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A Pole-Zero Placement Technique for Designing Second-Order IIR Parametric Equalizer Filters[1]

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A Pole-Zero Placement Technique for Designing Second-Order IIR Parametric Equalizer Filters

Toon van Waterschoot, *Student Member, IEEE,*
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***Abstract*— A new procedure is presented for designing second-order parametric equalizer filters. In contrast to the traditional approach, in which the design is based on a bilinear transform of an analog filter, the presented procedure allows for designing the filter directly in the digital domain. A rather intuitive technique known as pole-zero placement, is treated here in a quantitative way. It is shown that by making some meaningful approximations, a set of relatively simple design equations can be obtained. Design examples of both notch and resonance filters are included to illustrate the performance of the proposed method, and to compare with state-of-the-art solutions.**

***Index Terms*— Filter design, parametric equalizer filters, notch filters, resonance filters, pole-zero placement.**

I. Introduction

Many signal processing applications involve enhancing or attenuating only a small portion of a signal's frequency spectrum, while leaving the remainder of the spectrum unaffected. This effect is obtained by using bandpass or bandstop filters, that have a frequency response which is characterized by a gain increase or decrease around a specified center frequency f_c. In digital audio equalization, any desired frequency response may be realized by cascading such bandpass/-stop filters with different center frequencies, which are then often referred to as parametric equalizer filters or presence filters. Such a cascade may moreover include high-pass and low-pass filters, also known as shelving filters, which are in fact special (degenerate) cases of the bandpass/-stop filter, having one real pole-zero pair instead of two complex conjugate pole-zero pairs.

Apart from the center frequency, a parametric equalizer filter is also characterized by its bandwidth (which we will define later on). Filters having a small bandwidth B relative to their center frequency (i.e., having a high Q-factor, defined as $Q = f_c/B$), are better known as notch and resonance (or peaking) filters. Notch filters appear in numerous applications where a (nearly) sinusoidal interference has to be cancelled from a broadband signal, e.g., suppressing a 50/60 Hz AC interference in a low-voltage measurement signal, or cancelling acoustic feedback oscillations in audio amplification systems. Resonance filters are typically used to recover sinusoids buried in noise (so-called line enhancement), e.g., in communications and sonar applications.

It is well-known that a bandpass or bandstop characteristic around a specified center frequency can be realized efficiently using a second-order infinite impulse response (IIR) filter [1]-[11], also known as a biquadratic filter, with transfer function

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}. \tag{1}$$

Traditionally, the five filter coefficients are calculated so as to satisfy a set of five design equations [7]:

$$|H(e^{j0})| = G_0 \quad \text{(gain at DC)} \tag{2a}$$
$$|H(e^{j\pi})| = G_\pi \quad \text{(gain at Nyquist frequency)} \tag{2b}$$
$$\left(\frac{\partial}{\partial\omega}|H(e^{j\omega})|\right)\Big|_{\omega=\omega_c} = 0 \quad \text{(center frequency)} \tag{2c}$$
$$|H(e^{j\omega_c})| = G_c \quad \text{(gain at resonance)} \tag{2d}$$
$$|H(e^{j(\omega_c \pm B/2)})| = G_B \quad \text{(bandwidth)} \tag{2e}$$

where $\omega = 2\pi f$ represents radial frequency. These design equations incorporate the following design variables: (radial) center frequency ω_c, (radial) bandwidth B, gain at band edges G_B, gain at resonance G_c, gain at DC G_0, and gain at Nyquist frequency G_π. Typically, the DC and Nyquist gain are chosen to be equal (except in [10], with the aim of digitally emulating an analog equalizer) and are set to 0 dB, i.e., $G_0 = G_\pi = 1$, which facilitates the cascading of several parametric equalizer filters. Unfortunately, there is little agreement in the literature on how to appropriately define bandwidth [7]. We will adopt Moorer's bandwidth definition [2], which is found to be mathematically the most consistent one [7]. If the resonance gain relative to the gain at DC, i.e., G_c/G_0 (called the 'boost' in a resonance filter and the 'cut' in a notch filter), exceeds 6 dB in absolute value, then the band edges are defined as the frequencies at which the gain is 3dB below/above the peak/notch. For a boost/cut less than 6dB in absolute value, the band edges are found at the so-called midpoint gain, which is defined as the geometric mean $\sqrt{G_c G_0}$, i.e., $1/2(G_{c,\text{dB}} - G_{0,\text{dB}})$ dB below/above the peak/notch.

