Novel Orthogonal Codes for Spectrally-Encoded CDMA Systems in Fading Channels

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***Abstract*—Orthogonal spreading codes play an essential role in code-division multiple-access (CDMA) systems by eliminating multiple-access interference (MAI). In this paper, a novel orthogonal spreading code has been proposed for spectrally-encoded (SE) CDMA, a.k.a., spread-time (ST) CDMA with arbitrary pulse shape. It has been shown that it is possible to retain the orthogonality of the code in the presence of tail truncation by time windowing and in a general multipath fading channel in which users experience different frequency selectivity just by modifying the user codewords. Simulation results show that the proposed codes can achieve single user performance when the code length is twice the number of users.**

***Index Terms*—Code-division multiple-access, indoor radio communication, interference suppression, orthogonal spreading codes, spectrally-encoded CDMA, spread-time CDMA.**

I. Introduction

A code-division multiple-access (CDMA) scheme is attractive for personal mobile communications for many reasons, including more efficient use of the spectrum, sharing the spectrum fairly among the users, higher security, and more resistance to jamming as compared to other schemes. Over the last few decades a significant amount of research has been carried out on CDMA particularly direct-sequence (DS) and frequency-hopping (FH) CDMA. Recently, spectrally-encoded (SE) CDMA, a.k.a. spread-time (ST) CDMA, has attracted growing attention. SE-CDMA is the time-frequency dual of DS-CDMA. Although not as famous as DS-CDMA, it provides several advantages over DS-CDMA such as power efficiency, easier rejection of narrowband interference, more resistance to fading, and the ability to accommodate discontinuous frequency bands [1]–[9].

SE-CDMA was originally proposed for femto-second optical CDMA in [10]. Its application to wireless communications was later proposed in [1]. It has been shown that SE-CDMA is 2.1 dB more power efficient than DS-CDMA in AWGN and fading channels in [1] and [2], respectively. The ability of SE-CDMA to co-exist with several conventional radio systems has been studied for ultra-wideband (UWB) impulse radios in [2], [3], and [4]. In [4] a technique for real-time implementation of these systems in impulse radios has been proposed using surface-acoustic wave models. In [5] the power level profile of SE-CDMA signals has been studied. The performance of the system under multiple Gaussian interferences has been studied in [6], and a three level {-1, 0, +1} coding scheme has been proposed which could achieve higher performance than binary codes. In [7], the performance of combined time-hopping and spread time CDMA systems in the presence of narrowband interference has been studied.

In a DS-CDMA system, the symbol duration is divided into N chips, where N is called the processing gain of the system. The chips are modulated by a spreading codeword with length N, which is unique for each user. Since the chip duration is $1/N$-th of the symbol duration, the signal bandwidth is extended by a factor of N. SE-CDMA is conceptually very similar to DS-CDMA except that the signal is modulated by a codeword in the frequency domain instead of the time domain. In a SE-CDMA system, the available bandwidth is divided into N frequency chips. The n-th frequency chip is modulated by the n-th element of a user dedicated codeword. The transmitted pulse in the time domain is obtained by the inverse Fourier transform, which is then modulated by the user's symbol stream. As the codewords are different for different users, the transmitted signal has a unique shape for each user in the frequency and time domains. Moreover, modulating the pulse in the frequency domain with chips having $1/N$-th of the signal bandwidth spreads the signal in time by a factor of N. Therefore, SE-CDMA is also known as spread-time (ST) CDMA, as opposed to spread-spectrum CDMA.

Using random spreading codes, CDMA systems reduce the multiple-access interference (MAI) proportionally to their processing gain. However, DS-CDMA is capable of further canceling or significantly alleviating MAI by means of smartly designed orthogonal or semi-orthogonal spreading codes. Using orthogonal codes, it is possible to mitigate MAI perfectly, allowing each user to achieve the bit-rate that can be achieved when no other user is active. Therefore, the design of orthog-

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onal spreading codes for CDMA is of a very high practical and theoretical interest.

Although, many codes have been proposed for DS-CDMA, no orthogonal code has been developed for SE-CDMA so far. In [8] the use of complete-complementary (CC) codes has been proposed for SE-CDMA. These spreading-codes are multiplied by the transmitted signal over time as well as frequency. The CC codes provide ideal auto- and cross-correlation properties [11]. That is the auto-correlation function of these codes is zero for all shifts except for the zero shift and their cross-correlation function is zero for all shifts. This makes CC codes a very attractive candidate for CDMA systems. Unfortunately, the number of codewords for a CC code is bounded by $\sqrt{G}$, where G is the code length. The code length determines the number of chips in the frequency domain. Each symbol consists of $\sqrt{G}$ consecutive pulses in the time domain. This means that the (congregated) processing gain of the system is $P_{\rm g} = G\sqrt{G}$. Therefore, the number of codewords as a function of processing gain is $\sqrt[3]{P_{\rm g}}$. This imposes a severe restriction on the maximum number of users supported by the system. In this paper, we propose a class of orthogonal codes for SE-CDMA where code multiplication is done entirely in the frequency domain. Moreover, they support many more users than the CC codes.

Multiplication of the signal with SE codewords with rectangular-shaped chips causes the signal to spread infinitely in the time domain, which necessitates time domain windowing [1]. Time windowing causes some energy drift from each chip to adjacent chips resulting in overlapping chips. We modify our technique to obtain orthogonal codes for SE systems with windowing. We also study the undesirable channel conditions under which the orthogonality of the code could be lost. We consider a very general quasi-static frequency selective fading channel, which can be different for different users. Maintaining the orthogonality of codes for DS-CDMA systems may require chip filters which adapt to the channel conditions, yielding an extremely difficult implementation. However, we show that for SE-CDMA systems, we only need to modify the spreading codewords of the users to restore the orthogonality of the codes in such frequency selective fading channels. We propose chip and vector normalizing schemes for SE-CDMA systems without and with windowing, respectively, to restore the orthogonality of the code in these cases.

This paper is organized as follows. We describe the system and channel assumptions in the next section. We analyze the system performance followed by the code design in three different scenarios in Secs. III, IV, and V. Simulation results are provided in Sec. VI. Finally, we conclude this work in Sec. VII.

II. System Description

We consider disjoint downlink (DL) and uplink (UL) frequency bands, where all users communicate with a single base station. We assume a memoryless linear modulation of the data. In a SE-CDMA communication system, a spreading codeword is dedicated to each user which encodes the transmitted pulse $p_{\rm tr}(t)$ in the frequency domain [1]. In both the DL and UL directions, the transmitted signal for user k can be written in the following form

$$s_{\rm tr}^{(k)}(t) = \sum_{\ell=-\infty}^{+\infty} d_\ell^{(k)} \tilde{p}_{\rm tr}^{(k)}(t - \ell T_{\rm s}), \tag{1}$$

where $\left\{ d_\ell^{(k)};\ \text{for } \ell = -\infty \text{ to } +\infty \right\}$ is the sequence of real-valued information symbols for user k, $T_{\rm s}$ is the symbol period, and $\tilde{p}_{\rm tr}^{(k)}(\cdot)$ denotes the time windowed version of $p_{\rm tr}^{(k)}(\cdot)$ defined by $\tilde{p}_{\rm tr}^{(k)}(t) \triangleq p_{\rm tr}^{(k)}(t) w(t)$, where $p_{\rm tr}^{(k)}(\cdot)$ is the ST pulse assigned to user k and $w(\cdot)$ is a time domain window with length $T_{\rm w}$. We assume a rectangular time window $w(t) = \text{rect}\left(t/T_{\rm w}\right)$, where $\text{rect}(x) \triangleq \begin{cases} 1, & |x| < \frac{1}{2} \\ 0, & \text{otherwise.} \end{cases}$

The pulse $p_{\rm tr}^{(k)}(\cdot)$ is encoded in the frequency domain with its Fourier transform (FT) defined by

$$P_{\rm tr}^{(k)}(\omega) = \sum_{n=1}^{N} a_n^{(k)} \sqrt{p_{\rm PSD}[n]}\, \text{rect}\left(\tfrac{\omega}{\Omega_{\rm c}} - n\right), \tag{2}$$

where $\Omega_{\rm c}$ is the chip width in the frequency domain, $p_{\rm PSD}[\cdot]$ indicates samples of the power spectral density (PSD) of the pulse, and $\left\{ a_n^{(k)};\ \text{for } n = 1 \text{ to } N \right\}$ is the spreading sequence of user k. For now, we assume $a_n^{(k)} = \pm 1$. If we assume $p_{\rm PSD}[n] = 1$ for all n, (2) reduces to the standard form of SE modulation defined in [1]. The sequence $p_{\rm PSD}[\cdot]$ can be used to shape the PSD to (a piecewise constant approximation of) the desired spectrum. For example, a narrowband interfering signal located at $n\Omega_{\rm c}$ can be avoided by setting $p_{\rm PSD}^{(k)}[n] = 0$ [4],[5]-[7], or it can be used to shape the spectrum to the waterfilling solution as suggested in [1]. For the transmitted signal to be causal, we need to transmit it with some delay, which we ignore here without loss of generality. From (2), $p_{\rm tr}^{(k)}(\cdot)$ is calculated using the inverse FT (IFT)

$$p_{\rm tr}^{(k)}(t) = \tfrac{\Omega_{\rm c}}{2\pi} \text{sinc}\left(\tfrac{\Omega_{\rm c}}{2\pi} t\right) \sum_{n=1}^{N} a_n^{(k)} \sqrt{p_{\rm PSD}[n]}\, \exp(jn\Omega_{\rm c} t), \tag{3}$$

where $\text{sinc}(x) \triangleq \frac{\sin(\pi x)}{\pi x}$. The FT of $\tilde{p}_{\rm tr}^{(k)}(\cdot)$ can be calculated using the IFT convolution property as follows

$$\begin{aligned} \tilde{P}_{\rm tr}^{(k)}(\omega) &= \tfrac{1}{2\pi} P_{\rm tr}^{(k)}(\omega) \otimes W(\omega) \\ &= \textstyle\sum_{n=1}^{N} a_n^{(k)} \sqrt{p_{\rm PSD}[n]}\, \tilde{S}(\omega - n\Omega_{\rm c}), \end{aligned} \tag{4}$$

where operator $\otimes$ denotes convolution, $W(\omega) \triangleq \mathcal{F}\{w(t)\} = \frac{T_{\rm w}}{2\pi} \text{sinc}\left(\frac{T_{\rm w}}{2\pi}\omega\right)$, and

$$\begin{aligned} \tilde{S}(\omega) \triangleq\ & \tfrac{1}{2\pi} \text{rect}\left(\tfrac{\omega}{\Omega_{\rm c}}\right) \otimes W(\omega) \\ =\ & \pi^{-1} \left\{ \text{Si}\left(\tfrac{1}{2} \text{T}_{\rm w} \left[\omega + \tfrac{1}{2}\Omega_{\rm c}\right]\right) \right. \\ & \left. -\text{Si}\left(\tfrac{1}{2} \text{T}_{\rm w} \left[\omega - \tfrac{1}{2}\Omega_{\rm c}\right]\right) \right\}, \end{aligned} \tag{5}$$

where $\text{Si}(x) \triangleq \int_0^x \frac{\sin(u)}{u} \text{d}u$ is the sine integral. As it can be seen, the effect of time windowing can be taken into account by modifying the shape of the frequency chip. Since, $w(t)$ has an infinite length FT, $\tilde{S}(\omega)$ has an infinite length as well. This causes overlapping of all chips on each other. In this paper, we first ignore windowing and discuss the code design for systems with non-overlapping rectangular chips. Then, we generalize

our technique to systems with overlapping chips where we consider the effect of windowing on rectangular shaped chips.

