Joint Level 2 and 3 Dynamic Spectrum Management for Upstream VDSL

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***Abstract*—Dynamic spectrum management (DSM) refers to a wide range of techniques for counteracting crosstalk in digital subscriber line (DSL) networks. DSM is categorized into three levels based on the degree of coordination among users. In this article, we investigate optimal joint level 2 and 3 DSM for upstream DSL. We will discuss the difficulties of finding the universally optimal solution and we propose an optimal algorithm, referred to as IF/MAC-OSB, under some practical and implementation assumptions for this problem. Using computer simulations, we show that IF/MAC-OSB is capable of increasing the user bit rates considerably compared to several other DSM techniques. The proposed algorithm involves using the minimum mean squared error (MMSE)-generalized decision feedback equalizer (GDFE) together with Lagrange dual optimization. We address several aspects of the problem including the optimal decoding order in the GDFE receiver, GDFE error propagation, and the computational complexity of the algorithm. We also study effects of channel model randomness and upstream power back-off utilization on the performance of the algorithm.**

***Index Terms*—Digital subscriber line (DSL), dynamic spectrum management (DSM), interference channel (IF), multiple access channel (MAC), generalized decision feedback equalizer (GDFE), resource allocation, vectoring.**

I. Introduction

In recent years, dynamic spectrum management (DSM) has attracted a lot of attention to counteract crosstalk in digital subscriber line (DSL) networks. DSM is categorized into three levels, namely, DSM 1, DSM 2, and DSM 3, based on the degree of coordination among the users. In DSM 1, the modems work autonomously, however, a limited amount of information such as the target bit rates of the users is shared among the modems and is used in a distributed fashion to improve the overall performance of the network.

In DSM 2, the users are coordinated at the power spectral density (PSD) level. The PSD of the users is controlled and optimized jointly by a spectrum management center to reduce the mutual crosstalk among them. The performance of DSM 2 can be considerably better than DSM 1, however, the achievable bit rates by DSM 2 are limited to those of the interference channel (IF). In DSM 3, the users are coordinated at the signal level. Unlike DSM 1 and DSM 2, the coordinated loops must be terminated at the same place at least in one end of the loops so that a MIMO modem can be used to encode or decode the signal of the users jointly. This is mostly possible at the line termination (LT) side. Then, in the upstream (US) direction, the loops form a multiple-access channel (MAC) and in the downstream (DS) direction, the loops form a broadcast channel (BC).

The achievable bit rates by DSM 3 are usually much higher than those by DSM 1 and 2. However, there are some scenarios for which it is not possible to implement DSM 3 for all users. For example DSM 3 is not possible in joint central office (CO)/remote terminal (RT) deployments or when the large number of users makes DSM 3 too complex to be implemented for all loops. In these scenarios, which are referred to as "grouped DSL", the users may be divided into a few groups and DSM 3 is only possible amongst the users inside the same group.

When DSM 3 is only possible for groups of users, the best performance is achieved when DSM 3 (i.e. signal and spectrum coordination) is used to cancel crosstalk inside each group and DSM 2 (i.e. spectrum coordination) is used to avoid crosstalk among the groups. This technique is known as joint spectrum balancing and grouped vectoring and is referred to here as joint DSM 2/3. The joint DSM 2/3 problem is different in US and DS directions basically because vectoring is only possible at the receiver and transmitter sides for US and DS directions, respectively. Consequently, vector decoding schemes can be used in the US direction, while vector precoding schemes can be used in the DS direction.

In this paper, we study joint DSM 2/3 techniques for US DSL. From the information theoretic point of view, each group of users forms a multiple access channel (MAC). However, each group acts as an interfering source to the other

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groups. Therefore, the scenario can be considered as a mixed interference/multiple-access channel (IF/MAC). Finding the rate-region (RR) for this scenario is a complicated and non-convex problem. When each group consists of only one user for instance, the problem reduces to finding the RR of a Gaussian interference channel which is an open problem [1]–[3]. However, for practical DSL receivers, in which the multiuser interference is treated as Gaussian noise, the achievable RR is obtained by the optimal spectrum balancing (OSB) algorithm [4]. On the other hand, when there is only one group of users, the problem reduces to a MAC problem [1], [5], [6]. The MAC-OSB algorithm proposed in [7] achieves the largest achievable RR for practical DSL systems.

As it can be realized, finding the RR for the IF/MAC in the general case is not possible as the RR for the IF channel is unknown. In this paper, we propose an IF/MAC-OSB algorithm that reaches the largest possible achievable RR for joint DSM2/3 DSL systems by treating the interference received by each MAC group from other groups as (spatially colored Gaussian) noise. Using comprehensive simulations, we compare the performance of several techniques, namely, static spectrum management (SSM), OSB [4], zero-forcing (ZF) equalization [8], minimum mean squared error (MMSE) linear equalization [9], ZF-generalized decision feedback equalization (GDFE) [10], MMSE-GDFE, grouped ZF-OSB [11], grouped MMSE-OSB, and grouped ZF-GDFE-OSB, and compare them with the performance of IF/MAC-OSB. All these algorithms are investigated with and without upstream power back-off (UPBO).

Our proposed technique uses the MMSE-GDFE receiver structure for decoding the received signals in each group. Unlike the ideal case, the decoding order in the GDFE cannot be determined by the bit rate weight factors for the different users. This leads to an extra complexity with an exponential factor in the number of users. Our results indicate that it is possible to avoid this extra complexity for IF/MAC-OSB.

Additionally a practical GDFE receiver may not achieve the expected bit-error rate (BER) due to error propagation. We propose a simple sub-optimal solution for this problem and we show that the effect of our solution on the performance of the system is negligible.

We present simulation results using the standard 1% worst-case crosstalk model as well as a recently proposed statistical crosstalk model [12]. We also study the effect of a random channel phase on our algorithm. We show that a random phase could have opposite effects on the user bit rates depending on whether the disturber and victim loops are in the same or different MAC groups.

This paper is organized as follows. In Sec. II, we describe the grouped DSL transmission model. Then, we explain our joint DSM 2/3 algorithm in Sec. III. Simulation results are provided in Sec. IV. Finally the paper is concluded in Sec. V.

II. Formulation of Grouped DSL Transmission

We consider discrete multi-tone (DMT) transmission with K tones. We assume that N managed users form G groups with N_g users in group g ($g = 1 \ldots G$), such that $\sum_{g=1}^{G} N_g = N$. We assume that the users in each group are coordinated at the signal level and all managed users are coordinated with each other at the transmit spectrum (i.e., PSD) level. All of the managed users are assumed to be DMT frame synchronized at the receiver side [10]. Therefore, the transmission over tone k can be modeled by [11]:

$$\begin{bmatrix} \boldsymbol{y}_1^k \\ \vdots \\ \boldsymbol{y}_G^k \end{bmatrix} = \begin{bmatrix} \boldsymbol{H}_{11}^k & \cdots & \boldsymbol{H}_{1G}^k \\ \vdots & \ddots & \vdots \\ \boldsymbol{H}_{G1}^k & \cdots & \boldsymbol{H}_{GG}^k \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_1^k \\ \vdots \\ \boldsymbol{x}_G^k \end{bmatrix} + \begin{bmatrix} \boldsymbol{z}_1^k \\ \vdots \\ \boldsymbol{z}_G^k \end{bmatrix}, \tag{1}$$

where $\boldsymbol{x}_g^k$, $\boldsymbol{y}_g^k$, and $\boldsymbol{z}_g^k$ are the transmitted, received, and noise vectors for group g, respectively, and $\boldsymbol{H}_{gg'}^k$ is the crosstalk transfer matrix from group g' to group g on tone k. Each group g forms the following MAC channel

$$\boldsymbol{y}_g^k = \boldsymbol{H}_{gg}^k \boldsymbol{x}_g^k + \boldsymbol{v}_g^k, \tag{2}$$

where

$$\boldsymbol{v}_g^k = \boldsymbol{z}_g^k + \sum_{g' \neq g} \boldsymbol{H}_{gg'}^k \boldsymbol{x}_{g'}^k. \tag{3}$$