Nearly all existing design procedures start from the design of an analog parametric equalizer filter, followed by a bilinear transform that maps the analog frequency axis $[0, \infty)$ onto the digital frequency axis $[0, f_s/2]$, with f_s the sampling frequency [1],[3]-[7],[10]. To this end, the digital design variables ω_c and B should be "prewarped" to analog variables, which can however not be done in an exact way for the bandwidth [5],[7]. As an alternative, the parametric equalizer filter can also be designed directly in the digital domain. A first approach was suggested in [2] and starts by designing a digital parametric equalizer filter with center frequency $\omega_c = \pi/2$ (which is the only digital center frequency that allows for a truly symmetric frequency response, leading to $b_1 = a_1 = 0$). The filter centered at $\pi/2$ is then transformed to an arbitrarily centered filter using an appropriate bilinear transform. A second approach is more intuitive and based on a technique known as pole-zero placement. It was shown earlier how a resonance filter with a specified center frequency and bandwidth can be designed in the z-plane by placing two complex conjugate poles (inside, but close to the unit circle) on the radial lines from the origin to $e^{\pm j\omega_c}$, and two zeros at the origin [8, Ch. 6],[9, Ch. 6],[11, Ch. 11]. For this design procedure, several approximate relations between the bandwidth and the so-called pole radius (i.e., the distance from the origin to the pole) have been suggested [8, Ch. 6],[11, Ch. 11]. Also, it was noted in [9, Ch. 6] that by moving the zeros from the origin towards the poles, and even beyond, the resonance characteristic can be converted into a notch characteristic.

The aim of this correspondence is to present a more quantitative treatment of the pole-zero placement approach. We will derive exact relations between the pole and zero positions on the one hand, and the design variables on the other hand. These relations will

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however appear to be impractical for implementation, and hence we will suggest some useful approximations. The pole-zero placement design procedure is outlined in Section II, where we will make a distinction between notch filter and resonance filter design. Some design examples are given in Section III, and finally, Section IV concludes the correspondence.

II. Design Procedure

The pole-zero placement design procedure is based on a radial representation of the biquadratic filter transfer function, in which poles and zeros are constrained to lie on the radial lines from the origin to $e^{\pm j\omega_c}$ [9, Ch. 6],[11, Ch. 11],[12], i.e.,

$$\boxed{H(z) = K\frac{(1 - r_z e^{j\omega_c} z^{-1})(1 - r_z e^{-j\omega_c} z^{-1})}{(1 - r_p e^{j\omega_c} z^{-1})(1 - r_p e^{-j\omega_c} z^{-1})},} \tag{3}$$

which can equivalently be written in direct form as

$$H(z) = K\frac{1 - 2r_z \cos\omega_c z^{-1} + r_z^2 z^{-2}}{1 - 2r_p \cos\omega_c z^{-1} + r_p^2 z^{-2}}. \tag{4}$$

The zero radius $r_z \in [0, 1]$ is defined as the distance from the origin to each of the complex conjugate zeros, and likewise the pole radius $r_p \in [0, 1)$ is defined as the distance from the origin to the poles. A broadband gain factor K is also included.

In contrast to the original biquadratic filter transfer function in (1), the radial representation contains only four distinct parameters and, as a consequence, only four design equations can be fulfilled. Therefore, the design equations (2a) and (2b) may be replaced by only one equation that specifies the filter response at an arbitrary frequency. For convenience, however, we will just omit equation (2b) and retain equation (2a). Furthermore, since the center frequency can be specified directly in the radial representation in (3) or (4), design equation (2c) can also be omitted. Note that equation (2c) would however only be exactly fulfilled for a desired center frequency at $\omega_c = \{0, \pi/2, \pi\}$, since the maximum/minimum in the frequency response of the constrained resonance/notch filter in (3) does not generally appear at the resonance frequency, which is due to the influence of the complex conjugate pole-zero pair [11, Ch. 11]. This effect decreases as the poles approach the unit circle.