We assume a quasi-static channel and we study the performance of the system for both AWGN and multipath frequency selective fading channels. For the AWGN channel the normalized channel impulse response is the same for all of the users. In contrast, the frequency selective fading channel can differ between users. We assume synchronized users at the receiver side. Using a synchronization channel, this condition can be achieved when all of the users are communicating with a single base station.

Under the aforementioned assumptions, the received signal at the front end of the i-th receiver's antenna can be written as

$$r^{(i)}(t) = \sum_{k=1}^{K} s_{\rm rec}^{(k,i)}(t) + n^{(i)}(t), \tag{6}$$

where K is the number of users, $n^{(i)}(t)$ is the i-th receiver's noise and

$$s_{\rm rec}^{(k,i)}(t) = \sum_{\ell=-\infty}^{+\infty} d_\ell^{(k)} p_{\rm rec}^{(k,i)}(t-\ell T_{\rm s}), \tag{7}$$

where $p_{\rm rec}^{(k,i)}(\cdot)$ is the k-th user's received pulse at the i-th receiver. In the DL direction, all signals undergo the same channel $H_{\rm DL}^{(i)}(\omega)$ from the base station's transmit antenna to the i-th user's receive antenna. Then the FT of $p_{\rm rec}^{(k,i)}(\cdot)$ is

$$P_{\rm rec}^{(k,i)}(\omega) = H_{\rm DL}^{(i)}(\omega) \sum_{n=1}^{N} a_n^{(k)} \sqrt{p_{\rm PSD}[n]}\tilde{S}(\omega - n\Omega_{\rm c}), \tag{8}$$

where $H_{\rm DL}^{(i)}(\omega)$ is the downlink channel transfer function for user i. In the UL direction, the k-th user's signal travels the k-th uplink channel $H_{\rm UL}^{(k)}(\omega)$. Therefore

$$P_{\rm rec}^{(k,i)}(\omega) = H_{\rm UL}^{(k)}(\omega) \sum_{n=1}^{N} a_n^{(k)} \sqrt{p_{\rm PSD}[n]}\tilde{S}(\omega - n\Omega_{\rm c}). \tag{9}$$

Note that, there is only one base station receiver at the UL direction and we arbitrarily set i = 1.

In this paper we assume the correlation receiver. The correlator for detecting symbol 0 (ℓ=0) of user i takes the following form [12]:

$$\begin{aligned} x_0 &= \int_{-\infty}^{+\infty} r^{(i)}(t)\overline{p_{\rm rec}^{(i,i)}(t)}{\rm d}t \\ &= \mathcal{F}_{t=0}^{-1}\left\{R^{(i)}(\omega)\overline{P_{\rm rec}^{(i,i)}(\omega)}\right\}, \end{aligned} \tag{10}$$

where $\overline{(\cdot)}$ denotes the complex conjugate operation, $R^{(i)}(\omega)$ is the FT of $r^{(i)}(t)$, and $\mathcal{F}_{t=\tau}^{-1}\{\cdot\}$ denotes the inverse FT (IFT) evaluated at $t=\tau$. In this paper, we try to design orthogonal spreading codes without violating the structure of the correlation receiver as far as possible. That guarantees that we achieve the single user performance in the presence of multiple-access interference. However, it is impossible to do so in some undesirable channel conditions such as frequency selective channels. For those cases, we will propose a minimal change of system structure, which is the use of different spreading codes at the transmitter or receiver sides.

III. Systems without Windowing in AWGN Channel

In this section, we analyze the output of the correlation receiver and propose appropriate methods to annihilate the MAI in the AWGN channel when the time windowing is ignored.

A. System Analysis

Regardless of the direction of the transmission, we denote by A_k and τ_k the attenuation and delay of user k. In other words, we assume $H_{\rm DL}^{(k)}(\omega) = H_{\rm UL}^{(k)}(\omega) = A_k e^{-j\omega\tau_k}$. We assume synchronized users, and without loss of generality, we assume $\tau_k = 0$ for all users. Since the users are independent, we can calculate the contribution of the desired signal, MAI, and channel noise independently. By ignoring time windowing, the k-th user's received signal can be obtained from (2) and (3) in the frequency and time domains, respectively.

Desired User Signal: The FT of the desired user's received signal is

$$\begin{aligned} R^{(i)}(\omega) = \; & A_i \sum_{\ell=-\infty}^{+\infty} d_\ell^{(i)} e^{-j\omega\ell T_{\rm s}} \\ & \times \sum_{n=1}^{N} a_n^{(i)} \sqrt{p_{\rm PSD}[n]}{\rm rect}(\tfrac{\omega}{\Omega_{\rm c}} - n). \end{aligned}$$

From (10), the contribution of the desired user's signal at the correlator output can be calculated as

$$\begin{aligned} s_{\rm A,\cancel{W}} = \; & |A_i|^2 \sum_{\ell=-\infty}^{+\infty} d_\ell^{(i)} \mathcal{F}_{t=0}^{-1}\left\{ e^{-j\omega\ell T_{\rm s}} \sum_{n=1}^{N} a_n^{(i)}\overline{a_n^{(i)}} \right. \\ & \left. \times \left|{\rm rect}(\tfrac{\omega}{\Omega_{\rm c}} - n)\right|^2 \left|\sqrt{p_{\rm PSD}[n]}\right|^2 \right\} \\ = \; & \tfrac{\Omega_{\rm c}}{2\pi} |A_i|^2 \sum_{\ell=-\infty}^{+\infty} d_\ell^{(i)} {\rm sinc}(-\tfrac{\Omega_{\rm c}}{2\pi}\ell T_{\rm s}) \\ & \times \sum_{n=1}^{N} p_{\rm PSD}[n] e^{-jn\Omega_{\rm c}\ell T_{\rm s}}, \end{aligned}$$

where we have used A in the subscript for AWGN, and $\cancel{\rm W}$ to indicate that windowing is not used. With $T_{\rm s} = m2\pi/\Omega_{\rm c}$ (for an integer $m > 0$), the ISI terms will go zero and $s_{\rm A,\cancel{W}}$ reduces to

$$s_{\rm A,\cancel{W}} = d_0^{(i)} \tfrac{\Omega_{\rm c}}{2\pi} |A_i|^2 \sum_{n=1}^{N} p_{\rm PSD}[n].$$

Multiple Access Interference: In this section, we calculate the contribution of an interfering user $k \neq i$ to the decision variable. Using (10), for UL direction we have

$$\begin{aligned} I_{\rm UL,A,\cancel{W}}^{(k)} = \; & \overline{A_i} A_k \sum_{\ell=-\infty}^{+\infty} d_\ell^{(k)} \\ & \times \mathcal{F}_{t=\ell T_{\rm s}}^{-1} \sum_{n=1}^{N} a_n^{(k)} \sqrt{p_{\rm PSD}[n]}{\rm rect}(\tfrac{\omega}{\Omega_{\rm c}} - n) \\ & \times \sum_{n'=1}^{N} \overline{a_{n'}^{(i)} \sqrt{p_{\rm PSD}[n']}{\rm rect}(\tfrac{\omega}{\Omega_{\rm c}} - n')} \Big\}. \end{aligned}$$

Since ${\rm rect}(\frac{\omega}{\Omega_{\rm c}} - n)\,\overline{{\rm rect}(\frac{\omega}{\Omega_{\rm c}} - n')} = 0$ for $n \neq n'$, we can write $I_{\rm UL,A,\cancel{W}}^{(k)} = \overline{A_i} A_k \sum_{\ell=-\infty}^{+\infty} d_\ell^{(k)} \sum_{n=1}^{N} p_{\rm PSD}[n]\overline{a_n^{(i)}} a_n^{(k)} \mathcal{F}_{t=-\ell T_{\rm s}}^{-1}\left\{\left|{\rm rect}(\frac{\omega}{\Omega_{\rm c}} - n)\right|^2\right\}$. Taking the IFT, we get $I_{\rm UL,A,\cancel{W}}^{(k)} = \overline{A_i} A_k \sum_{\ell=-\infty}^{+\infty} d_\ell^{(k)} {\rm sinc}(-\frac{\Omega_{\rm c}}{2\pi}\ell T_{\rm s}) \sum_{n=1}^{N} e^{-jn\Omega_{\rm c}\ell T_{\rm s}} \overline{a_n^{(i)}} a_n^{(k)} p_{\rm PSD}[n]$. Assuming $T_{\rm s} = m2\pi/\Omega_{\rm c}$, the factor ${\rm sinc}(-\frac{\Omega_{\rm c}}{2\pi}\ell T_{\rm s})$ will be zero for $\ell \neq 0$ and one for $\ell = 0$. Thus $I_{\rm UL,A,\cancel{W}}^{(k)} = d_0^{(k)} \frac{\Omega_{\rm c}}{2\pi} \overline{A_i} A_k \sum_{n=1}^{N} \overline{a_n^{(i)}} a_n^{(k)} p_{\rm PSD}[n]$.

The MAI can be calculated similarly for the DL direction as follows $I^{(k)}_{\mathrm{DL,A,}\not{\mathrm{W}}} = d^{(k)}_0 \frac{\Omega_c}{2\pi} |A_i|^2 \sum_{n=1}^{N} \overline{a^{(i)}_n} a^{(k)}_n p_{\mathrm{PSD}}[n]$. Note that both formulas can be represented as follows

$$I^{(k)}_{d_x,\mathrm{A},\not{\mathrm{W}}} = d^{(k)}_0 \eta_{d_x} \sum_{n=1}^{N} \overline{a^{(i)}_n} a^{(k)}_n p_{\mathrm{PSD}}[n], \tag{11}$$

where d_x indicates transmission direction (either UL or DL) and η_{d_x} is equal to $\frac{\Omega_c}{2\pi}\overline{A_i}A_k$ and $\frac{\Omega_c}{2\pi}|A_i|^2$ for UL and DL, respectively.

B. Orthogonal SE Code Design

In this section, we propose our method to make $I^{(k)}_{\mathrm{MA}}$ zero for SE systems without time windowing.