Since in the US direction the transmit modems are at different locations and the transmit signal of different users are uncorrelated, the noise covariance matrix for group g on tone k is calculated by

$$\boldsymbol{N}_g^k = \mathrm{E}\left\{\boldsymbol{v}_g^k [\boldsymbol{v}_g^k]^{\mathrm{H}}\right\} = \boldsymbol{Z}_g^k + \sum_{g' \neq g} \boldsymbol{H}_{gg'}^k \boldsymbol{S}_{g'}^k [\boldsymbol{H}_{gg'}^k]^{\mathrm{H}}, \tag{4}$$

where $\mathrm{E}\{\cdot\}$ and $[\cdot]^{\mathrm{H}}$ denote the expected value and transpose conjugate operations, $\boldsymbol{Z}_g^k = \mathrm{E}\left\{\boldsymbol{z}_g^k [\boldsymbol{z}_g^k]^{\mathrm{H}}\right\}$ is the covariance matrix of $\boldsymbol{z}_g^k$, $\boldsymbol{S}_{g'}^k = \mathrm{diag}\{\boldsymbol{s}_{g'}^k\}$ is the covariance matrix of the transmit signals in group g', $\boldsymbol{s}_{g'}^k = (s_{(g',1)}^k, \ldots, s_{(g',N_{g'})}^k)^{\mathrm{T}}$ is the vector of transmit powers on tone k for group g', and $\mathrm{diag}\,(\boldsymbol{a})$ denotes a diagonal matrix with diagonal elements equal to the elements of vector $\boldsymbol{a}$. Note that $\boldsymbol{Z}_g^k$ depends on the receiver background noise as well as received power from any unmanaged users and is assumed to be known.

III. Joint DSM 2/3 for Grouped DSL

The joint DSM 2/3 problem can be stated as follows:

$$\begin{aligned} \text{maximize} \quad & R_{(1,1)} \\ \text{subject to} \quad & R_{(g,n)} \geq R_{(g,n)}^{\min}, \ \forall g, n, (g,n) \neq (1,1) \\ \text{and} \quad & P_{(g,n)} \leq P_{(g,n)}^{\max}, \ \forall g, n, \end{aligned} \tag{5}$$

where $R_{(g,n)}$ is the total bit rate, $P_{(g,n)}$ is the total transmit power, $R_{(g,n)}^{\min}$ is the target bit rate, and $P_{(g,n)}^{\max}$ is the maximum total transmit power for the n-th user of group g. The total bit rate is obtained by

$$R_{(g,n)} = f_{\mathrm{s}} \sum_{k=1}^{K} b_{(g,n)}^k, \tag{6}$$

where f_{s} is the DMT symbol rate and $b_{(g,n)}^k$ is the bit rate on tone k for the n-th user of group g. The total transmit power is

$$P_{(g,n)} = \Delta_{\mathrm{f}} \sum_{k=1}^{K} s_{(g,n)}^{k}, \tag{7}$$

where Δ_{f} is the DMT tone spacing and $s_{(g,n)}^{k}$ is the transmit power on tone k for the n-th user of group g.

The maximization in (5) takes place over all feasible (or practical) transmission schemes and the bit-rates $b_{(g,n)}^{k}$ and transmit powers $s_{(g,n)}^{k}$ that they support. In this paper, we focus on finding a practical transmission scheme which may result in the largest RR for any set of $P_{(g,n)}^{\max}$. We will also discuss how we can calculate $b_{(g,n)}^{k}$ and $s_{(g,n)}^{k}$ over all tones and users for a particular set of $R_{(g,n)}^{\min}$ and $P_{(g,n)}^{\max}$ in a computationally efficient way.

A. Weighted Sum Rate Maximization Approach and Lagrange Dual Optimization

The non-convex constrained optimization problem in (5) can be transformed into the following unconstrained problem by using weighted sum rate maximization and Lagrange dual optimization techniques [4], [13]:

$$\underset{\{s_{(g,n)}^{k};\forall k,g,n\}}{\text{maximize}} \; \mathcal{L} \equiv \sum_{g=1}^{G}\sum_{n=1}^{N_g} w_{(g,n)} R_{(g,n)}/f_{\mathrm{s}} - \lambda_{(g,n)} P_{(g,n)}/\Delta_{\mathrm{f}}, \tag{8}$$

where $w_{(g,n)}$ and $\lambda_{(g,n)}$ are called the weight factor and Lagrange multiplier for the n-th user of group g, respectively. The weight factors and Lagrange multipliers should be selected appropriately to satisfy the constraints in (5). Substituting (6) and (7) into (8) the problem is restated as

$$\text{maximize} \sum_{k=1}^{K} \mathcal{L}_k, \tag{9}$$

where

$$\mathcal{L}_k \equiv \sum_{g=1}^{G}\sum_{n=1}^{N_g} w_{(g,n)} b_{(g,n)}^{k} - \lambda_{(g,n)} s_{(g,n)}^{k}. \tag{10}$$

Since the bit rates $b_{(g,n)}^{k}$ and transmit powers $s_{(g,n)}^{k}$ are independent for different tones, the problem in (9) can be decoupled into the following independent per-tone problems:

$$\text{maximize } \mathcal{L}_k \text{ (for } k = 1, \ldots, K). \tag{11}$$

The per-tone maximization in (11) is now limited to the transmission scheme and bit rate and transmit powers on tone k only. To solve (11), we need to know the relationship between the bit rates and the required transmit power. On top of that we need to design an efficient technique which could maximize the bit rates with given transmit powers or could minimize the required powers for given bit rates. In the following sub-sections, we propose an efficient receiver structure for this purpose and we explain the relationship between bit rates and transmit powers for this receiver in a general IF/MAC channel. Since (11) is a non-convex optimization problem, we need to find the optimal point by running an exhaustive search over all feasible bit rate and transmit powers on tone k.

The resulting algorithm is referred to as IF/MAC-OSB. An overview of IF/MAC-OSB is enlisted in Algorithm 1. Overall, the algorithm consists of an outer loop in which the optimal values of $w_{(g,n)}$ and $\lambda_{(g,n)}$ are found and an inner loop in which we solve (11) for a particular set of $w_{(g,n)}$ and $\lambda_{(g,n)}$ over all tones. A few efficient techniques for optimizing $w_{(g,n)}$ and $\lambda_{(g,n)}$ are discussed in [4], [13]–[15], and [16].

Algorithm 1: An Overview of IF/MAC-OSB Algorithm

repeat
  Set/update $w_{(g,n)}$ and $\lambda_{(g,n)}$;
  for $k = 1$ **to** K **do**
    $(b_{(g,n)}^{k}, s_{(g,n)}^{k}) \leftarrow \arg\max \mathcal{L}_k$ [Solve using exhaustive search. See Sec. III-B/III-C for the relationship between $b_{(g,n)}^{k}$ and $s_{(g,n)}^{k}$];
  $R_{(g,n)} \leftarrow f_{\mathrm{s}} \sum_{k=1}^{K} b_{(g,n)}^{k}$;
  $P_{(g,n)} \leftarrow \Delta_{\mathrm{f}} \sum_{k=1}^{K} s_{(g,n)}^{k}$;
until *convergence* ;

B. Grouped Vectoring

Each group of users forms a MAC. Standard algorithms to decode the received signals in each group are the ZF equalizer [8], MMSE equalizer [9], ZF-GDFE using QR decomposition [10], and MMSE-GDFE [7]. Here we explain the MMSE-GDFE in the ideal case and then we discuss it for DSL systems.