We will now derive exact and approximate relations between the remaining three filter parameters $\{r_z, r_p, K\}$ and the design variables $\{\omega_c, B, G_0, G_c\}$, by evaluating the remaining design equations (2a), (2d), and (2e). We will generally assume that the poles are close to the unit circle, i.e., $0 << r_p < 1$, resulting in narrowband parametric equalizer filters. To obtain a broader bandpass/bandstop characteristic, a cascade of shelving filters should be used instead. Below, the design of notch and resonance filters is treated separately, since different assumptions can be made in either case.

A. Notch Filters

To obtain a notch characteristic, the zeros should be placed between the poles and the unit circle, i.e., $0 << r_p < r_z \leq 1$. The gain at resonance can be calculated by using the so-called graphical method for evaluating a frequency response [8, Ch. 4],[11, Ch. 9], see Fig. 1,

$$|H(e^{j\omega_c})| = K\frac{|Z_1R||Z_2R|}{|P_1R||P_2R|}. \tag{5}$$

Note that the notation $|\cdot|$ is used in two different ways: $|XY|$ denotes the distance between two points X and Y in the imaginary plane, whereas the $|H(e^{j\omega})|$ stands for the frequency magnitude response of the biquadratic filter.

Since the poles are close to the unit circle, and hence close to the zeros (in practice even much closer than in Fig. 1), we can assume

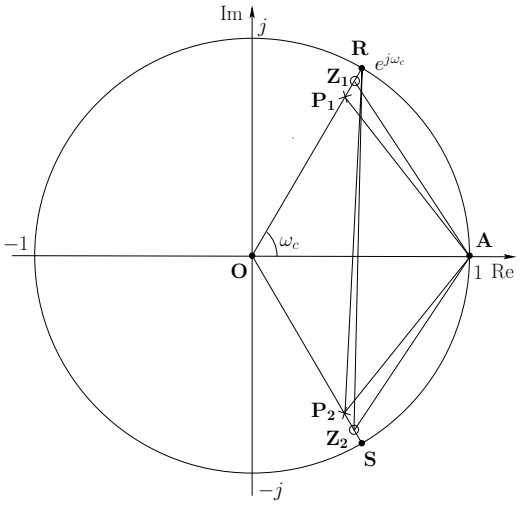


Fig. 1. Pole-zero plot for notch filter (x = poles, o = zeros).

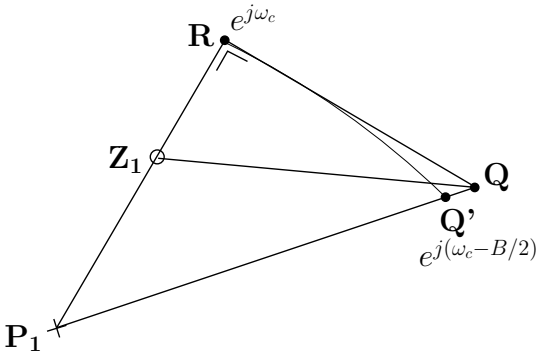


Fig. 2. Bandwidth calculation for notch filter.

the distances from the complex conjugate pole and zero to $e^{j\omega_c}$ to be nearly equal, i.e. $|Z_2R| \approx |P_2R|$, as compared to the distances $|Z_1R|$ and $|P_1R|$. With this approximation, (5) can be rewritten as

$$|H(e^{j\omega_c})| = K\frac{(1 - r_z)}{(1 - r_p)}. \tag{6}$$

The gain at DC can be evaluated by using the same approach, i.e.,

$$|H(e^{j0})| = K\frac{|Z_1A||Z_2A|}{|P_1A||P_2A|}. \tag{7}$$

Because the distance between the pole at P_1 and the zero at Z_1 is much smaller than the distance of P_1 and Z_1 to the point e^{j0}, we may state that $|Z_1A| \approx |P_1A|$ (this may appear to be a bad approximation for $\omega_c \to 0$, however in that case a highpass characteristic will be obtained, unless $r_p, r_z \to 1$, which still leads to $|Z_1A| \approx |P_1A|$). The same reasoning results in $|Z_2A| \approx |P_2A|$ for the complex conjugate pole-zero pair, such that (7) can be rewritten as

$$|H(e^{j0})| = K. \tag{8}$$

Dividing (6) and (8), and applying design equations (2a) and (2d), leads to

$$\frac{G_c}{G_0} = \frac{(1 - r_z)}{(1 - r_p)}. \tag{9}$$

We will now identify the lower band edge $e^{j(\omega_c - B/2)}$ and calculate its distance from the poles and zeros. Since we are dealing with narrowband filters, the distance between $e^{j\omega_c}$ and $e^{j(\omega_c - B/2)}$ will be relatively small, hence the influence of the complex conjugate pole-zero pair on the change in frequency response from $e^{j\omega_c}$ to