Problem Definition: From (11), the MUI is zero if

$$\sum_{n=1}^{N} \overline{a^{(i)}_n} a^{(k)}_n p_{\mathrm{PSD}}[n] = 0, \text{ for any } k \neq i; 1 \leq k \leq K. \tag{12}$$

We call an ST code that satisfies (12) (for any i; $1 \leq i \leq K$) an orthogonal ST code.

Code Design: Here we propose a technique to design a set of orthogonal codes based on binary orthogonal Walsh sequences. A set of Walsh sequences consists of M codewords of length M, where M is a power of 2. The k-th codeword $\left(c^{(k)}_1, c^{(k)}_2, \ldots, c^{(k)}_M\right)$ consists of M binary 1 and -1 elements. The orthogonality property of Walsh codes states that

$$\sum_{m=1}^{M} c^{(i)}_m c^{(k)}_m = \begin{cases} M, & k = i \\ 0, & k \neq i \end{cases} \tag{13}$$

where $1 \leq k, i \leq M$.

Consider M sets, G_1 to G_M, which contain the indices of the available chips and make a partition over $\{1, 2, \ldots, N\}$, that is $G_m \bigcap G_{m'} = \{\}$ for all $m \neq m'$ $(1 \leq m, m' \leq M)$ and $\bigcup_{m=1}^{M} G_m = \{1, 2, \ldots, N\}$. The subset G_m is attributed to the m-th element of the Walsh codeword. We say chip n is attributed to the m-th element of the Walsh codeword, if $n \in G_m$. The total power of the chips attributed to G_m is simply calculated by $g_m = \sum_{n \in G_m} p_{\mathrm{PSD}}[n]$. We show that the ST code constructed using the following two steps is an orthogonal ST code:

Step 1) Partition the N frequency chips into M subsets, G_m, for m = 1, ..., M, such that the sum received energy in the m-th subset, g_m, is equal to the total received energy divided by M, i.e.:

$$g_m = \sum_{n \in G_m} p_{\mathrm{PSD}}[n] = \tfrac{1}{M} \sum_{n=1}^{N} p_{\mathrm{PSD}}[n]. \tag{14}$$

Step 2) Set $a^{(k)}_n$ (the n-th element of the k-th ST codeword) equal to $c^{(k)}_m$ (the m-th element of the k-th Walsh codeword), where m is the index of the subset that the n-th chip belongs to, i.e.:

$$a^{(k)}_n = c^{(k)}_m \qquad \text{for } n \in G_m. \tag{15}$$

A set partition of a set S is a collection of disjoint subsets of S whose union is S [13].

Note that, since subsets $\{G_m; m = 1, \ldots, M\}$ form a partition for $\{1, \ldots, N\}$, all of the elements of the ST codeword dedicated to user k can be determined in (15) without any ambiguity. Figure 1 shows an illustration of the proposed code design for N = 22 and M = 4.

To prove the orthogonality we write

$$\begin{aligned} \sum_{n=1}^{N} a^{(i)}_n a^{(k)}_n p_{\mathrm{PSD}}[n] &= \sum_{m=1}^{M} \sum_{n \in G_m} a^{(i)}_n a^{(k)}_n p_{\mathrm{PSD}}[n] \\ &= \sum_{m=1}^{M} c^{(i)}_m c^{(k)}_m \tfrac{1}{M} \sum_{n=1}^{N} p_{\mathrm{PSD}}[n] \\ &= \begin{cases} \sum_{n=1}^{N} p_{\mathrm{PSD}}[n], & k = i \\ 0, & k \neq i. \end{cases} \end{aligned}$$

where we have used (14), (15), and (13) in the first, second, and third equalities, respectively.

C. Channel Partitioning

Step 1 of the proposed algorithm involves partitioning the signal PSD in N frequency chips into M subsets. This partitioning problem is a variation of a well-known problem in computer science called the number partitioning problem (NPP) [14], which involves partitioning a given set of positive integer numbers into two subsets such that the sum of the numbers in the subsets are equal. Unfortunately, exact partitioning as in (14) is not possible in general. That is, it is not possible to find a partition which satisfies (14) precisely with an arbitrary sequence of PSD values, $p_{\mathrm{PSD}}[\cdot]$. In this paper, we consider an approximate algorithm for chip partitioning, taking into account that no precise partitioning is possible in general. However, we propose a chip normalizing scheme (CNS) to restore the orthogonality property of the codes. With this modification the multiple-access noise is eliminated at the receiver.

A Simple Heuristic Algorithm for NPP: In this algorithm, we assign the N available channel gains to M subsets G_m (m = 1, ...,M) iteratively. First, we set $G_m = \{\}$ for all m. Then, we iterate through the channel gains N times. In each iteration, we assign the largest remaining channel gain to the subset with the smallest sum and omit the gain from the list of remaining channel gains until all of the gains are assigned to a subset. A pseudo-code representation of the algorithm is as follows, where R is the set of remaining chips and $\setminus$ denotes set subtraction.

Algorithm 1: Channel Partitioning Using the Proposed Heuristic Solution for the NPP

```
G_m ← {} and g_m ← 0 (for m = 1, ..., M);
R ← {1, ..., N};
while R ≠ {} do
    n ← index max_{n' ∈ R} p_PSD[n'];
    m ← index min_{m'} g_{m'};
    G_m ← G_m ⋃ {m};
    g_m ← g_m + p_PSD[n];
    R ← R \ {n};
```

The algorithm performs better for larger numbers of chips N and smaller lengths of Walsh code M. When precise partitioning is not possible, (14) will not hold and each receiver receives some MAI. Since M is lower-bounded by

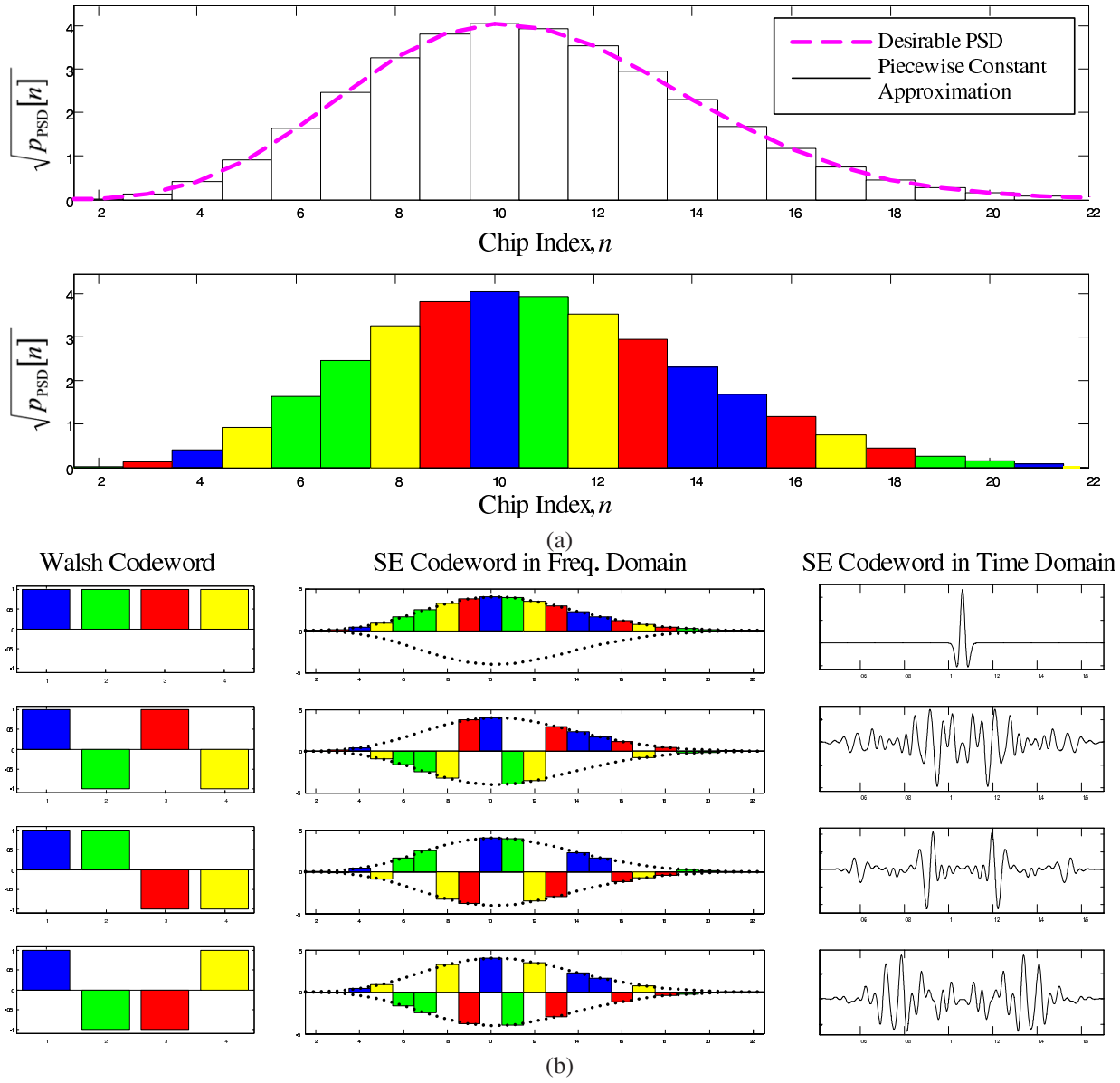

Fig. 1. An illustration of SE-code design for $N = 22$ and $M = 4$. Partitions 1 to 4 are indicated by colors blue, green, red, and yellow, respectively. (a) Step 1 of the algorithm: Partitioning the frequency chips into M subsets with equal aggregate gains. In this plot the sum square area in each color is (approximately) equal to that of other colors (b) Step 2 of the algorithm: Modulating the spectrum with Walsh codewords and the resultant codewords in the time domain obtained by IFT.

the total number of users, we predict that the performance will be degraded by increasing the total number of users (total number of required codewords). Furthermore, the MAI will be increased when the number of *active* users increases.

SNR with Imperfect Partitioning: If we define $E_{\rm p} \triangleq \sum_{n=1}^{N} p_{\rm PSD}[n]$ and $e_{i,k} \triangleq \sum_{n=1}^{N} a_n^{(i)} a_n^{(k)} p_{\rm PSD}[n]$, the multi-user (MU) signal-to-noise power ratio (SNR) with K active users has the following form

$$\mathrm{SNR}_{\rm MU} = \left[\mathrm{SNR}_1^{-1} + \sum_{k \neq i} \left| \frac{A_i}{A_k} \right|^2 \left(\frac{e_{i,k}}{E_{\rm p}} \right)^2 \right]^{-1} \tag{16}$$

where SNR_1 is the single-user SNR. Therefore, the MU-SNR is upper-bounded by $\left[\sum_{k \neq i} \left| \frac{A_i}{A_k} \right|^2 \left(\frac{e_{i,k}}{E_{\rm p}} \right)^2 \right]^{-1}$. It can be shown that $\mathrm{SNR}_{\rm MU}$ is the same for all of the users when we have perfect power control ($A_k = A_1$ for all k) and $K = M$.