Consider group g and assume that the transmit power of all users is fixed. The sum capacity for this MAC is given by

$$C_g^k = \log_2\left(\det\left(\boldsymbol{I}_{N_g} + [\boldsymbol{N}_g^k]^{-1}\boldsymbol{H}_{gg}^k \boldsymbol{S}_g^k [\boldsymbol{H}_{gg}^k]^{\mathrm{H}}\right)\right), \tag{12}$$

where $\log_2(\cdot)$ and $\det(\cdot)$ denote the logarithm base 2 and matrix determinant operations and $\boldsymbol{I}_{N_g}$ is the identity matrix of size N_g. The sum capacity of the MAC channel can be achieved by a MMSE-GDFE receiver [17]. The MMSE-GDFE receiver structure decodes the received signals one by one. When decoding the data from a particular user, the crosstalk of the previously decoded users is first removed from the received signal vector to improve its SNR. Hence

$$C_g^k = \sum_{n=1}^{N_g} b_{(g,n)}^{k}\Big|_{\text{no xtalk from users 1 to } n-1}, \tag{13}$$

where $b_{(g,n)}^{k}\Big|_{\text{no xtalk from users 1 to } n-1}$ denotes the bit rate on tone k for the n-th user of group g when crosstalk from users 1 to $n-1$ has been removed and is obtained by

$$b_{(g,n)}^{k}\Big|_{\text{no xtalk from users 1 to } n-1} = \log_2\left(1 + s_{(g,n)}^{k} \times [h_{(gg,n)}^{k}]^{\mathrm{H}}\left(\boldsymbol{N}_g^k + \boldsymbol{H}_{gg}^k \bar{\boldsymbol{S}}_{(g,n+1:N_g)}^{k} [\boldsymbol{H}_{gg}^k]^{\mathrm{H}}\right)^{-1} h_{(gg,n)}^{k}\right). \tag{14}$$

where $h_{(gg,n)}^{k}$ is the n-th column of $\boldsymbol{H}_{gg}^k$ and $\bar{\boldsymbol{S}}_{(g,n+1:N_g)}^{k}$ $= \mathrm{diag}\{\overbrace{0, \ldots, 0}^{n\text{times}}, s_{(g,n+1)}^{k}, s_{(g,n+2)}^{k}, \ldots, s_{(g,N_g)}^{k}\}$.

By changing the order of decoding we obtain a maximum of $N_g!$ different bit rates for the users in group g which are all feasible and optimal in the sense that they are all located on the outer boundary of the rate-region (RR)[1]. The RR for a MAC has a polymatroid structure [5], [6]. The MMSE-GDFE structure reaches an optimal vertex of the RR for a particular set of weight factors if the users are sorted based on the weight factors and decoded from the user with the smallest weight to the user with the largest weight [7].

The capacity formula in (12) does not hold for practical DSL systems. This is because DSL systems are not capable of achieving the Shannon capacity, i.e. there exists a gap between the achievable bit rates by a DSL receiver structure and the Shannon capacity. The RR then does not necessarily have a polymatroid structure. However, the GDFE structure can still be used to obtain a set of achievable (but not necessarily optimal) bit rates [7]. Assume the decoding order in group g is from the user 1 to user N_g. The bit rates are obtained by

$$b^k_{(g,n)}\Big|_{\text{no xtalk from users 1 to } n-1} = \begin{cases} 0; & \bar{b}^k_{(g,n)} < b_{\min}, \\ \left\lfloor \bar{b}^k_{(g,n)} \right\rfloor; & b_{\min} \le \bar{b}^k_{(g,n)} < b_{\max}, \\ b_{\max}; & \bar{b}^k_{(g,n)} \ge b_{\max}, \end{cases} \tag{15}$$

where $b_{\min}$ and $b_{\max}$ are the minimum and maximum number of bits that can be loaded on a tone, $\lfloor \cdot \rfloor$ denotes the floor function, and

$$\bar{b}^k_{(g,n)} = \log_2 \left(1 + \tfrac{1}{\Gamma} s^k_{(g,n)} [h^k_{(gg,n)}]^{\mathrm{H}} \times \left(\boldsymbol{N}^k_g + \boldsymbol{H}^k_{gg} \bar{\boldsymbol{S}}^k_{(g,n)} [\boldsymbol{H}^k_{gg}]^{\mathrm{H}} \right)^{-1} h^k_{(gg,n)} \right), \tag{16}$$

where Γ is the so-called SNR gap. Once again, we obtain $N_g!$ different sets of bit rates by changing the order of decoding. In [7] the order of decoding is chosen to be the same as the order of weight factors. However, since the structure of the RR may not be a polymatroid as for the ideal MAC receiver, the optimal point and decoding order are not necessarily determined by the order of weight factors anymore. By comparing the weighted sum for all $N_g!$ different sets of bit rates that are achieved by all decoding orders, we obtain a modified MMSE-GDFE receiver structure which theoretically achieves a better performance than the MMSE-GDFE structure with decoding order based on the order of the weight factors.

Note that in (13), we consider the capacity of the channel, which implies a zero BER. However, (15) gives the bit rate at a non-zero BER. When a decoding error has occurred for a previously decoded user, this may cause error propagation for subsequently decoded users and hence a higher BER. In Sec. III-D, we propose a technique to overcome the GDFE error propagation problem.

[1]The outer boundary of the RR is the set of points in the RR for which no other point in the RR can be found with all elements greater than or equal to its elements [14], [15].

C. Bit Loading versus Power Loading

In (15), we calculate bit rates as a function of transmit powers which is referred to as power loading. To reduce the computational complexity of the exhaustive search, it is desirable to calculate transmit powers as a function of the bit rates, which is referred to as bit loading. Unfortunately, it is not possible to calculate the transmit powers explicitly as a function of the bit rates for a MMSE-GDFE receiver. Here we propose two techniques for this purpose. The first technique is an iterative technique and the second technique is an explicit technique obtained by approximating the MMSE-GDFE operation by a ZF-GDFE.

The iterative technique is as follows: Consider group g. Assume the optimal order of decoding is known and (as in (15) and (16)) that the users are sorted with the first user decoded first. From (16) we have

$$s^k_{(g,n)} = \Gamma(2^{b^k_{(g,n)}} - 1) \Big/ \left[[h^k_{(gg,n)}]^{\mathrm{H}} \times \left(\boldsymbol{N}^k_g + \boldsymbol{H}^k_{gg} \bar{\boldsymbol{S}}^k_{(g,n+1:N_g)} [\boldsymbol{H}^k_{gg}]^{\mathrm{H}} \right)^{-1} h^k_{(gg,n)} \right], \tag{17}$$

If we know the noise covariance matrix $\boldsymbol{N}^k_g$, we can calculate $s^k_{(g,n)}$ using a simple loop starting from the last user in the group down to the first user. Therefore, the transmit powers of the users in group g can be calculated explicitly in terms of their bit rates. However, $\boldsymbol{N}^k_g$ depends on the spectra of users in other groups as well. To overcome this difficulty, we propose the iterative Algorithm 2. Since the transmit powers are always smaller than the final solution and they are monotonically increasing, it is easy to show that the algorithm either converges to the optimal solution or reports the set of bit rates as unachievable. We detect convergence when for all users n we have $s^n_{(k,g)}(i) - s^n_{(k,g)}(i-1) < \frac{1}{1000} s^n_{(k,g)}(i)$.