$e^{j(\omega_c - B/2)}$ will be negligible. It then suffices to consider the region depicted in Fig. 2 for calculating the lower band edge, and

$$|H(e^{j(\omega_c - B/2)})| \approx K\frac{|Z_1Q'|}{|P_1Q'|} \approx K\frac{|Z_1Q|}{|P_1Q|}, \tag{10}$$

where the second approximation is justified by the fact that all points shown in Fig. 2 are close to the unit circle, so that the unit circle may be locally approximated by its tangent in $e^{j\omega_c}$, and hence $Q \approx Q'$. Furthermore, the arc length between R and Q' is nearly equal to $|RQ|$. Since the frequency response is approximately symmetric around the center frequency if the poles are close to the unit circle [11, Ch. 11], the bandwidth can be calculated as $B \approx 2|RQ|$.

Recall our definition of bandwidth,

$$\frac{|H(e^{j\omega_c})|}{|H(e^{j(\omega_c - B/2)})|} = \begin{cases} \frac{1}{\sqrt{2}} & \text{if} \quad \frac{G_c}{G_0} \leq \frac{1}{2}, \\ \sqrt{\frac{G_c}{G_0}} & \text{if} \quad \frac{G_c}{G_0} \geq \frac{1}{2}. \end{cases} \tag{11}$$

Moreover, combining (6) and (10) yields

$$\frac{|H(e^{j\omega_c})|}{|H(e^{j(\omega_c - B/2)})|} = \frac{(1-r_z)|P_1Q|}{(1-r_p)|Z_1Q|}. \tag{12}$$

With the Pythagorean theorem applied to the triangles P_1RQ and Z_1RQ, we obtain two more equations,

$$|P_1Q|^2 = |P_1R|^2 + |RQ|^2 = (1-r_p)^2 + |RQ|^2, \tag{13}$$
$$|Z_1Q|^2 = |Z_1R|^2 + |RQ|^2 = (1-r_z)^2 + |RQ|^2, \tag{14}$$

which can be solved for $|RQ|$, together with (11) and (12), resulting in

$$B \approx 2|RQ| = \begin{cases} 2\sqrt{\frac{(1-r_p)^2(1-r_z)^2}{(1-r_p)^2 - 2(1-r_z)^2}} & \text{if} \quad \frac{G_c}{G_0} \leq \frac{1}{2}, \\ 2\sqrt{\frac{(\frac{G_c}{G_0}-1)(1-r_p)^2(1-r_z)^2}{(1-r_z)^2 - \frac{G_c}{G_0}(1-r_p)^2}} & \text{if} \quad \frac{G_c}{G_0} \geq \frac{1}{2}. \end{cases} \tag{15}$$

Dividing both the numerator and denominator in the above square root expressions by either $(1-r_p)^2$ or $(1-r_z)^2$, and subsequently using the result in (9), yields the following expressions for the zero and pole radius:

$$\boxed{r_z = \begin{cases} 1 - \frac{B}{2}\sqrt{1 - 2\frac{G_c^2}{G_0^2}} & \text{if} \quad \frac{G_c}{G_0} \leq \frac{1}{2}, \\ 1 - \frac{B}{2}\sqrt{\frac{G_c}{G_0}} & \text{if} \quad \frac{G_c}{G_0} \geq \frac{1}{2}, \end{cases}} \tag{16}$$

$$\boxed{r_p = \begin{cases} 1 - \frac{B}{2}\sqrt{\frac{G_0^2}{G_c^2} - 2} & \text{if} \quad \frac{G_c}{G_0} \leq \frac{1}{2}, \\ 1 - \frac{B}{2}\sqrt{\frac{G_0}{G_c}} & \text{if} \quad \frac{G_c}{G_0} \geq \frac{1}{2}. \end{cases}} \tag{17}$$

These expressions, together with $\boxed{K = G_0}$, constitute the notch filter design procedure. Note that, while the expressions in (16) and (17) may theoretically yield negative results, this should never occur for the narrowband filters we are dealing with (i.e., for sufficiently small B).