D. Chip Normalizing to Counteract Imperfect Partitioning

As explained before, imperfect channel partitioning results in the non-orthogonality of the code. In this section, we propose a chip normalizing scheme (CNS) to preserve the orthogonality of our code. To do this, we modify the receiver structure by normalizing the signal in each chip n by $1/g_m$, i.e., the inverse of the gain corresponding to its subset G_m ($n \in G_m$). That is, we calculate the decision variable using

$$x_0^{\rm mdfd} = \mathcal{F}_{t=0}^{-1} \left\{ R^{(i)}(\omega) \, \overline{P_{\rm mdfd}^{(i)}(\omega)} \right\}, \tag{17}$$

where $P_{\rm mdfd}^{(i)}(\omega)$ is the modified spread-time filter matched to user k, defined as

$$P^{(i)}_{\text{mdfd}}(\omega) = \sum_{n=-N}^{N} \nu_n a_n^{(i)} \text{rect}\left(\tfrac{\omega}{\Omega_c} - n\right) p_{\text{PSD}}[n] \qquad (18)$$

where ν_n is the normalizing factor for chip n, defined as $\nu_n = 1/g_m$; for all $n \in G_m$.

Here, we prove that CNS restores the orthogonality property of the code. We have

$$\begin{aligned} I^{(k)}_{d_x,\text{A},\cancel{\text{W}}} &= d_0^{(k)} \eta_{d_x} \textstyle\sum_{n=1}^{N} \nu_n a_n^{(i)} a_n^{(k)} p_{\text{PSD}}[n] \\ &= d_0^{(k)} \eta_{d_x} \textstyle\sum_{m=1}^{M} \frac{1}{g_m} \sum_{n\in G_m} a_n^{(i)} a_n^{(k)} p_{\text{PSD}}[n] \\ &= d_0^{(k)} \eta_{d_x} \textstyle\sum_{m=1}^{M} \frac{1}{g_m} c_m^{(i)} c_m^{(k)} \sum_{n\in G_m} p_{\text{PSD}}[n] \\ &= d_0^{(k)} \eta_{d_x} \textstyle\sum_{m=1}^{M} c_m^{(i)} c_m^{(k)} \\ &= 0 \qquad \text{for } k \neq k'. \end{aligned}$$

As an unfortunate byproduct, chip normalizing at the receiver side can lead to noise enhancement as will be calculated in the next section. Note that our proposed zero-forcing (ZF) CNS does not maximize the SNR. Instead, a minimum mean-square error (MMSE) technique could be used to maximize the SNR. However, the ZF scheme is discussed here as it perfectly restores the orthogonality of the proposed SE-CDMA code.

Noise Power with CNS: Now, we calculate the effect of white Gaussian noise with two-sided power spectral density N_0 at the output of the modified receiver structure. From (17), we can write the contribution of the thermal noise to x_ℓ as follows

$$N_{\text{th}} = \mathcal{F}^{-1}_{t=0}\left\{N(\omega)\overline{P^{(i)}_{\text{mdfd}}(\omega)}\right\} \qquad (19)$$

where $N(\omega)$ is the FT of the equivalent lowpass thermal noise. Since the IFT is a linear operation, N_{th} is a Gaussian random variable. The mean of N_{th} is zero. The variance of N_{th} is calculated as

$$\begin{aligned} \sigma^2_{\text{th}} = \; & \underline{\text{E}\left\{|N_{\text{th}}|^2\right\}} = \tfrac{1}{4\pi^2}\textstyle\int_{\omega_1}\int_{\omega_2} \text{E}\left\{N(\omega_1)\overline{N(\omega_2)}\right\} \\ & \times P^{(i)}_{\text{mdfd}}(\omega_1) P^{(i)}_{\text{mdfd}}(\omega_2) \text{d}\omega_1 \text{d}\omega_2. \end{aligned}$$

Applying $\text{E}\left\{N(\omega_1)\overline{N(\omega_2)}\right\} = 2\pi N_0 \delta(\omega_1 - \omega_2)$, the noise variance simplifies to

$$\begin{aligned} \sigma^2_{\text{th}} &= \tfrac{1}{2\pi} N_0 \textstyle\int \sum_{n=1}^{N} p_{\text{PSD}}[n] v_n^2 \left|\text{rect}\left(\tfrac{\omega}{\Omega_c} - n\right)\right|^2 \text{d}\omega \\ &= \tfrac{1}{2\pi} N_0 \Omega_c \textstyle\sum_m [g_m]^{-2} \sum_{n\in G_m} p_{\text{PSD}}[n] \\ &= \tfrac{\Omega_c}{2\pi} N_0 \textstyle\sum_m 1/g_m. \end{aligned} \qquad (20)$$

SNR with CNS: Using CNS, the MAI is zero. Therefore, from (20) we have

$$\text{SNR}_{\text{CNS}} = \text{SNR}_1 \times \frac{M^2}{E_{\text{p}} \sum_{m=1}^{M} 1/g_m}. \qquad (21)$$

We see that the SNR is unbounded compared to the case without CNS (Eqn. (16)), but we have an SNR loss equal to $M^{-2} E_{\text{p}} \sum_{m=1}^{M} 1/g_m$.

IV. Systems with Windowing in AWGN Channel

For these systems, we consider the same assumptions as Sec. III. However, we assume that the transmitted signal is truncated with a time window with duration T_{w}. To avoid ISI, it is enough to select the symbol duration T_{s} greater than or equal to T_{w}.

A. System Analysis

In this section, we analyze the desired signal and MAI. We then propose orthogonal SE spreading codes for these systems using eigenvalue decomposition.

Desired Signal: The desired user's signal can be calculated by setting $k \leftarrow i$ in (8) or (9) and substituting into (10) as follows:

$$\begin{aligned} s_{\text{A,W}} = \; & d_0^{(i)} |A_i|^2 \textstyle\sum_{n=1}^{N}\sum_{m=1}^{N} a_n^{(i)} \overline{a_m^{(i)}} \\ & \times \sqrt{p_{\text{PSD}}[n]}\sqrt{p_{\text{PSD}}[m]}\varphi(0,n,m), \end{aligned} \qquad (22)$$

where $\varphi(\tau,n,m) \triangleq \mathcal{F}^{-1}_{t=\tau}\left\{\tilde{S}(\omega - n\Omega_c)\,\overline{\tilde{S}(\omega - m\Omega_c)}\right\}$. In the appendix, we have calculated $\varphi(\tau,n,m)$ as an explicit function of τ, n, and m.

MAI: To calculate the MAI, we substitute (8) or (9) into (10). By doing some algebra, we simply get

$$\begin{aligned} I^{(k)}_{d_x,\text{A,W}} = \; & d_0^{(k)} \eta_{d_x} \textstyle\sum_{n=1}^{N}\sum_{m=1}^{N} a_n^{(k)} \overline{a_m^{(i)}} \\ & \times \sqrt{p_{\text{PSD}}[n]}\sqrt{p_{\text{PSD}}[m]}\varphi(0,n,m), \end{aligned} \qquad (23)$$

where $\eta_{d_x} = \overline{A_i} A_k$ in the UL direction and $|A_i|^2$ in the DL direction. As it can be seen, unlike the result that we get by ignoring time windowing, terms corresponding to $n \neq m$ are not necessarily zero.

B. Code Design

By defining an $N \times N$ matrix $\boldsymbol{B}$ with elements

$$b_{nm} \triangleq [\boldsymbol{B}]_{nm} \triangleq \sqrt{p_{\text{PSD}}[n]}\sqrt{p_{\text{PSD}}[m]}\varphi(0,n,m), \qquad (24)$$

$I^{(k)}_{d_x,\text{ A, W}}$ can be re-written in the following matrix form

$$\begin{aligned} I^{(k)}_{d_x,\text{ A, W}} &= d_0^{(k)} \eta_{d_x} \textstyle\sum_{m=1}^{N}\sum_{n=1}^{N} \overline{a_m^{(i)}} a_n^{(k)} b_{mn} \\ &= d_0^{(k)} \eta_{d_x} \left[\boldsymbol{a}^{(i)}\right]^{\text{H}} \boldsymbol{B}\boldsymbol{a}^{(k)}, \end{aligned} \qquad (25)$$

where $\boldsymbol{a}^{(k)} = \left(a_1^{(k)}, a_2^{(k)}, \ldots, a_N^{(k)}\right)^{\text{T}}$ is the codeword for user k ($1 \le k \le K$), and $(\cdot)^{\text{T}}$ and $(\cdot)^{\text{H}}$ denote the transpose and conjugate transpose operations, respectively. To achieve a set of orthogonal codes, we take advantage of the eigenvalue decomposition of matrix $\boldsymbol{B}$. Let $\boldsymbol{q}_n$ and λ_n for $1 \le n \le N$ denote the n-th eigenvector and eigenvalue of $\boldsymbol{B}$, respectively. Since $\boldsymbol{B}$ is Hermitian, its eigenvalues are real but not necessarily positive. Once again we consider a set of orthogonal Walsh codes with M members and the chip partitioning scheme proposed in the previous section. Instead of partitioning the set of chips, however, we partition over the absolute eigenvalues of matrix $\boldsymbol{B}$. We partition the N eigenvectors into M subsets G_m with equal aggregate absolute eigenvalues corresponding to each subset. That is

$$\sum_{n\in G_m}|\lambda_n| = \tfrac{1}{M}\sum_{n=1}^{N}|\lambda_n| \tag{26}$$

Then, two SE codewords are calculated for each user k:

$$\boldsymbol{a}^{(k)} = \sum_{m=1}^{M} c_m^{(k)} \sum_{n\in G_m} \boldsymbol{q}_n \tag{27}$$

$$\tilde{\boldsymbol{a}}^{(k)} = \sum_{m=1}^{M} c_m^{(k)} \sum_{n\in G_m} \operatorname{sgn}(\lambda_n)\, \boldsymbol{q}_n \tag{28}$$

where $c_m^{(k)}$ is the m-th element of the k-th Walsh codeword and the sign function $\operatorname{sgn}(x)$ is 1 for $x \geq 0$ and -1 for $x < 0$. The SE codewords are real because the eigenvectors of $\boldsymbol{B}$ are real. One of the codewords should be used to encode the signal at the transmitter side while the other should be used to decode the signal at the receiver side.