Algorithm 2: Calculation of Transmit Powers from Bit Rates

```
s^k_(g,n)(0) ← 0;
i ← 0;
repeat
    i ← i + 1;
    for g = 1 to G do
        N^k_g(i) ← Z^k_g + Σ_{g'=1}^{g-1} H^k_{gg'} S^k_{g'}(i)[H^k_{gg'}]^H +
        Σ_{g'=g+1}^{G} H^k_{gg'} S^k_{g'}(i-1)[H^k_{gg'}]^H ;
        for n = N_g downto 1 do
            s^k_(g,n) ← Γ(2^{b^k_(g,n)} − 1) / [ [h^k_(gg,n)]^H
            × ( N^k_g + H^k_gg S̄^k_(g,n+1:N_g) [H^k_gg]^H )^{-1} h^k_(gg,n) ];
            if s^k_(g,n)(i) > PSD mask then
                Report the bit rates as unachievable and
                exit;
until convergence ;
```

In the second technique, we exploit the fact that the ZF-GDFE is near-optimal in the US direction due to the column-wise diagonal dominance of the US DSL channel [8]. In fact,

as we will see in Sec. IV, our simulation results do not totally agree with this statement and we will see that the MMSE-GDFE performs better than the ZF-GDFE in the presence of colored noise.

As we noticed before it is not possible to calculate the transmit powers as a function of the bit rates for the MMSE-GDFE receiver. This is because the MMSE-GDFE receiver for each group is a complicated function of the crosstalk power received from other groups. The ZF-GDFE receiver, on the other hand, depends only on the channel matrix $\boldsymbol{H}_{gg}^{k}$, which is known. Assume we implement the ZF-GDFE using QR decomposition as proposed in [10]. Let

$$\boldsymbol{H}_{gg}^{k} = \boldsymbol{Q}_{g}^{k}\boldsymbol{R}_{g}^{k} \tag{18}$$

denote the QR decomposition for the g-th group, where $\boldsymbol{Q}_{g}^{k}$ is a unitary matrix and $\boldsymbol{R}_{g}^{k}$ is an upper triangular matrix. By multiplying the received vector in group g, $\boldsymbol{y}_{g}^{k}$, by $[\boldsymbol{Q}_{g}^{k}]^{\mathrm{H}}$, we obtain:

$$\tilde{\boldsymbol{y}}_{g}^{k} = [\boldsymbol{Q}_{g}^{k}]^{\mathrm{H}}\boldsymbol{y}_{g}^{k} = \boldsymbol{R}_{g}^{k}\boldsymbol{x}_{g}^{k} + \boldsymbol{u}_{g}^{k}, \tag{19}$$

where $\boldsymbol{u}_{g}^{k} = [\boldsymbol{Q}_{g}^{k}]^{\mathrm{H}}\boldsymbol{v}_{g}^{k}$ and $\boldsymbol{v}_{g}^{k}$ is defined in (3). Since $\boldsymbol{R}_{g}^{k}$ is an upper triangular matrix, the users in the group can be decoded iteratively from the last user to the first user by removing the interference originated from the previously decoded users as explained in [10]. Assuming that this operation is error free, the decision variables for group g can be written in the following matrix formula:

$$\hat{\boldsymbol{y}}_{g}^{k} = \left[\boldsymbol{I}_{N_g} \odot \boldsymbol{R}_{g}^{k}\right]\boldsymbol{x}_{g}^{k} + \boldsymbol{u}_{g}^{k}, \tag{20}$$

where $\odot$ denotes the element wise matrix multiplication operation. Rearranging the reception over all groups using (3), we obtain

$$\hat{\boldsymbol{y}}^{k} = \hat{\boldsymbol{H}}^{k}\boldsymbol{x}^{k} + \hat{\boldsymbol{z}}^{k}, \tag{21}$$

where $\hat{\boldsymbol{y}}^{k} \equiv \left([\hat{\boldsymbol{y}}_{1}^{k}]^{\mathrm{T}}, \ldots, [\hat{\boldsymbol{y}}_{G}^{k}]^{\mathrm{T}}\right)^{\mathrm{T}}$, $\hat{\boldsymbol{z}}^{k} \equiv \left([\hat{\boldsymbol{z}}_{1}^{k}]^{\mathrm{T}}, \ldots, [\hat{\boldsymbol{z}}_{G}^{k}]^{\mathrm{T}}\right)^{\mathrm{T}}$, $\hat{\boldsymbol{z}}_{g}^{k} \equiv \left[\boldsymbol{Q}_{g}^{k}\right]^{\mathrm{H}}\boldsymbol{z}_{g}^{k}$, and

$$\hat{\boldsymbol{H}}^{k} \equiv \begin{bmatrix} \boldsymbol{I}_{N_1} \odot \boldsymbol{R}_{1}^{k} & \hat{\boldsymbol{H}}_{12}^{k} & \cdots & \hat{\boldsymbol{H}}_{1G}^{k} \\ \hat{\boldsymbol{H}}_{21}^{k} & \boldsymbol{I}_{N_2} \odot \boldsymbol{R}_{2}^{k} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\boldsymbol{H}}_{G1}^{k} & \cdots & \cdots & \boldsymbol{I}_{N_G} \odot \boldsymbol{R}_{G}^{k} \end{bmatrix}, \tag{22}$$

where $\hat{\boldsymbol{H}}_{gg'}^{k} = \left[\boldsymbol{Q}_{g}^{k}\right]^{\mathrm{H}}\boldsymbol{H}_{gg'}^{k}$. Note that $\hat{y}^{k}$ is the vector of decision variables, i.e. no further vectoring operation is applied, and decisions are made on the elements separately. Therefore, Eqn. (21) describes an equivalent interference channel for the grouped ZF-GDFE. Thus, we can use the following formula to calculate the transmit powers from the bit rates for this interference channel [4], [14]:

$$\boldsymbol{s}^{k} = \left[\left(\boldsymbol{I}_{N} + [\Gamma\boldsymbol{B}^{k}]^{-1}\right)\left(\boldsymbol{I}_{N} \odot \hat{\boldsymbol{G}}^{k}\right) - \hat{\boldsymbol{G}}^{k}\right]^{-1}\hat{\boldsymbol{\sigma}}^{k}, \tag{23}$$

where $\boldsymbol{B}^{k}$ is a diagonal matrix with diagonal elements equal to ($2^{b_{(1,1)}^{k}}$-1, …, $2^{b_{(1,N_1)}^{k}}$-1, $2^{b_{(2,1)}^{k}}$-1,…, $2^{b_{(G,N_G)}^{k}}$-1), and $\hat{\boldsymbol{\sigma}}^{k}$ is a column vector consisting of the diagonal elements of $\mathrm{E}\{\hat{\boldsymbol{z}}^{k}[\hat{\boldsymbol{z}}^{k}]^{\mathrm{H}}\}$. Finally, $\hat{\boldsymbol{G}}^{k} \equiv \hat{\boldsymbol{H}}_{k} \odot \hat{\boldsymbol{H}}_{k}^{*}$, where $(\cdot)^{*}$ denotes the complex conjugate operation. The obtained transmit power vector should be checked to be positive and below the nominal PSD mask to guarantee that the bit rates are achievable.

Note that both of the proposed techniques in this section assume a fixed decoding order. However, the transmit powers depend on the decoding order. Therefore, to find the optimal solution we should test all decoding orders inside the groups. Since the decoding order inside each group affects the transmit powers and hence the crosstalk into other groups, each particular decoding order for a group should be tested with all possible decoding orders of other groups. As for a group g we have $N_g!$ orders, the total number of decoding orders is $\prod_g N_g!$.