B. Resonance Filters

A similar procedure can be used to design resonance filters, but then the zeros should be placed between the origin and the poles, i.e., $0 \leq r_z < r_p < 1$, see Fig. 3. Note that in this case, no assumption can be made about the position of the zeros, i.e., depending on the specified gain at resonance, the zeros may be close to the origin, or close to the poles (near the unit circle), or anywhere in between. We will however assume that the distance between Z_1 and P_1 is significantly larger than the distance from P_1 to the unit circle, and the same for Z_2 and P_2, such that the zeros can be neglected when calculating the bandwidth. As in the notch filter case, we will also assume that the influence of the complex conjugate pole-zero pair on the change in frequency response from $e^{j\omega_c}$ to $e^{j(\omega_c - B/2)}$ is negligible. Hence the bandwidth, which in this case is defined by

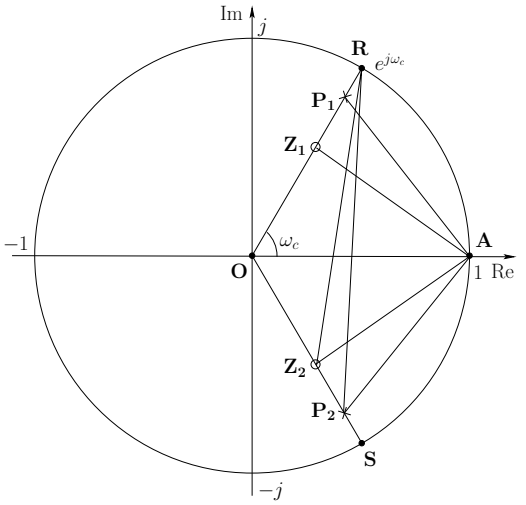


Fig. 3. Pole-zero plot for resonance filter.

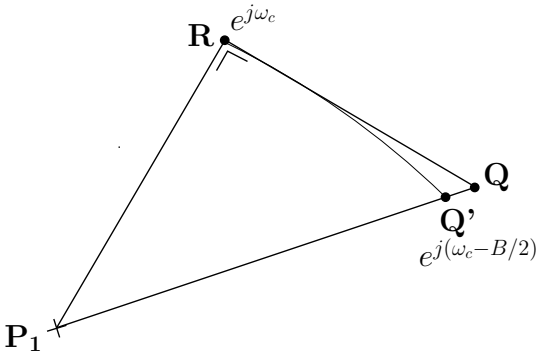


Fig. 4. Bandwidth calculation for resonance filter.

$$\frac{|H(e^{j\omega_c})|}{|H(e^{j(\omega_c - B/2)})|} = \begin{cases} \sqrt{2} & \text{if} \quad \frac{G_c}{G_0} \geq 2, \\ \sqrt{\frac{G_c}{G_0}} & \text{if} \quad \frac{G_c}{G_0} \leq 2, \end{cases} \tag{18}$$

only depends on the pole position, see Fig. 4, i.e.,

$$\frac{|H(e^{j\omega_c})|}{|H(e^{j(\omega_c - B/2)})|} \approx \frac{|P_1Q|}{(1-r_p)}. \tag{19}$$

This expression, together with the Pythagorean theorem in (13), results in

$$B \approx 2|RQ| = \begin{cases} 2(1-r_p) & \text{if} \quad \frac{G_c}{G_0} \geq 2, \\ 2(1-r_p)\sqrt{\frac{G_c}{G_0} - 1} & \text{if} \quad \frac{G_c}{G_0} \leq 2, \end{cases} \tag{20}$$

so that the pole radius can be calculated as

$$\boxed{r_p = \begin{cases} 1 - \frac{B}{2} & \text{if} \quad \frac{G_c}{G_0} \geq 2, \\ 1 - \frac{B}{2\sqrt{\frac{G_c}{G_0} - 1}} & \text{if} \quad \frac{G_c}{G_0} \leq 2. \end{cases}} \tag{21}$$

The upper part of (21) was also derived in [8, Ch. 6].