Assuming that the codewords $\boldsymbol{a}^{(k)}$ are used at the transmitters and the codewords $\tilde{\boldsymbol{a}}^{(k)}$ are used at the receivers, here, we prove that the MAI due to any user k is zero. From (25), the contribution of the signal from user k is

$$\begin{aligned} I_{d_{\mathrm{x}},\,\mathrm{A},\,\mathrm{W}}^{(k)} &= d_0^{(k)}\eta_{d_{\mathrm{x}}}\left[\tilde{\boldsymbol{a}}^{(i)}\right]^{\mathrm{H}}\boldsymbol{B}\boldsymbol{a}^{(k)} \\ &= d_0^{(k)}\eta_{d_{\mathrm{x}}}\textstyle\sum_{m=1}^{M} c_m^{(i)}\sum_{n\in G_m}\operatorname{sgn}(\lambda_n)\,[\boldsymbol{q}_n]^{\mathrm{H}} \\ &\quad \times\boldsymbol{B}\textstyle\sum_{m'=1}^{M} c_{m'}^{(k)}\sum_{n'\in G_{m'}}\boldsymbol{q}_{n'} \\ &= d_0^{(k)}\eta_{d_{\mathrm{x}}}\textstyle\sum_{m=1}^{M}\sum_{m'=1}^{M} c_m^{(i)}c_{m'}^{(k)} \\ &\quad \times\textstyle\sum_{n\in G_m}\sum_{n'\in G_{m'}}\operatorname{sgn}(\lambda_n)\,[\boldsymbol{q}_n]^{\mathrm{H}}\boldsymbol{B}\boldsymbol{q}_{n'}. \end{aligned}$$

The term $[\boldsymbol{q}_n]^{\mathrm{H}}\boldsymbol{B}\boldsymbol{q}_{n'}$ is equal to λ_n for $n = n'$ and is zero otherwise. Since the subsets $\{G_m; 1 \leq m \leq M\}$ form a partition for the set {1, 2,..., N}, the condition $n = n'$ merely happens when $m = m'$. Thus, from the orthogonality of the Walsh codes and (26) we have

$$\begin{aligned} I_{d_{\mathrm{x}},\,\mathrm{A},\,\mathrm{W}}^{(k)} &= d_0^{(k)}\eta_{d_{\mathrm{x}}}\textstyle\sum_{m=1}^{M} c_m^{(1)}c_m^{(k)}\sum_{n\in G_m}\operatorname{sgn}(\lambda_n)\,\lambda_n \\ &= 0, \qquad \text{for } k \neq 1. \end{aligned}$$

C. Vector Normalizing to Counteract Imperfect Partitioning

In case (26) does not hold precisely, we can use a technique similar to the CNS to reach perfect orthogonal codes. The vector normalizing scheme (VNS) is implemented by using a modified codeword at the receiver side. The modified codeword for user k is defined by

$$\boldsymbol{a}_{\mathrm{mdfd}}^{(k)} \triangleq \sum_{m=1}^{M} c_m^{(k)}\tfrac{1}{\gamma_m}\sum_{n\in G_m}\operatorname{sgn}(\lambda_n)\,\boldsymbol{q}_n, \tag{29}$$

where $\gamma_m \triangleq \sum_{n\in G_m}|\lambda_n|$. We now show that the MAI goes to zero using the VNS. Substituting (29) and (27) into (25) and doing some algebra we get

$$\begin{aligned} I_{d_{\mathrm{x}},\,\mathrm{A},\,\mathrm{W}}^{(k)} &= d_0^{(k)}\eta_{d_{\mathrm{x}}}\textstyle\sum_{m=1}^{M}\sum_{m'=1}^{M} c_m^{(i)}c_{m'}^{(k)}\gamma_m^{-1} \\ &\quad \times\textstyle\sum_{n\in G_m}\sum_{n'\in G_{m'}}\operatorname{sgn}(\lambda_n)\,[\boldsymbol{q}_n]^{\mathrm{H}}\boldsymbol{B}\boldsymbol{q}_{n'} \\ &= d_0^{(k)}\eta_{d_{\mathrm{x}}}\textstyle\sum_{m=1}^{M}\sum_{m'=1}^{M} c_m^{(i)}c_{m'}^{(k)}\gamma_m^{-1}\sum_{n\in G_m}|\lambda_n| \\ &= d_0^{(k)}\eta_{d_{\mathrm{x}}}\textstyle\sum_{m=1}^{M}\sum_{m'=1}^{M} c_m^{(i)}c_{m'}^{(k)} \\ &= 0, \qquad \text{for } k \neq i. \end{aligned}$$

SNR with VNS: The contribution of the thermal noise to the decision variable is a zero-mean Gaussian random variable. The variance of the thermal noise for the DL direction depends on the noise power spectral density and the receiver structure and is calculated using

$$\sigma_{\mathrm{th}}^2 = |A_i|^2 N_0 \left[\boldsymbol{a}_{\mathrm{mdfd}}^{(i)}\right]^{\mathrm{H}}\boldsymbol{B}\boldsymbol{a}_{\mathrm{mdfd}}^{(i)}. \tag{30}$$

The contribution of the desired signal is

$$s_{\mathrm{A},\,\mathrm{W}} = |A_i|^2\left[\boldsymbol{a}_{\mathrm{mdfd}}^{(i)}\right]^{\mathrm{H}}\boldsymbol{B}\boldsymbol{a}^{(i)}. \tag{31}$$

Finally, the SNR of the modified receiver can be expressed in terms of the single user SNR using

$$\mathrm{SNR}_{\mathrm{mdfd}} = \frac{\mathrm{SNR}_1\left(\left[\boldsymbol{a}_{\mathrm{mdfd}}^{(i)}\right]^{\mathrm{H}}\boldsymbol{B}\boldsymbol{a}^{(i)}\right)^2}{\left(\left[\boldsymbol{a}_{\mathrm{mdfd}}^{(i)}\right]^{\mathrm{H}}\boldsymbol{B}\boldsymbol{a}_{\mathrm{mdfd}}^{(i)}\right)\left(\left[\boldsymbol{a}^{(i)}\right]^{\mathrm{H}}\boldsymbol{B}\boldsymbol{a}^{(i)}\right)}. \tag{32}$$

V. Systems With Windowing In Frequency Selective Fading Channels

In the general case, we consider time truncation by windowing and a random frequency selective multipath fading channel that can be different for different users. To avoid ISI for these systems, we require $T_{\mathrm{s}} = T_{\mathrm{w}} + T_{\mathrm{g}}$, where the guard time T_{g} is greater than the length of all of the users' channels. SE-CDMA systems demonstrate great flexibility in dealing with this complicated type of channel, and we will show that, depending on the transmission direction, cancellation of MAI can be done by modification of the spreading codes at the receiver or transmitter side. We first analyze the MAI and propose the required modifications, then we calculate the desired signal.

MAI: As shown in (8) and (9) the received pulse depends on the direction of transmission in frequency selective channels. Consequently, the required modifications are different based on the direction. For the DL, the transmitter side is left intact, but we use modified spreading codewords at the receiver side. For the UL, on the other hand, the modified spreading codewords are used at the transmitter side. This requires the base station to send channel state information (CSI) to the users using a feedback loop. We let $\boldsymbol{a}^{(k)}$ and $\boldsymbol{a}_{\mathrm{mdfd}}^{(k)}$ denote the spreading codewords of user k.

DL Direction: In the DL direction the MAI due to user k at the i-th user's receiver is

$$\begin{aligned} I_{\mathrm{DL},\,\mathrm{F},\,\mathrm{W}}^{(k)} &= d_0^{(k)}\textstyle\sum_{n=1}^{N}\sum_{n'=1}^{N} a_n^{(k)}\overline{a_{\mathrm{mdfd},\,n'}^{(i)}} \\ &\quad \times\sqrt{p_{\mathrm{PSD}}[n]}\sqrt{p_{\mathrm{PSD}}[n']} \\ &\quad \times\mathcal{F}_{t=0}^{-1}\left\{H_{\mathrm{DL}}^{(i)}(\omega)\,\tilde{S}(\omega - n\Omega_{\mathrm{c}})\,\overline{\tilde{S}(\omega - n'\Omega_{\mathrm{c}})}\right\} \\ &= d_0^{(k)}\left[\boldsymbol{a}_{\mathrm{mdfd}}^{(i)}\right]^{\mathrm{H}}\boldsymbol{B}_{\mathrm{DL}}^{(i)}\boldsymbol{a}^{(k)} \end{aligned} \tag{33}$$

where we have used F in the subscript for fading, and W to indicate that windowing is used. The i-th user's DL channel matrix $\boldsymbol{B}_{\mathrm{DL}}^{(i)}$ is defined by

$$\left[\boldsymbol{B}_{\mathrm{DL}}^{(i)}\right]_{nm} \triangleq \sqrt{p_{\mathrm{PSD}}[n]}\sqrt{p_{\mathrm{PSD}}[m]}\mathcal{F}_{t=0}^{-1}\left\{H_{\mathrm{DL}}^{(i)}(\omega) \times \tilde{S}(\omega - n\Omega_{\mathrm{c}})\,\overline{\tilde{S}(\omega - m\Omega_{\mathrm{c}})}\right\} \tag{34}$$

We need to determine $\boldsymbol{a}_{\mathrm{mdfd}}^{(i)}$ and $\boldsymbol{a}^{(k)}$ such that $I_{\mathrm{MA}}^{(k)}$ goes to zero for all values of $k \neq i$. We set $\boldsymbol{a}^{(k)}$ to be as defined in (27). However, the modified codeword for user i, $\boldsymbol{a}_{\mathrm{mdfd}}^{(i)}$, is defined by

$$\boldsymbol{a}_{\mathrm{mdfd}}^{(i)} \triangleq \sum_{m=1}^{M} c_m^{(i)} \frac{1}{\gamma_m^{(i)}} \sum_{n \in G_m} \boldsymbol{z}_n^{(i)}, \tag{35}$$

where

$$\gamma_m^{(i)} = \sum_{n \in G_m} [\boldsymbol{z}_n^{(i)}]^{\mathrm{H}} \boldsymbol{p}_n^{(i)}, \tag{36}$$

the N-dimensional vectors $\boldsymbol{p}_n^{(i)}$ are defined by

$$\boldsymbol{p}_n^{(i)} = \boldsymbol{B}_{\mathrm{DL}}^{(i)} \boldsymbol{q}_n; \text{ for } 1 \leq n \leq N, \tag{37}$$

and $\boldsymbol{z}_n^{(i)}$ is in the null space of the matrix $\boldsymbol{P}_n^{(i)} = \left[\boldsymbol{p}_1^{(i)} \middle| \cdots \middle| \boldsymbol{p}_{n-1}^{(i)} \middle| \boldsymbol{p}_{n+1}^{(i)} \middle| \cdots \middle| \boldsymbol{p}_N^{(i)}\right]^{\mathrm{T}}$. Note that since $\boldsymbol{P}_n^{(i)}$ is an $(N-1) \times N$ matrix, $\boldsymbol{z}_n^{(i)}$ will be an N-dimensional vector. Without loss of generality, we assume that the Euclidian norm of $\boldsymbol{z}_n^{(i)}$ is one and $[\boldsymbol{z}_n^{(i)}]^{\mathrm{H}} \boldsymbol{p}_n^{(i)} \geq 0$.