D. Addressing the GDFE Error Propagation Problem

Consider a MAC with N users. Assume the users are decoded using a MMSE- or ZF-GDFE receiver and are sorted according to the decoding order where user 1 is decoded first and user N is decoded last. Let $P_{\mathrm{e}}(n)$ denote the symbol error rate (SER) for the n-th user. We have

$$\begin{aligned} P_{\mathrm{e}}(n) &= \textstyle\sum_{m=1}^{n} \Pr\{e_m\}\, P_{\mathrm{e}}(n)|_{e_m} \\ &< \textstyle\sum_{m=1}^{n} \Pr\{e_m\}. \end{aligned} \tag{24}$$

where e_m denotes the event of user m being the first wrongly decoded user. Now assume the modulation gap is the same for all users such that the SER for any user is equal to p when all previously decoded users are decoded correctly. We have $\Pr\{e_m\} = p(1-p)^{m-1}$. By substituting in (24) and ignoring the $(1-p)^{m-1}$ term for small p, we get

$$P_{\mathrm{e}}(n) < np. \tag{25}$$

The SER per dimension ($P_{\mathrm{e}}/2$) should be smaller than 10^{-7} for DSL systems [18]. Obviously, the last decoded user experiences the largest BER. Therefore, the modulation gap should be calculated such that $p \leq \frac{2}{N} \times 10^{-7}$. The SER for QAM modulation is given in [19] as a function of bit rates and SNR. If we calculate the inverse function $\rho(b, P_{\mathrm{e}})$ (i.e., the required SNR for bit rate = b at SER = P_{e}), the modulation gap for bit rate = b is obtained by $\gamma_{\mathrm{mod}}(b) = \rho(b, P_{\mathrm{e}}) / (2^{b}-1)$. The modulation gap, $\gamma_{\mathrm{mod}}(b)$, is an almost constant function of b, particularly for large b. In Table I, we have calculated the modulation gap as a function of the number of users in the MAC group by averaging $\gamma_{\mathrm{mod}}(b)$ in dBs for b = 2 to 15. As it can be seen, the BER requirements can be met in the presence of GDFE error propagation by a small increase in the modulation gap.

E. Computational Complexity

The IF/MAC-OSB algorithm can be implemented by exhaustively searching over the space of transmit powers or by searching over the space of bit rates.

When the search is carried out over the space of transmit powers, the computational complexity is calculated as follows.

TABLE I
REQUIRED MODULATION GAP TO COUNTERACT GDFE ERROR PROPAGATION VS. NUMBER OF USERS IN THE MAC GROUP

Number of Users in the MAC	Modulation Gap [dB]
1	9.75
2	9.96
3	10.07
4	10.15
5	10.21
6	10.26
7	10.30
8	10.33
9	10.37
10	10.39

The computational complexity of (16) for a particular decoding order is $\mathcal{O}(N_g^4)$, due to N_g matrix inversions (with complexity $\mathcal{O}(N_g^3)$) required to calculate the bit rates for all users in group g. However, using the matrix inversion lemma [20], this can be reduced to $\mathcal{O}(N_g^3)$[28]. Since there are $N_g!$ different orderings, the total computational complexity associated with group g for each set of transmit powers is $\mathcal{O}(N_g! N_g^3)$. We also have to calculate the noise covariance matrix for each group, which can be safely ignored here as the required number of operations for group g is much smaller than $N_g! N_g^3$. If we assume that each transmit power can take α levels, the size of the search space for each tone is α^N. Therefore, the computational complexity of the algorithm is $\mathcal{O}(\beta K \alpha^N \sum_g N_g! N_g^3)$, where K is the number of tones and β is the number of iterations required for finding the optimal value of the weight factors and Lagrange multipliers. Parameter β depends on the technique used for updating the weight factors and Lagrange multipliers. A possible algorithm for this purpose is the iterative facet dividing algorithm (IFDA) [14], [15] for which we have $\beta = \mathcal{O}(N \log \frac{1}{\varepsilon})$, where ε is the desired precision.

The computational complexity of the algorithm when the search is carried out over the space of bit rates is calculated as follows. If we use the first technique, for each set of bit rates we execute Algorithm 2 $\prod_g N_g!$ times corresponding to all possible decoding orders. Using the matrix inversion lemma, the number of operations in each iteration of the main loop of Algorithm 2 is $\mathcal{O}(\sum_g N_g^3)$. We denote by β' the number of iterations needed for Algorithm 2 to converge. Our simulation results show that two to three iterations are enough for the convergence of Algorithm 2. Since in a typical scenario most of the multiuser bit rate vectors are identified as unachievable in the first iteration, the average value for β' is very close to one. Finally, the size of the search space is $b_{\max}^N$. Therefore, the computational complexity of the algorithm is $\mathcal{O}(\beta\beta' K b_{\max}^N (\prod_g N_g!)(\sum_g N_g^3))$, where again β is the number of iterations required for finding the optimal value of the weight factors and Lagrange multipliers. Since the number of transmit power levels α is usually much larger than $b_{\max}$ in DSL systems, the computational complexity of the algorithm will be much smaller when the search is carried out over the space of the bit rates.

If we use the second technique, the main share of complexity for each set of bit rates is due to the matrix inversion in (23) with $\mathcal{O}(N^3)$. Considering the complexity factor due to ordering, the total computational complexity is $\mathcal{O}(\beta K b_{\max}^N (\prod_g N_g!) N^3)$.

We can obtain a simplified IF/MAC-OSB algorithm with lower computational complexity by setting the decoding order in the MMSE-GDFE receiver based on the order of the weight factors and ignore other decoding orders. The computational complexity of this simplified IF/MAC-OSB is $\mathcal{O}(\beta K \alpha^N \sum_g N_g^3)$ when the search is carried out over the space of transmit powers and $\mathcal{O}(\beta\beta' K b_{\max}^N \sum_g N_g^3)$ or $\mathcal{O}(\beta K b_{\max}^N N^3)$ (depending on the employed technique) when the search is carried out over the space of bit rates. Obviously, we may not achieve the bit rates obtained by the original IF/MAC-OSB algorithm. We will study the performance of the simplified algorithm using computer simulations in Sec. IV.

The computational complexity of the algorithm is large when the number of users is larger than four. However, several techniques, which provide near-optimal performance, have been proposed in the literature to reduce the computational complexity of the per-tone exhaustive search required by the OSB algorithm, which can readily be extended to IF/MAC-OSB. Examples include the iterative spectrum balancing (ISB) algorithm [21], [22], a branch and bound based algorithm proposed in [16], and 2SB [23]. Using these techniques, the factor α^N which is exponential in the number of users N is replaced by a factor which is polynomial in α and N. This reduces the computational complexity of the algorithm significantly.

IV. SIMULATION RESULTS

We have simulated our technique on six scenarios. The first five scenarios consist of three to six VDSL 2 997E17 loops in two MAC groups. The sixth scenario consists of twelve VDSL 2 997E17 loops in three MAC groups. The first scenario has three 800 m loops in the first MAC group and two 400 m loops in the second MAC group and is referred to as the 3x800-2x400 scenario. The second scenario has two 800 m loops in the first MAC group, two 400 m loops in the second MAC group, and an unmanaged 400 m loop and is referred to as the scenario with an unmanaged loop. The third scenario has one 800 m loop in the first MAC group and two 400 m loops in the second MAC group and is referred to as the 1x800-2x400 scenario. The fourth scenario consists of two MAC groups each with an 800 m loop and a 400 m loop and is referred to as the mixed length scenario. The fifth scenario consists of two symmetric MAC groups each with three 500 m loops and is referred to as the symmetric scenario. Finally, the sixth scenario consists of three MAC groups in a joint CO/RT deployment. The first group consists of two 500 m loops terminated at the CO. The second and third groups consist of, respectively, four and six 400 m loops terminated at a RT. The RT is located 300 m away from the CO. This scenario is called the 12-user CO/RT scenario. These scenarios are illustrated in Fig. 1. Simulation parameters are listed in Table II. Unless explicitly stated otherwise, in all simulations the standard 1% worst-case crosstalk model [24] without considering FSAN power sum rule has been used .