Determining the zero radius for the resonance filter is somewhat more difficult, as no assumption can be made about the position of the zeros. Graphically evaluating the gain at resonance and at DC

yields

$$|H(e^{j\omega_c})| = K\frac{(1-r_z)|Z_2R|}{(1-r_p)|P_2R|}, \tag{22}$$

$$|H(e^{j0})| = K\frac{|Z_1A||Z_2A|}{|P_1A||P_2A|} = K\frac{|Z_1A|^2}{|P_1A|^2}. \tag{23}$$

Eliminating K from (22) and (23), and taking into account design equations (2a) and (2d), leads to

$$\frac{(1-r_z)|Z_2R||P_1A|^2}{(1-r_p)|P_2R||Z_1A|^2} = \frac{G_c}{G_0}, \tag{24}$$

where we can make the following approximations. Since the poles are close to the unit circle, $|P_2R| \approx |SR| = 2\sin\omega_c$, and $|P_1A| \approx |RA| = 2(1-\cos\omega_c)$. Furthermore, the fraction

$$\frac{|Z_1A|^2}{|Z_2R|} = \frac{r_z^2 - 2r_z\cos\omega_c + 1}{\sqrt{r_z^2 - 2r_z\cos 2\omega_c + 1}}, \tag{25}$$

may be approximated by a first-order polynomial in r_z. As r_z varies between 0 and 1, the fraction in (25) takes on a value between 1 and $(1-\cos\omega_c)/\sin\omega_c$, which leads to the following approximation:

$$\frac{|Z_1A|^2}{|Z_2R|} \approx (1-r_z) + \Big(\frac{1-\cos\omega_c}{\sin\omega_c}\Big)r_z. \tag{26}$$

Substituting the above approximations in (24), results in a first-order equation in r_z,

$$\frac{(1-r_z)}{(1-r_p)} = \frac{G_c}{G_0}\Big(r_z + (1-r_z)\frac{\sin\omega_c}{1-\cos\omega_c}\Big), \tag{27}$$

which can readily be solved by using (21), i.e.,

$$\boxed{r_z = \begin{cases} \frac{1-\Gamma}{1-\Gamma+\frac{B}{2}\frac{G_c}{G_0}} & \text{if} \quad \frac{G_c}{G_0} \geq 2, \\ \frac{1-\Gamma}{1-\Gamma+\frac{B}{2\sqrt{G_c/G_0-1}}\frac{G_c}{G_0}} & \text{if} \quad \frac{G_c}{G_0} \leq 2, \end{cases}} \tag{28}$$

with

$$\boxed{\Gamma = \begin{cases} \frac{B}{2}\frac{G_c}{G_0}\frac{\sin\omega_c}{1-\cos\omega_c} & \text{if} \quad \frac{G_c}{G_0} \geq 2, \\ \frac{B}{2\sqrt{G_c/G_0-1}}\frac{G_c}{G_0}\frac{\sin\omega_c}{1-\cos\omega_c} & \text{if} \quad \frac{G_c}{G_0} \leq 2. \end{cases}} \tag{29}$$

It should be noted that, once the bandwidth is specified, the boost G_c/G_0 cannot take on an arbitrarily large value. The maximum boost, given B, is obtained when the zeros are placed at the origin, and can be calculated from (29) with $\Gamma = 1$.

Finally, the broadband gain factor K can be calculated by evaluating (4) at e^{j0}, i.e.,

$$\boxed{K = G_0\frac{1-2r_p\cos\omega_c + r_p^2}{1-2r_z\cos\omega_c + r_z^2}.} \tag{30}$$