In the DL direction, the spreading code at the transmitter side is obtained by partitioning the power of the SE pulse without taking into account the user dependent frequency selective channels. As before, perfect partitioning might not be possible. At the receiver side, each user i compensates for the effect of its own channel as well as imperfect partitioning using $\boldsymbol{a}_{\mathrm{mdfd}}^{(i)}$ defined in (35). That is, VNS is implied in the construction of $\boldsymbol{a}_{\mathrm{mdfd}}^{(i)}$.

We now show that the MAI goes to zero using the proposed technique. Substituting (35) and (36) into (33) and doing some algebra we get

$$\begin{aligned} I_{\mathrm{DL,\,F,\,W}}^{(k)} = \; & d_0^{(k)} \sum_{m=1}^{M} \sum_{m'=1}^{M} c_m^{(i)} c_{m'}^{(k)} \frac{1}{\gamma_m^{(i)}} \\ & \times \sum_{n \in G_m} \sum_{n' \in G_{m'}} [\boldsymbol{z}_n^{(i)}]^{\mathrm{H}} \overbrace{\boldsymbol{B}_{\mathrm{DL}}^{(i)} \boldsymbol{q}_{n'}}^{\boldsymbol{p}_{n'}^{(i)}}. \end{aligned} \tag{38}$$

Since $\boldsymbol{z}_n^{(i)}$ is in the null space of the matrix $\boldsymbol{P}_n^{(i)} = \left[\boldsymbol{p}_1^{(i)} \middle| \cdots \middle| \boldsymbol{p}_{n-1}^{(i)} \middle| \boldsymbol{p}_{n+1}^{(i)} \middle| \cdots \middle| \boldsymbol{p}_N^{(i)}\right]^{\mathrm{T}}$, $[\boldsymbol{z}_n^{(i)}]^{\mathrm{H}} \boldsymbol{p}_{n'}^{(i)}$ is non-zero merely for $n = n'$. Note that with the partitioning made in (26), $n = n'$ necessitates $m = m'$. Therefore

$$\begin{aligned} I_{\mathrm{DL,\,F,\,W}}^{(k)} &= d_0^{(k)} \sum_{m=1}^{M} c_m^{(k)} c_m^{(i)} \frac{1}{\gamma_m^{(i)}} \sum_{n \in G_m} [\boldsymbol{z}_n^{(i)}]^{\mathrm{H}} \boldsymbol{p}_n^{(i)} \\ &= d_0^{(k)} \sum_{m=1}^{M} c_m^{(k)} c_m^{(i)} = 0, \end{aligned} \tag{39}$$

where we have used (36) and the orthogonality property of the Walsh codes in the last two equalities.

Note that we can calculate the elements of $\boldsymbol{B}_{\mathrm{DL}}^{(i)}$ explicitly for multipath fading channels using the results obtained in the appendix. The baseband impulse response of a multipath fading channel with L paths can be represented by [12]:

$$h_{\mathrm{DL}}^{(i)}(t) = \sum_{\ell=1}^{L} \alpha_\ell^{(i)} e^{-j2\pi f_{\mathrm{c}} \tau_\ell^{(i)}} \delta\left(t - \tau_\ell^{(i)}\right), \tag{40}$$

where $\alpha_\ell^{(i)}$ and $\tau_\ell^{(i)}$ are the amplitude and delay of the ℓ-th component, respectively, and $\delta(\cdot)$ is the Dirac's impulse function. Taking the FT, we obtain

$$H_{\mathrm{DL}}^{(i)}(\omega) = \sum_{\ell=1}^{L} \alpha_\ell^{(i)} \exp\left(-j[2\pi f_{\mathrm{c}} + \omega]\tau_\ell^{(i)}\right). \tag{41}$$

By substituting (40) into (34) we obtain

$$\begin{aligned} \left[\boldsymbol{B}_{\mathrm{DL}}^{(i)}\right]_{nm} = \; & \sqrt{p_{\mathrm{PSD}}[n]}\sqrt{p_{\mathrm{PSD}}[m]} \\ & \times \textstyle\sum_{\ell=1}^{L} \alpha_\ell^{(i)} e^{-j2\pi f_{\mathrm{c}} \tau_\ell^{(i)}} \varphi(-\tau_\ell^{(i)}, n, m), \end{aligned} \tag{42}$$

where $\varphi(\tau, n, m) \triangleq \mathcal{F}_{t=\tau}^{-1}\left\{\tilde{S}(\omega - n\Omega_{\mathrm{c}})\,\overline{\tilde{S}(\omega - m\Omega_{\mathrm{c}})}\right\}$ is calculated in the appendix.

UL Direction: In this case, a modified code should be used at the transmitter side. The MAI due to user k at the i-th user's receiver is

$$\begin{aligned} I_{\mathrm{UL,\,F,\,W}}^{(k)} = \; & d_0^{(k)} \textstyle\sum_{n=1}^{N} \sum_{n'=1}^{N} a_{\mathrm{mdfd},n}^{(k)} \overline{a_{n'}^{(i)}} \\ & \times \sqrt{p_{\mathrm{PSD}}[n]}\sqrt{p_{\mathrm{PSD}}[n']} \\ & \times \mathcal{F}_{t=0}^{-1}\left\{H_{\mathrm{UL}}^{(k)}(\omega)\,\tilde{S}(\omega - n\Omega_{\mathrm{c}})\,\overline{\tilde{S}(\omega - n'\Omega_{\mathrm{c}})}\right\} \\ = \; & d_0^{(k)} \left[\boldsymbol{a}^{(i)}\right]^{\mathrm{H}} \boldsymbol{B}_{\mathrm{UL}}^{(k)} \boldsymbol{a}_{\mathrm{mdfd}}^{(k)}, \end{aligned} \tag{43}$$

where the k-th user's UL channel matrix $\boldsymbol{B}_{\mathrm{UL}}^{(k)}$ is defined by

$$\left[\boldsymbol{B}_{\mathrm{UL}}^{(k)}\right]_{nm} \triangleq \sqrt{p_{\mathrm{PSD}}[n]}\sqrt{p_{\mathrm{PSD}}[m]}\mathcal{F}_{t=0}^{-1}\left\{H_{\mathrm{UL}}^{(k)}(\omega) \times \tilde{S}(\omega - n\Omega_{\mathrm{c}})\,\overline{\tilde{S}(\omega - m\Omega_{\mathrm{c}})}\right\} \tag{44}$$

The similarity of (43) and (33) is rather obvious. Likewise, the modified codeword can be designed by:

$$\boldsymbol{a}_{\mathrm{mdfd}}^{(k)} = \sum_{m=1}^{M} c_m^{(k)} \frac{1}{\gamma_m^{(k)}} \sum_{n \in G_m} \boldsymbol{z}_n^{(k)}, \tag{45}$$

where $\gamma_m^{(k)} = \sum_{n \in G_m} [\boldsymbol{p}_n^{(k)}]^{\mathrm{H}} \boldsymbol{z}_n^{(k)}$, the N-dimensional vectors $\boldsymbol{p}_n^{(k)}$ are defined by $\boldsymbol{p}_n^{(k)} = \left[\boldsymbol{B}_{\mathrm{UL}}^{(k)}\right]^{\mathrm{H}} \boldsymbol{q}_n$; for $1 \leq n \leq N$, and $\boldsymbol{z}_n^{(k)}$ is in the null space of the matrix $\boldsymbol{P}_n^{(i)} = \left[\boldsymbol{p}_1^{(i)} \middle| \cdots \middle| \boldsymbol{p}_{n-1}^{(i)} \middle| \boldsymbol{p}_{n+1}^{(i)} \middle| \cdots \middle| \boldsymbol{p}_N^{(i)}\right]^{\mathrm{T}}$. Following the same steps as (38) and (39), it is easy to show that the MAI is zero using the proposed codes. The proof is omitted here for brevity. Note that each transmitter k only requires its own CSI $H_{\mathrm{UL}}^{(k)}(\omega)$ to calculate the modified codeword $\boldsymbol{a}_{\mathrm{mdfd}}^{(k)}$. For a multipath fading channel with L paths (see (40)), this reduces to L real-valued numbers for path delays and L complex-valued numbers for path amplitudes. Alternatively, the base station can calculate $\boldsymbol{a}_{\mathrm{mdfd}}^{(k)}$ and send it to the transmitter using the downlink channel as a feedback loop.

Desired Signal: By setting $k = i$ in (33) and (43) we can easily compute the desired signal for the DL and UL directions as $s_{\text{DL, F, W}} = d_0^{(i)} \left[\boldsymbol{a}_{\text{mdfd}}^{(i)}\right]^{\text{H}} \boldsymbol{B}_{\text{DL}}^{(i)} \boldsymbol{a}^{(i)}$ and $s_{\text{UL, F, W}} = d_0^{(i)} \left[\boldsymbol{a}^{(i)}\right]^{\text{H}} \boldsymbol{B}_{\text{UL}}^{(i)} \boldsymbol{a}_{\text{mdfd}}^{(i)}$, respectively.

VI. Simulation Results

We have simulated the proposed systems in both AWGN and frequency-selective multipath fading channels. We have studied the effects of windowing and channel estimation error in the simulations as well. We have simulated the systems for different numbers of frequency chips N and different Walsh code lengths M. For the AWGN channel, we assume a Gaussian-shaped transmit spectrum $p_{\text{PSD}}[\cdot]$ with a 3 dB bandwidth of 20 MHz located at $f_{\text{c}} = 12$ GHz. To receive 99% of the spectral energy, a total bandwidth of *BW*=43.753 MHz is considered.

For the multipath fading channel, we assume the channel model in (40) with L = 3. We assume $\tau_2^{(k)} - \tau_1^{(k)}$ and $\tau_3^{(k)} - \tau_1^{(k)}$ have independent uniform distributions over 0 to 100 ns. We assume that the amplitude of the ℓ-th component $\alpha_\ell^{(k)}$ has a Rayleigh distribution with a negative exponentially-decaying power with delay governed by $\text{E}\{|\alpha_\ell^{(k)}|^2\} = 10^{-(\tau_\ell - \tau_1)/50 \text{ ns}}$. This means that the average power of a component arriving at $\tau_\ell^{(k)} - \tau_1^{(k)}$ = 100 ns is one-hundredth of that of the first component. We assume a quasi-static channel such that the delay and amplitude of the components are constant during a symbol period but vary independently from symbol to symbol. Other channel parameters such as the carrier frequency, bandwidth, and the received pulse shape are assumed to be the same as those described for the AWGN channel. For both channels, we assume that the transceivers have perfect CSI.