Figure 2 compares the performance of IF/MAC-OSB to several other algorithms and techniques for the 3x800-2x400

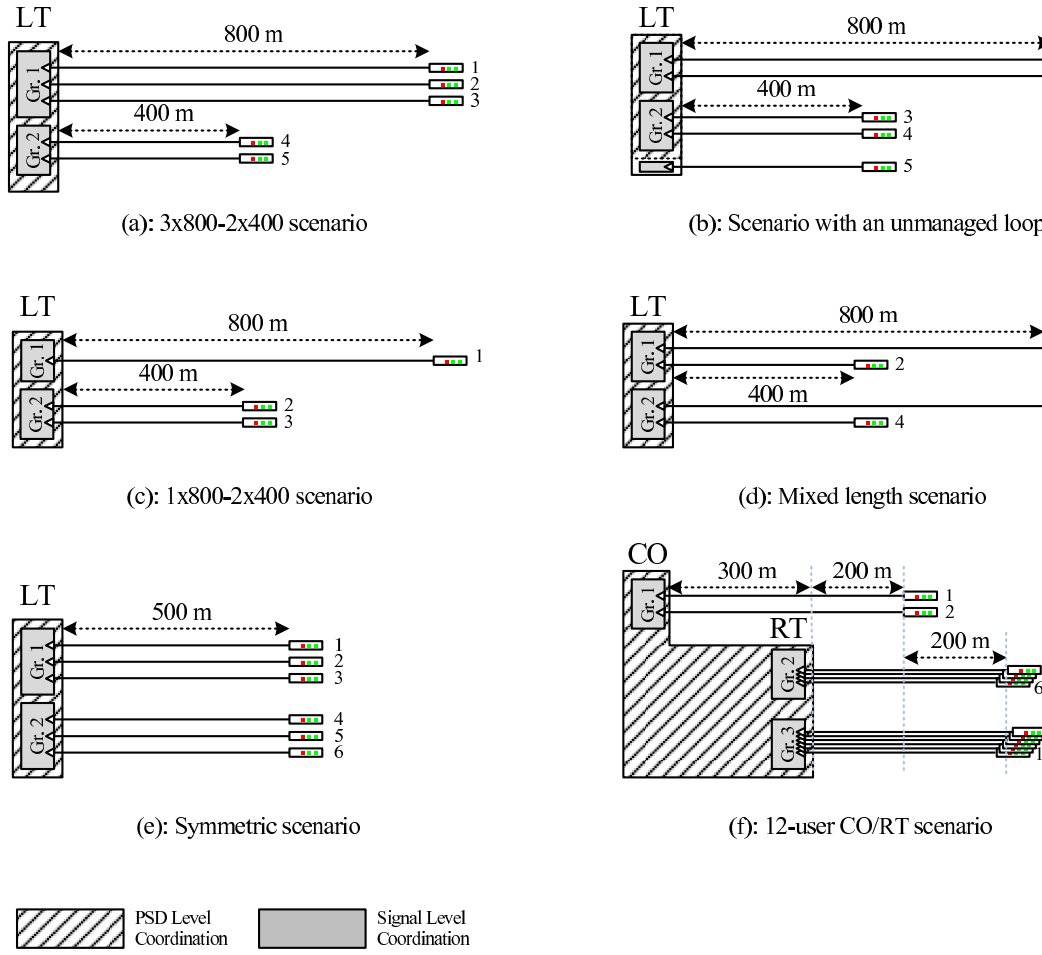

Fig. 1. Simulated scenarios.

TABLE II
SIMULATION PARAMETERS

Parameter	Value
Bandplan and PSD mask	US VDSL2E17 B7-9 bandplan (3-5.1, 7.05-12, 14-17.67 MHz) [25], [26]
Cable type	26 AWG [24]
Noise	White noise, -140 dBm/Hz
Tone spacing, Δ_f	4.3125 kHz
Symbol rate, f_s	4 kHz
b_{max}	15
b_{min}	2
Noise margin	6 dB
Coding gain	6.3 dB

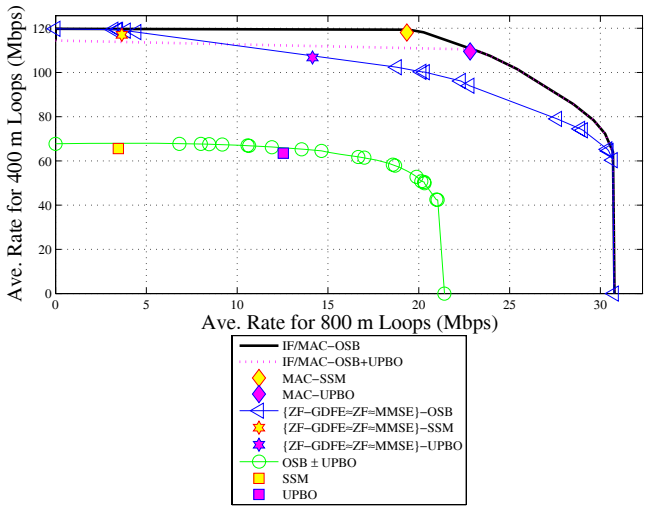

Fig. 2. Average achievable bit rates of 400 m loops vs. average achievable bit rates of 800 m loops for the 3x800-2x400 scenario using different crosstalk mitigation schemes.

scenario. For four techniques we do not apply any vectoring for users inside the groups. These techniques are "SSM" for which the transmit powers of all users are set at the nominal PSD mask, "UPBO" for which the transmit powers are obtained by upstream power back-off (UPBO), "OSB with UPBO" for which the OSB algorithm is used when the transmit powers are constrained to UPBO masks, and "OSB without UPBO" for which the OSB algorithm is used when the transmit powers are constrained by nominal PSD masks. For the rest of the algorithms a form of vectoring is applied for the users in the same MAC groups. The first part/parts of the names indicates/indicate the employed vectoring technique as follows: ZF: linear ZF equalizer, MMSE: linear MMSE equalizer, ZF-GDFE: ZF-GDFE equalizer, and MAC: MMSE-GDFE equalizer. The last part/parts of the names indicates/indicate the spectrum balancing technique used for the groups. For example, OSB+UPBO indicates that OSB with UPBO has been used.

In this figure, we have plotted the average achievable bit rate for the 400 m loops vs. the average achievable bit rate for the 800 m loops. To obtain the plotted RRs in this figure and the following figures, the weight factors for the 800 m loops are set to the same value w while the weight factors for the 400 m loops are set to 1-w. Then, w is swept from 0 to 1 in 0.05 steps and the algorithms are executed for each value of w. For faster simulations, all techniques which require an exhaustive search are implemented with ISB type iterations. For better readability of the figures, the techniques that show the same performance are represented by only one of them. As indicated in the figure legend, ZF-SSM, MMSE-SSM, and ZF-GDFE-SSM reach the same performance. Similarly, grouped ZF-OSB, grouped MMSE-OSB, and grouped ZF-GDFE-OSB offer (almost) the same bit rates. Finally, OSB with or without UPBO achieve the same performance.

