$$
\begin{aligned}
p(\mathcal{R},\, \mathcal{C} \,|\, \mathbf{D}) =& \int_{\boldsymbol{\Theta}} \int_{\boldsymbol{\phi}} \int_{\lambda_\mathrm{r}} \int_{\lambda_\mathrm{c}} p(\mathcal{R},\, \mathcal{C},\, \lambda_\mathrm{r},\, \lambda_\mathrm{c},\, \boldsymbol{\Theta},\, \boldsymbol{\phi} \,|\, \mathbf{D})\, \mathrm{d}\boldsymbol{\Theta}\, \mathrm{d}\boldsymbol{\phi}\, \mathrm{d}\lambda_\mathrm{r}\, \mathrm{d}\lambda_\mathrm{c} \\
=& \frac{1}{p(\mathbf{D})} \cdot \int_{\boldsymbol{\Theta}} \int_{\boldsymbol{\phi}} p(\mathbf{D} \,|\, \mathcal{R},\, \mathcal{C},\, \boldsymbol{\Theta},\, \boldsymbol{\phi}) \cdot p(\boldsymbol{\Theta}) \mathrm{d}\boldsymbol{\Theta} \cdot p(\boldsymbol{\phi})\, \mathrm{d}\boldsymbol{\phi} \\
& \int_{\lambda_\mathrm{r}} \cdot P(\mathcal{R} \,|\, \lambda_\mathrm{r}) \cdot p(\lambda_\mathrm{r})\, \mathrm{d}\lambda_\mathrm{r} \cdot \int_{\lambda_\mathrm{c}} P(\mathcal{C},\, |\, \lambda_\mathrm{c}) \cdot p(\lambda_\mathrm{c})\, \mathrm{d}\lambda_\mathrm{c}.
\end{aligned} \tag{1}
$$

$$
\begin{aligned}
\gamma_i^r =& \frac{p(\mathbf{D},\, r_i = 1,\, \mathcal{R}_{\bar{i}},\, \mathcal{C})}{p(\mathbf{D},\, r_i = 0,\, \mathcal{R}_{\bar{i}},\, \mathcal{C})} \\
=& \frac{\int_{\boldsymbol{\Theta}} \int_{\boldsymbol{\phi}} p(\mathbf{D} \,|\, r_i = 1,\, \mathcal{R}_{\bar{i}},\, \mathcal{C},\, \boldsymbol{\Theta},\, \boldsymbol{\phi}) \cdot p(\boldsymbol{\Theta}) \mathrm{d}\boldsymbol{\Theta} \cdot p(\boldsymbol{\phi}) \mathrm{d}\boldsymbol{\phi}}{\int_{\boldsymbol{\Theta}} \int_{\boldsymbol{\phi}} p(\mathbf{D} \,|\, r_i = 0,\, \mathcal{R}_{\bar{i}},\, \mathcal{C}, \boldsymbol{\Theta}, \boldsymbol{\phi}) \cdot p(\boldsymbol{\Theta}) \mathrm{d}\boldsymbol{\Theta} \cdot p(\boldsymbol{\phi}) \mathrm{d}\boldsymbol{\phi}} \\
& \cdot \frac{v_{\bar{i}} + \xi_{\mathrm{r}1}}{N - 1 - v_{\bar{i}} + \xi_{\mathrm{r}0}}
\end{aligned} \tag{2}
$$

$$
\begin{aligned}
& P(\mathbf{D} \,|\, r_i = 1,\, \mathcal{R}_{\bar{i}},\, \mathcal{C},\, \boldsymbol{\Theta},\, \boldsymbol{\phi}) \\
=& \boldsymbol{\phi}^{c(\mathbf{D}[\bar{\boldsymbol{r}}_{\bar{i}}, \cdot]) + c(\mathbf{D}[\boldsymbol{r}_{\bar{i}}, \bar{\mathbf{c}}]) + c(\mathbf{D}[i, \bar{\mathbf{c}}])} \cdot \prod_{j \in \mathbf{c}} \boldsymbol{\Theta}_{\cdot,j}^{c(\mathbf{D}[r_{\bar{i}} \oplus i, j])}
\end{aligned} \tag{3}
$$

$$
\begin{aligned}
& P(\mathbf{D} \,|\, \mathcal{R},\, c_j = 1,\, \mathcal{C}_{\bar{j}},\, \boldsymbol{\Theta},\, \boldsymbol{\phi}) \\
=& P \boldsymbol{\phi}^{c(\mathbf{D}[\cdot, \bar{\boldsymbol{c}}_{\bar{j}}]) + c(\mathbf{D}[\bar{\boldsymbol{r}}_{\bar{j}}, \bar{\boldsymbol{r}}])} \\
\cdot \prod_{j \in \bar{\mathbf{c}}} \boldsymbol{\Theta}_{\cdot,k}^{c(\mathbf{D}[\bar{\boldsymbol{r}}, k])} & \cdot \boldsymbol{\Theta}_{\cdot,j}^{c(\mathbf{D}[\bar{\boldsymbol{r}}, j])}
\end{aligned} \tag{4}
$$