The performance of the system in the AWGN channel is studied in Figs. 2 to 6. Figure 2 illustrates the SNR upper-bound (signal-to-MAI ratio, see Eqn. (16)) for synchronous uncoded (i.e., coded using random sequences) SE-systems without windowing as a function of the number of chips N from 1 to 256 for K = 4, 8, 16, and 32 users. As expected, the SNR is linearly proportional to N and inversely proportional to the number of interfering users $K - 1$. Figure 3 shows a similar graph for a coded SE-system. As it can be seen, the SNR bound (in dB) increases exponentially with N, such that the SNR is almost unbounded when $N \geq 2M$. It can also be seen that increasing the Walsh code length M increases the number of chips N required to achieve a certain SNR linearly. For example, to achieve an SNR of 10 dB, we need almost N = 6, 12, 23, and 46 chips for code lengths of M = 4, 8, 16, and 32, respectively. Clearly, the proposed orthogonal codes improve the performance of the system drastically.

Figure 4 displays the SNR loss (Eqn. (21)) versus the number of chips using the CNS. As it can be seen, the SNR loss is below 4 dB in all cases, and increasing N rapidly decreases the loss to 0 dB. If the number of chips N is over twice the Walsh code length M, the SNR loss is almost negligible. From Figs. 3 and 4, it is clear that the smaller the Walsh code length, the better the performance. However, it is important to note that the maximum number of users

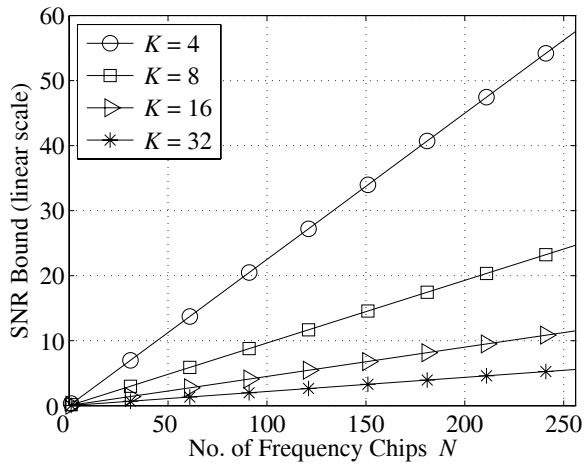

Fig. 2. Upper bound on SNR (linear scale) vs. number of frequency chips N for an uncoded SE-CDMA system without windowing in AWGN channel for various numbers of users K.

is bounded by M. Therefore, to support a higher number of users, we need to increase the Walsh code length, resulting in a SNR loss.

Figures 5 and 6 demonstrate the performance of coded systems with time windowing in the AWGN channel without and with the vector normalizing scheme (VNS), respectively. The window length is assumed to be $T_{\text{w}} = 2\pi/\Omega_{\text{c}}$. As it can be seen, unlike systems without windowing (Fig. 3), the SNR bound does not grow exponentially with the number of chips, particularly for SNR $>$ 30 dB where the curves are not monotonically increasing. This is because the absolute eigenvalues of R and the variations in the sum gains for each partition might not decrease evenly and monotonically with N. However, as it can be seen in Fig. 6, when VNS is employed the system performance is the same as systems without windowing using CNS. In addition, when $N \geq 2M$ the SNR loss is negligible, which is the same result as that for systems with non-overlapping chips.

Turning now to the frequency selective channel, Figs. 7 and 8 show the BER vs. SNR simulation results for BPSK SE-CDMA systems without windowing and with windowing, respectively, with N = 64 and K = 16. As it can be seen in Fig. 7, the proposed code considerably improves the performance of the system by decreasing the BER floor by about two orders of magnitude. More importantly, using CNS, the system removes the BER floor and shows only an approximate 1.5 dB loss compared to a single-user system with an optimal matched filter receiver.

The BER curves in Fig. 8 show the same behavior for a system with windowing. However, the gains of the proposed orthogonal code and VNS are smaller in this system compared to the system with non-overlapping chips. As it can be seen, using the proposed orthogonal codes, the BER floor decreases by less than one order of magnitude, and the SNR gap of the system using VNS is about 5.5 dB, which shows a

We can double the number of orthogonal codewords by mapping them to the imaginary domain as explained in [15]. In that case, the data symbols, however, should be real valued.

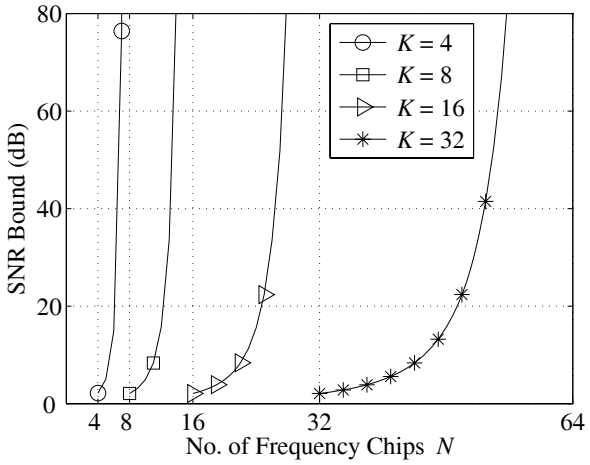

Fig. 3. Upper bound on SNR vs. number of frequency chips N for a coded system without windowing in AWGN channel for various numbers of users K. The Walsh code length M is assumed to be equal to K.

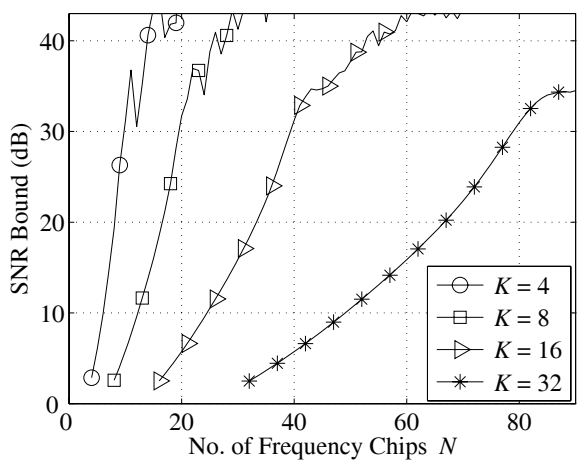

Fig. 5. Upper bound on SNR vs. number of frequency chips N for a windowed coded system with $T_{\rm w} = 2\pi/\Omega_{\rm c}$ in AWGN channel for various numbers of users K. The Walsh code length M is assumed to be equal to K.

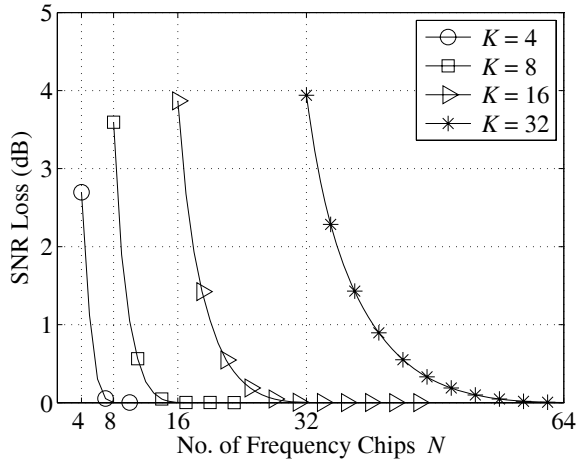

Fig. 4. SNR loss vs. number of frequency chips N using CNS for a coded system without windowing in AWGN channel for various numbers of users K. The Walsh code length M is assumed to be equal to K.

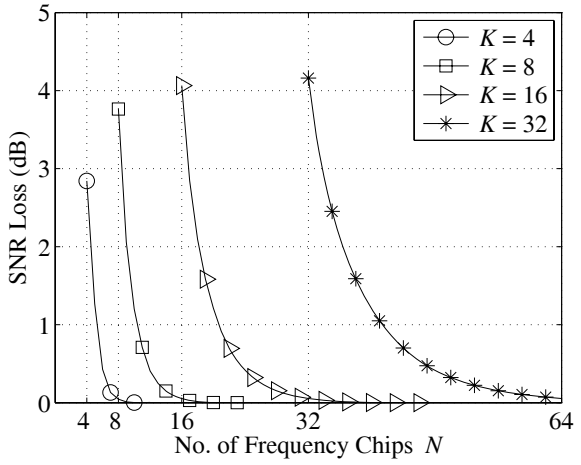

Fig. 6. SNR loss vs. number of frequency chips N using vector normalizing scheme for a windowed coded system with $T_{\rm w} = 2\pi/\Omega_{\rm c}$ in AWGN channel for various numbers of users K. The Walsh code length M is assumed to be equal to K.

4 dB inferior performance compared to a system with non-overlapping chips using CNS.

Comparing Figs. 7 and 8, we find that the BER of the randomly coded system in the frequency selective channel is smaller when windowing is used. However, this is the opposite for the coded system without CNS/VNS. This is because in fading channels, the ISI terms do not go to zero (except for the first multipath component) when windowing is not used. However, they are zero when windowing is used, because we assume that the guard time is longer than the channel impulse response. Therefore, an uncoded system with windowing does not see any ISI and achieves smaller BERs than an uncoded system without windowing. However, in the coded case, partitioning can be performed better for the system without windowing (as shown in Figs. 4 and 6) resulting in less multi-user interference. Since multi-user interference is the dominant term in the presence of $K-1=15$ interfering users, the BER is smaller for the system without windowing than the system with windowing.

Using our simulation results, we may also compare the performance of the CC-ST-CDMA system in [8] with our system. As explained in the introduction, the maximum number of supported users for CC-ST-CDMA is $\sqrt[3]{P_{\rm g}}$, where $P_{\rm g}$ is the (congregated) processing gain of the system. This is much smaller than the number of users that can be supported by our system which is about $N/2 = P_{\rm g}/2$ in AWGN channels. Alternatively, for K users the processing gain or the number of chips can be much larger than $2K$. Therefore, the system probably achieves the single user performance even without applying the CNS or VNS. Analytical results in [8] show that the use of CC codes reduces the multiuser interference in a flat fading channel by a factor of $4G\sqrt{G} = 4P_{\rm g}$ ([8] Eqn. (18)) for an asynchronous system in a flat fading channel. A random code reduces the multiuser interference by $P_{\rm g}$, which is a lower bound for the performance of our codes in

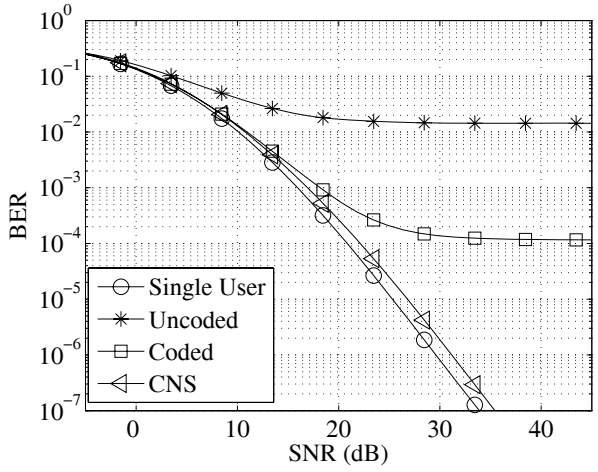

Fig. 7. BER vs. SNR for various BPSK SE-CDMA systems (Single User: Single-user; Uncoded: Uncoded system with K = 16; Coded: Coded system with $K = M$ = 16; CNS: Coded system using CNS with $K = M$ = 16) without windowing in an indoor multipath fading channel for N = 64.