It is seen that the largest RR is achieved by IF/MAC-OSB when UPBO is not used. The MAC-SSM technique reaches two points on the RR one for the case with UPBO and one for the case without UPBO. This technique can reach the maximum bit rate for the shorter loops, however, the bit rates for the longer loops are 26% and 37% less than the maximum achievable bit rate depending on UPBO being used or not.

As it can be seen, the proposed algorithm provides significantly higher bit-rates than SSM and OSB. It also provides noticeably higher bit rates than the grouped ZF-OSB, grouped MMSE-OSB, and grouped ZF-GDFE-OSB algorithms in the cases where the users in both groups are actively demanding higher bit rates (corresponding to the top right corner of the RR). This is in fact an interesting outcome and demands intricate explanation. The ZF-GDFE structure uses QR decomposition. The noise is multiplied by a unitary matrix, therefore there is no boost in the crosstalk power and there is no performance loss due to it. The ZF equalizer achieves the performance of ZF-GDFE. Therefore, the ZF equalizer does not increase the noise power either which, as proved in [8], is due to the column-wise diagonal dominance of the DSL channel. The MMSE equalizer provides the same performance. The MMSE structure takes into account the noise covariance matrix. This implies that using the information regarding the colored noise is not necessarily helpful to improve the performance. However, when the MMSE equalizer is used together with the GDFE structure as in IF/MAC-OSB, we obtain higher bit-rates in the corner of the RR.

The reason for this phenomenon is that this structure not only removes the crosstalk from the users in the same MAC group using the GDFE but it is also capable of canceling the crosstalk coming from the other groups using the information in the noise covariance matrix. Consider the reception in the 3x800 MAC group for the last decoded 800 m loop. For this user the crosstalk coming from the first two 800 m loops is removed using the GDFE feedback loop. For decoding of this user, we have three signals available, which convey combinations from the desired signal of the last 800 m loop and the crosstalk signals of the two 400 m loops in the second MAC group. Therefore, if we ignore the receiver noise, the crosstalk can be canceled perfectly. To make this point clearer, in Fig. 3, we have plotted the RR for the 1x800-2x400 scenario in which we only have one 800 m loop in the first MAC group. As it can be seen, the achievable bit rate for the 800 m loop is smaller than the average achievable bit rate for the 800 m loops in the 3x800-2x400 scenario. Obviously, two crosstalkers are omitted and the crosstalk level for the 800 m loop is smaller in this new scenario, however, it cannot achieve the same bit rate as in the 3x800-2x400 scenario because it does not have sufficient number of signals to remove the crosstalk completely.

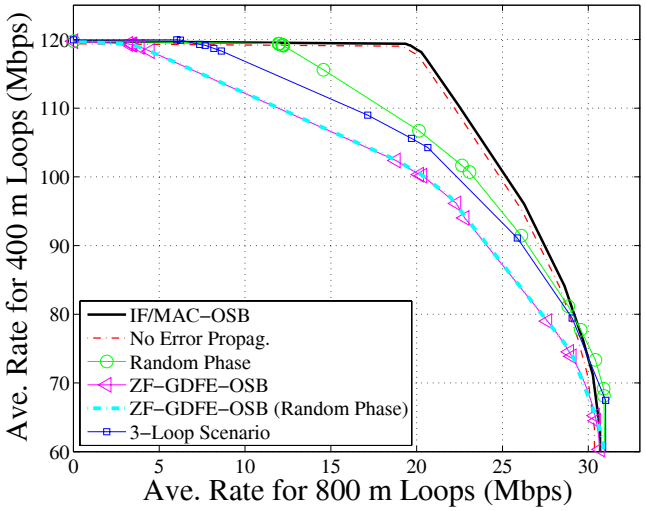

Fig. 3. Performance of the IF/MAC-OSB algorithm for the 3x800-2x400 scenario: Black solid line: Normal performance, Red dashed line: performance when GDFE error propagation is tackled, Green solid line marked with circles: performance when the entries of channel matrix have random phase, Magenta solid line marked with triangles: grouped ZF-GDFE-OSB, Cyan thick dashed line: grouped ZF-GDFE-OSB with random phased channel, Blue solid line marked with squares: performance of IF/MAC-OSB for the 1x800-2x400 scenario.

Related to the above discussion, it is important to note that the channel randomness plays an important role in the achievable bit rates of the IF/MAC-OSB algorithm. Consider the 3x800-2x400 scenario. When we assume the 1% worst case crosstalk model for all crosstalk couplings and we assume linear phase, the same crosstalk signal is received by the users in group 1 from users in group 2. The sum of these signals can then be removed as if they are originating from a single crosstalker. So using the same argument as in the previous paragraph, we can show that by only decoding the first user we can actually decode the second user as well as the third user as if there is no crosstalk. In Fig. 3, we have provided simulation results for the 3x800-2x400 scenario assuming i.i.d channel phase with uniform distribution over [0,2π) for all channel entries. A meaningful reduction in the bit rates can be seen, however, the performance is still better than the performance for the case with a single 800 m loop and the performance of the grouped ZF-GDFE-OSB algorithm, which means that , using the MMSE-GDFE structure, the algorithm is still capable of removing crosstalk more effectively than other techniques. Interestingly, our simulations for scenarios with a single MAC group show a slight increase in the bit rates when random phase is added to the channel. This is probably because randomness increases the rank of a crosstalk channel matrix.

Also, the grouped ZF-GDFE-OSB algorithm does not show

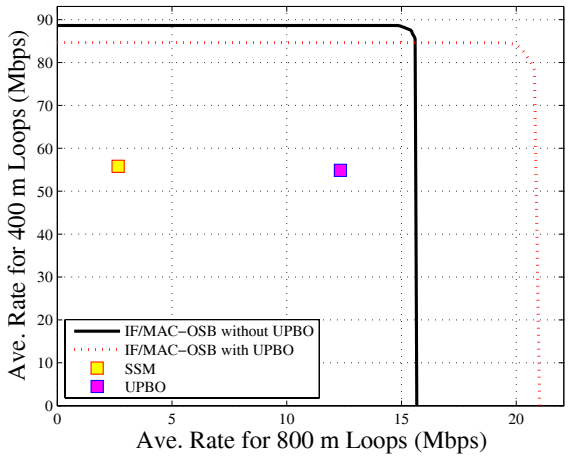

Fig. 4. Simulation results for the scenario with an unmanaged loop.

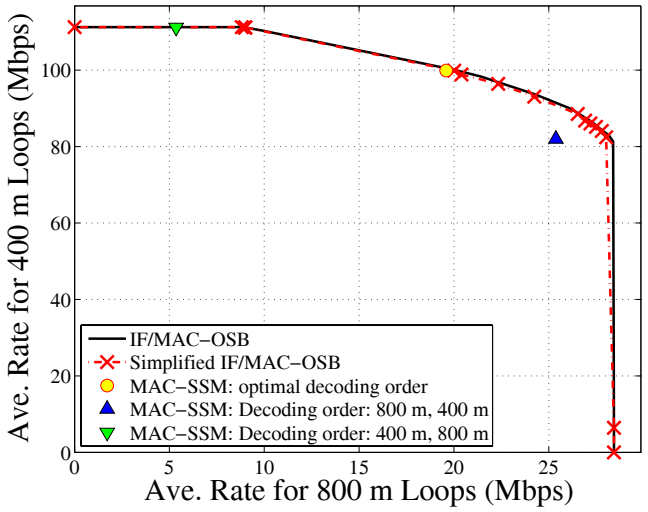

Fig. 5. Effect of decoding order on the performance of IF/MAC-OSB and MAC-SSM for the mixed length scenario.

any sensitivity to the channel phase in Fig. 3, which is an expected behavior based on the above discussions. In Fig. 3, we have also plotted the achievable RR for the 3x800-2x400 scenario by the algorithm when the GDFE error propagation is addressed using the technique proposed in Sec. III-D. As expected the bit rate losses are very small.