III. Design Examples

We have applied the above design procedure to four design examples. In the first example, the bandwidth of a resonance filter with $\omega_c = 2$ rad (0.64π rad), $G_0 = 1$ (0 dB), and $G_c = 4$ (12 dB) takes on different values $B = \{0.01, 0.1, 0.2\}$ rad $\big(\{0.0032, 0.032, 0.064\}\pi$ rad$\big)$, see Fig. 5. The second example illustrates the effect of varying the boost of a resonance filter as $G_c/G_0 = \{2, 3, 4\}$ $\big(\{6, 9.5, 12\}$ dB$\big)$, with $\omega_c = 1$ rad (0.32π rad), $B = 0.05$ rad (0.016π rad), and $G_0 = 1$ (0 dB), see Fig. 6. In the third example, in Fig. 7, the center frequency of a notch filter with $B = 0.1$ rad (0.032π rad), $G_0 = 1$ (0 dB), and $G_c = 0.3$ (-10.5 dB) is set to $\omega_c = \{1, 1.5, 2\}$ rad $\big(\{0.32, 0.48, 0.64\}\pi$ rad$\big)$. The fourth example is included to illustrate the capability to fade from a resonance filter to a notch filter using the proposed design procedure. In Fig. 8, the boost/cut of a parametric equalizer filter with $\omega_c = 1.5$ rad (0.48π rad), $B = 0.1$ rad (0.032π rad), and $G_0 = 1$ (0 dB) is varied as $G_c/G_0 = \{1.1220, 1.0593, 1.0233, 1, 0.9772, 0.9441, 0.8913\}$ $\big(\{1, 0.5, 0.2, 0, -0.2, -0.5, -1\}$ dB$\big)$. Note that a flat magnitude response ($G_c/G_0 = 1$) is obtained by setting $r_z = r_p$ in (3), resulting in pole-zero cancellation, and $K = G_0$.

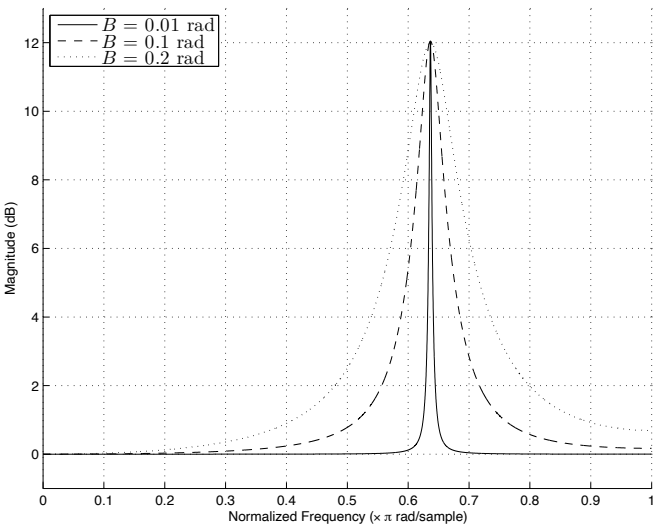


Fig. 5. Design example 1: resonance filter with varying bandwidth.

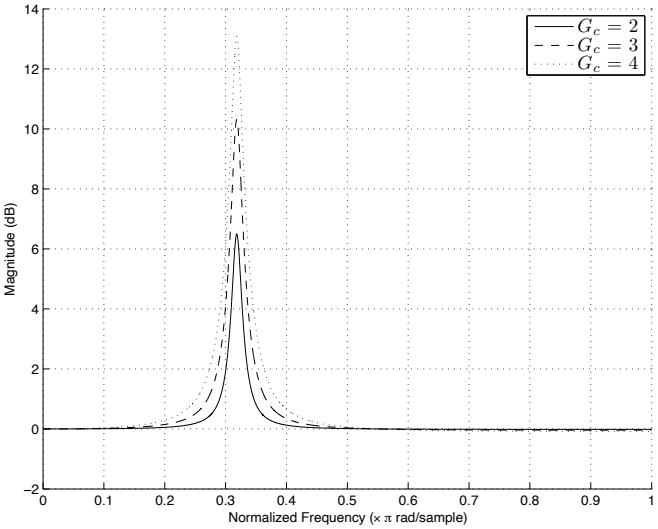


Fig. 6. Design example 2: resonance filter with varying boost.

Finally, we compare the new procedure to three state-of-the-art techniques for designing parametric equalizer filters. A cascade of a notch filter with $\omega_c = 1$ rad (0.32π rad), $B = 0.05$ rad (0.016π rad), $G_0 = 1$ (0 dB), and $G_c = 0.7$ (-3 dB), and a resonance filter with $\omega_c = 2$ rad (0.64π rad), $B = 0.1$ rad (0.032π rad), $G_0 = 1$ (0 dB), and $G_c = 4$ (12 dB), was designed using the proposed pole-zero placement technique, as well as using the techniques described by Regalia and Mitra [4], Bristow-Johnson [7], and Moorer [2]. From Fig. 9 it can be seen that the four methods perform equally well on the notch filter, whereas some discrepancies in the 3 dB-bandwidth appear for the resonance filter. More specifically, in Bristow-Johnson's method, the resonance filter bandwidth is somewhat underestimated.

