Review*, vol. 51, no. 3, pp. 455–500, Aug. 2009.

[9] N. Sidiropoulos, L. De Lathauwer, X. Fu, K. Huang, E. Papalexakis, and C. Faloutsos, "Tensor decomposition for signal processing and machine learning," *IEEE Transactions on Signal Processing*, 2017, (to appear).

[10] A. Cichocki, D. P. Mandic, L. De Lathauwer, G. Zhou, Q. Zhao, C. F. Caiafa, and A.-H. Phan, "Tensor decompositions for signal processing applications: From two-way to multiway component analysis," *IEEE Signal Processing Magazine*, vol. 32, no. 2, pp. 145–163, Mar. 2015.

[11] L. De Lathauwer and J. Vandewalle, "Dimensionality reduction in higher-order signal processing and rank-$(R_1, R_2, \ldots, R_N)$ reduction in multilinear algebra," *Linear Algebra and its Applications*, vol. 391, pp. 31–55, Nov. 2004.

[12] N. Vervliet, O. Debals, L. Sorber, and L. De Lathauwer, "Breaking the curse of dimensionality using decompositions of incomplete tensors: Tensor-based scientific computing in big data analysis," *IEEE Signal Processing Magazine*, vol. 31, no. 5, pp. 71–79, Sept. 2014.

[13] O. Debals and L. De Lathauwer, "Stochastic and deterministic tensorization for blind signal separation," in *Latent Variable Analysis and Signal Separation*, ser. Lecture Notes in Computer Science. Springer Berlin / Heidelberg, 2015, vol. 9237, pp. 3–13.

[14] A. L. Goldberger, L. A. N. Amaral, L. Glass, J. M. Hausdorff, P. C. Ivanov, R. G. Mark, J. E. Mietus, G. B. Moody, C.-K. Peng, and H. E. Stanley, "PhysioBank, PhysioToolkit, and PhysioNet: Components of a new research resource for complex physiologic signals," *Circulation*, vol. 101, no. 23, pp. e215–e220, June 2000, Circulation Electronic Pages: http://circ.ahajournals.org/content/101/23/e215.full PMID:1085218; doi: 10.1161/01.CIR.101.23.e215".

[15] N. Vervliet, O. Debals, L. Sorber, M. Van Barel, and L. De Lathauwer, "Tensorlab 3.0," Mar. 2016. [Online]. Available: http://www.tensorlab.net/

[16] N. Vervliet, O. Debals, and L. De Lathauwer, "Tensorlab 3.0 — Numerical optimization strategies for large-scale constrained and coupled matrix/tensor factorization," in *Proceedings of the 50th Asilomar Conference on Signals, Systems and Computers (Pacific Grove, CA)*, Nov. 2016, pp. 1733–1738.