As we mentioned, IF/MAC-OSB achieves higher bit rates for shorter loops when UPBO is not used compared to the case when UPBO is used for the 3x800-2x400 scenario. However, it is important to note that it is not always a good idea not to use UPBO when IF/MAC-OSB or other DSM techniques are used. Figure 4 illustrates simulation results for the scenario with an unmanaged loop. The achievable bit rates for the long managed loops when UPBO is not used are significantly smaller than when UPBO is used due to higher crosstalk originating from the unmanaged loop. The obvious conclusion is that it is safer to have UPBO enforced even if we are using the IF/MAC OSB algorithm, unless we are sure that all loops will be managed by the same spectrum management center.

We have studied the effect of the decoding order on the performance of IF/MAC-OSB and MAC-SSM in Fig. 5. In this figure we have simulated the mixed length scenario in which loops with different lengths co-exist in the same MAC group. As discussed in Sec. III-B, the optimal decoding order for a MMSE-GDFE receiver cannot be determined by the order of the weight factors in practical DSL systems. Consider we want to find the sum rate maximization point. To find this point we set all weight factors to one. Since the weight factors are equal, the order of decoding does not matter for the ideal MMSE-GDFE receiver.

Now let us study simulation results for the MAC-SSM technique as illustrated in Fig. 5. If we decode the 400 m loops first and then the 800 m loops on all tones, we obtain 25.4 Mbps for the long loops and 82.0 Mbps for the short loops. Therefore, the sum bit rate for the users in the same MAC group is 107.4 Mbps. If we switch the order of decoding on all tones, we obtain 5.4 Mbps for the long loops, 111.2 Mbps for the shorter loops, and a sum bit rate of 116.6 Mbps. If we test both decoding orders in order to find the optimal order of decoding for each tone, we obtain 19.6 Mbps, 99.9 Mbps, and 119.5 Mbps for the long loops, short loops, and the sum bit rate, respectively.

Clearly, we obtain different sum bit rates depending on the decoding order and the optimal decoding order is different on different tones. Therefore, to reach the optimal performance it is required to test all decoding orders. The number of decoding orders for group g is $N_g!$. As studied in Sec. III-E, checking all decoding orders to find the optimal solution could increase the computational complexity of the algorithm considerably. Two algorithms with polynomial complexity in N_g for determining the decoding order for the ZF-GDFE have been proposed in [27]. In [28], the authors have proposed a nearly optimal algorithm with polynomial complexity in N_g for determining the decoding order for the MMSE-GDFE. However, this is not required for IF/MAC-OSB. We have plotted the achievable RR by IF/MAC-OSB and the simplified version in which the decoding order is determined by the order of weight factors in Fig. 5 as well. The simplified IF/MAC-OSB achieves the performance of the IF/MAC-OSB for this scenario. We have tested this fact by simulating the two algorithms for several different scenarios and weight factors. For all cases studied, we found that the simplified algorithm achieves the same weighted sum bit rate as the original algorithm. This is practically very helpful as the simplified algorithm has a much smaller computational complexity than the original algorithm.

In Fig. 6, we have provided the simulation results for the symmetric scenario. As it can be seen, IF/MAC-OSB performs considerably better than the other algorithms again. Other results are essentially the same as the results obtained for the other simulated scenarios. The main difference is that the grouped MMSE-OSB performs better than the grouped ZF-OSB and grouped ZF-GDFE-OSB algorithms for this scenario.

As for the last simulation results, we have plotted the average achievable bit rates of the 400 m loops vs. that of the 500 m loops for the 12-user CO/RT scenario in Fig. 7. In this scenario, we have generated the crosstalk couplings using the statistical crosstalk model with beta distribution proposed in [12]. Channel entries are assumed to have i.i.d phases which are uniformly distributed over $[0, 2\pi)$. As expected, the

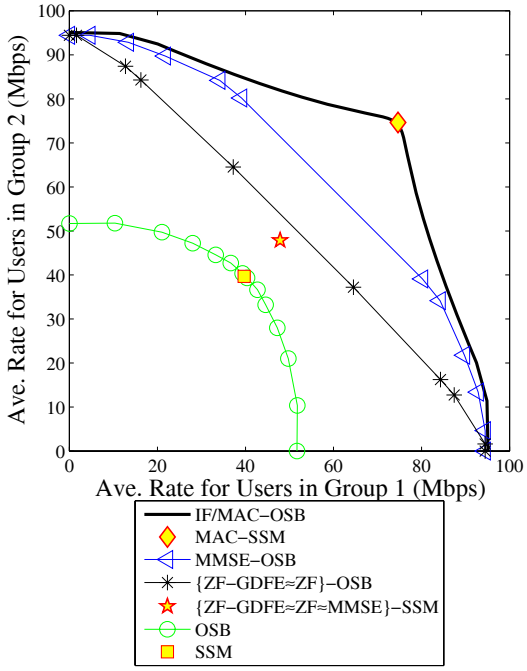

Fig. 6. Average achievable bit rates of the users in group 1 vs. average achievable bit rates of the users in group 2 for the symmetric scenario using different crosstalk mitigation schemes.

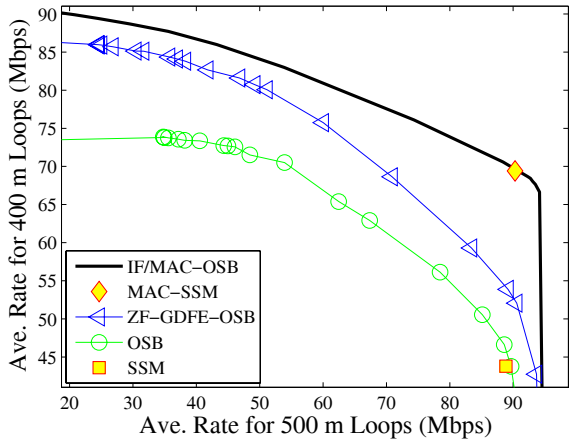

Fig. 7. Simulation results for the 12-user CO/RT scenario using the statistical crosstalk model in [12].

performance gains of the vectoring techniques are relatively smaller than the cases in which we had only two MAC groups and the crosstalk couplings were calculated by the 1% worst-case model. Aside from that we see the same trend in the simulation results as before. Particularly, IF/MAC-OSB peforms noticably better than all other techniques.

V. Conclusion

In this paper we have proposed an IF/MAC-OSB algorithm for joint DSM 2/3 in US grouped DSL scenarios. We have shown that it achieves considerably higher bit rates than several other algorithms and techniques. Our analysis shows that IF/MAC-OSB is capable of cancelling the crosstalk originating from the users in the same MAC group as well as the crosstalk originated from the users in other groups. This explains why this technique achieves higher bit rates than other techniques such as grouped ZF-GDFE-OSB and grouped MMSE-OSB. The achievable bit rates depend on the degree of the crosstalk channel randomness and ironically, assuming a 1% worst case channel model with linear phase may lead to optimistic performance predictions, unless all active users are in the same MAC group.

We have proposed a simplified version of IF/MAC-OSB in which the decoding order is determined by the order of the user weight factors. We have shown that this algorithm achieves the same performance as the original algorithm with less computational complexity.

We have also proposed a simple technique to avoid GDFE error propagation in practical DSL systems. The technique is based on determining the modulation gap based on the number of users in the MAC group. Simulation results show that the bit rate loss using this technique is negligible. This technique can be potentially extended to any decision feedback equalizer for practical systems in which the BER is non-zero.

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