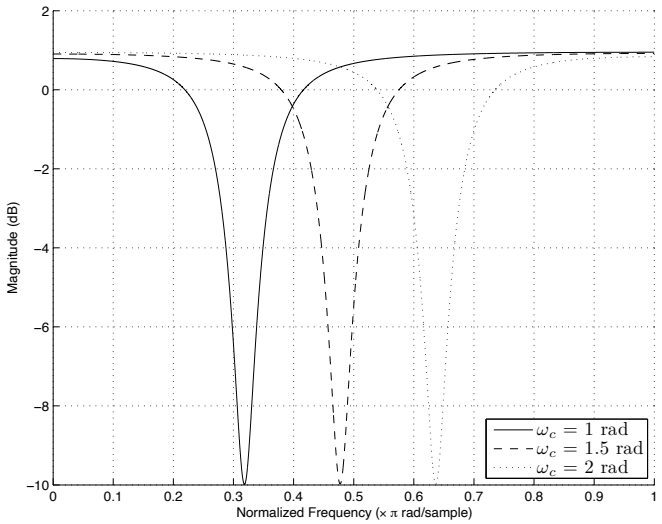


Fig. 7. Design example 3: notch filter with varying center frequency.

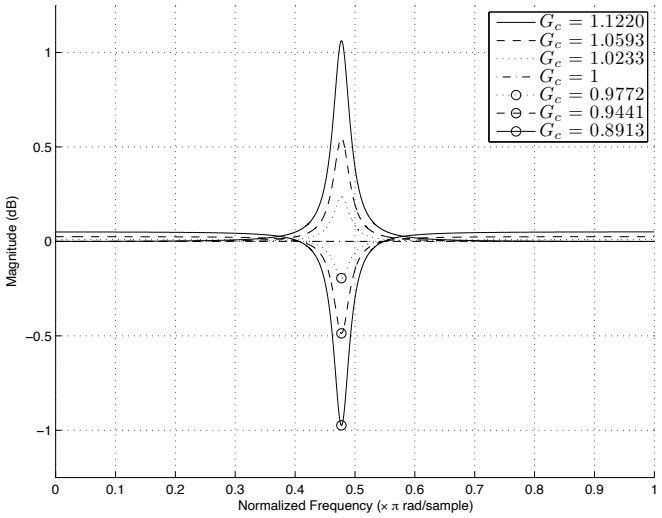


Fig. 8. Design example 4: parametric equalizer filter fading from boost to cut.

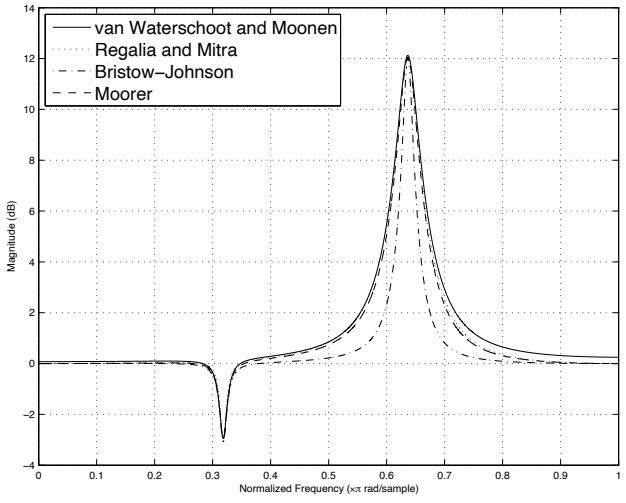


Fig. 9. Comparison with three state-of-the-art design procedures.

IV. Conclusion

In this correspondence, we have proposed a new pole-zero placement technique for designing second-order parametric equalizer filters. Depending on whether a notch or resonance filter characteristic is desired, certain approximations on the pole and zero positions can be made which simplify the filter design equations. Expressions were given for calculating the filter's pole radius, zero radius, and broadband gain factor, which, together with the specified center frequency, completely determine the filter transfer function. The effect of varying the design variables, i.e., center frequency, bandwidth, and boost/cut, was illustrated in four design examples. The proposed design procedure was also compared to three state-of-the-art algorithms and was found to be equally accurate, with the additional advantage of providing an intuitive design approach executed entirely in the digital domain.

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