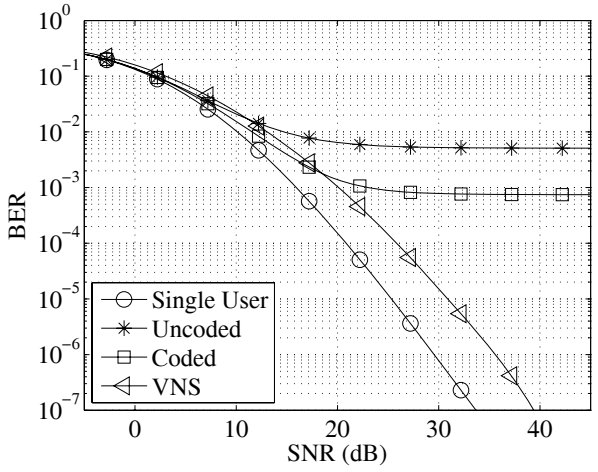

Fig. 8. BER vs. SNR for various BPSK SE-CDMA systems (Single User: Single-user; Uncoded: Uncoded system with K = 16; Coded: Coded system with $K = M$= 16; VNS: Coded system using VNS with $K = M$= 16) with windowing in an indoor multipath fading channel for N = 64.

the asynchronous scenarios. Therefore, CC codes reduce the multiuser interference 6 dB more than the proposed codes in the asynchronous case.

VII. Conclusion

In this paper, we have designed novel orthogonal spreading codes for SE-CDMA. Design of the proposed codes is based on binary orthogonal Walsh codes and a variation of the number partitioning problem (NPP). We have proposed a simple heuristic algorithm to solve the NPP for our application. Perfect channel partitioning is not possible when the channel is different for different users such as with time-varying fading channels. We have proposed chip and vector normalizing schemes to restore the orthogonality of our codes in the presence of imperfect partitioning for systems without or with windowing, respectively. Our simulation results show that, by increasing the number of frequency chips, both systems achieve the performance of an interference-free channel when CNS or VNS are used. Using the CNS, however, the system achieves the optimal performance using a smaller number of chips, resulting in a higher achievable bit-rate for the system than the unnormalized case. Finally, although SE-CDMA systems with overlapping chips can reach higher bit-rates using shorter time windows, SE-CDMA systems with non-overlapping chips perform better in cancelling MAI using the proposed codes.

Appendix: Calculation of $\varphi(\tau, n, m)$

By definition $\varphi(\tau,n,m) \triangleq \mathcal{F}_{t=\tau}^{-1}\left\{\tilde{S}\left(\omega - n\Omega_{\mathrm{c}}\right)\overline{\tilde{S}\left(\omega - m\Omega_{\mathrm{c}}\right)}\right\}$. From (3) and (4), the IFT of $\tilde{S}(\omega)$ is calculated by $\tilde{s}(t) = \frac{\Omega_{\mathrm{c}}}{2\pi}\mathrm{sinc}\left(\frac{\Omega_{\mathrm{c}}}{2\pi}t\right)\ \mathrm{rect}\left(\frac{t}{T_{\mathrm{w}}}\right)$. Using the convolution property of FT, we obtain (46) located at the top of the next page, where we obtain the last equality by setting the integral limits to the non-zero parts of $\mathrm{rect}\left(\frac{u}{T_{\mathrm{w}}}\right)$ and $\mathrm{rect}\left(\frac{t-u}{T_{\mathrm{w}}}\right)$. It is straightforward but tedious to show that the indefinite integral

$$I(u) = \int \mathrm{sinc}\left(\tfrac{\Omega_{\mathrm{c}}}{2\pi}u\right)\mathrm{sinc}\left(\tfrac{\Omega_{\mathrm{c}}}{2\pi}\left[t-u\right]\right)e^{j(n-m)\Omega_{\mathrm{c}}u}\mathrm{d}u$$

can be expressed in terms of the exponential integral $\mathrm{E}_1(x) \triangleq \int_x^\infty \frac{\exp(-t)}{t}\mathrm{d}t$ as follows:

$$\begin{aligned} I(u) \;=\; & \tfrac{1}{t\,\Omega_{\mathrm{c}}^2}\left\{\exp\left(\tfrac{1}{2}j\left[1+2\left(n-m\right)\right]\Omega_{\mathrm{c}}t\right)\right. \\ & \times \mathrm{E}_1\left(j\left(1+n-m\right)\Omega_{\mathrm{c}}\left(u-t\right)\right) \\ & -\exp\left(-\tfrac{1}{2}j\Omega_{\mathrm{c}}t\right)\mathrm{E}_1\left(j\left(-1-n+m\right)\Omega_{\mathrm{c}}u\right) \\ & -2\cos\left(\tfrac{1}{2}\Omega_{\mathrm{c}}t\right)\exp\left(j\left(n-m\right)\Omega_{\mathrm{c}}t\right) \\ & \times \mathrm{E}_1\left(j\left(n-m\right)\Omega_{\mathrm{c}}\left(u-t\right)\right) \\ & +2\cos\left(\tfrac{1}{2}\Omega_{\mathrm{c}}t\right)\mathrm{E}_1\left(j\left(-n+m\right)\Omega_{\mathrm{c}}u\right) \\ & +\exp\left(\tfrac{1}{2}j\left[-1+2\left(n-m\right)\right]\Omega_{\mathrm{c}}t\right) \\ & \times \mathrm{E}_1\left(j\left(1-n+m\right)\Omega_{\mathrm{c}}\left(u-t\right)\right) \\ & \left.-\exp\left(\tfrac{1}{2}j\Omega_{\mathrm{c}}t\right)\mathrm{E}_1\left(j\left(1-n+m\right)\Omega_{\mathrm{c}}u\right)\right\}. \end{aligned}$$

Thus

$$\varphi(\tau,n,m) = \tfrac{\Omega_{\mathrm{c}}^2}{4\pi^2}e^{jm\Omega_{\mathrm{c}}\tau} \times \begin{cases} I\left(\frac{T_{\mathrm{w}}}{2}\right) - I\left(\tau - \frac{T_{\mathrm{w}}}{2}\right), & 0 < \tau \le T_{\mathrm{w}} \\ I\left(\tau + \frac{T_{\mathrm{w}}}{2}\right) - I\left(-\frac{T_{\mathrm{w}}}{2}\right), & -T_{\mathrm{w}} \le \tau \le 0 \\ 0, & |\tau| > T_{\mathrm{w}}. \end{cases} \quad (47)$$

References

[1] P. M. Crespo, M. L. Honig, and J. A. Salehi, "Spread-time code-division multiple access," *IEEE Trans. Commun.*, vol. 43, no. 6, pp. 2139-2148, June 1995.

[2] M. G. Shayesteh, J. A. Salehi, and M. Nasiri-Kenari, "Spread-time CDMA resistance in fading channels," *IEEE Trans. Wireless Commun.*, vol. 2, no. 3, pp. 446-458, May 2003.

[3] M. Farhang and J. A. Salehi, "Spread-time/time-hopping UWB CDMA communication," in *Proc. IEEE Int'l. Symp. Commun. Inf. Tech.*, vol. 2, Oct. 2004, pp. 1047-1050.

[4] C. R. C. M. daSilva and L. B. Milstein, "Spectral-encoded UWB communication systems: real-time implementation and interference suppression," *IEEE Trans. Commun.*, vol. 53, no. 8, pp. 1391-1401, Aug. 2005.

[5] S. Mashhadi and J. A. Salehi, "Novel probabilistic bounds on power level profile of spectrally-encoded spread-time CDMA signals," *IEEE Trans. Wireless Commun.*, vol. 8, no. 5, pp. 2296-2301, May 2009.

[6] ——, "UWB spectrally-encoded spread-time CDMA in the presence of multiple Gaussian interferences: RAKE receiver and three-level codes," *IEEE Trans. Commun.*, vol. 56, no. 12, pp. 2178-2189, Dec. 2008.

$$\begin{aligned}
\mathcal{F}_t^{-1}\left\{\tilde{S}\left(\omega-n\Omega_{\mathrm{c}}\right)\overline{\tilde{S}\left(\omega-m\Omega_{\mathrm{c}}\right)}\right\} &= \left[\tfrac{\Omega_{\mathrm{c}}}{2\pi}\mathrm{sinc}\left(\tfrac{\Omega_{\mathrm{c}}}{2\pi}t\right)\exp\left(jn\Omega_{\mathrm{c}}t\right)\ \mathrm{rect}\left(\tfrac{t}{T_{\mathrm{w}}}\right)\right]\otimes\left[\tfrac{\Omega_{\mathrm{c}}}{2\pi}\mathrm{sinc}\left(-\tfrac{\Omega_{\mathrm{c}}}{2\pi}t\right)\exp\left(jm\Omega_{\mathrm{c}}t\right)\ \mathrm{rect}\left(-\tfrac{t}{T_{\mathrm{w}}}\right)\right] \\
&= \tfrac{\Omega_{\mathrm{c}}^2}{4\pi^2}\int_{u=-\infty}^{+\infty}\mathrm{sinc}\left(\tfrac{\Omega_{\mathrm{c}}}{2\pi}u\right)\exp\left(jn\Omega_{\mathrm{c}}u\right)\ \mathrm{rect}\left(\tfrac{u}{T_{\mathrm{w}}}\right)\mathrm{sinc}\left(\tfrac{\Omega_{\mathrm{c}}}{2\pi}\left[t-u\right]\right)\exp\left(jm\Omega_{\mathrm{c}}\left[t-u\right]\right)\ \mathrm{rect}\left(\tfrac{t-u}{T_{\mathrm{w}}}\right)\mathrm{d}u \\
&= \tfrac{\Omega_{\mathrm{c}}^2}{4\pi^2}e^{jm\Omega_{\mathrm{c}}t}\begin{cases}\int_{u=t-\frac{T_{\mathrm{w}}}{2}}^{\frac{T_{\mathrm{w}}}{2}}\mathrm{sinc}\left(\tfrac{\Omega_{\mathrm{c}}}{2\pi}u\right)\ \mathrm{sinc}\left(\tfrac{\Omega_{\mathrm{c}}}{2\pi}\left[t-u\right]\right)\exp\left(j\left(n-m\right)\Omega_{\mathrm{c}}u\right)\mathrm{d}u, & 0\le t\le T_{\mathrm{w}} \\ \int_{u=-\frac{T_{\mathrm{w}}}{2}}^{t+\frac{T_{\mathrm{w}}}{2}}\mathrm{sinc}\left(\tfrac{\Omega_{\mathrm{c}}}{2\pi}u\right)\ \mathrm{sinc}\left(\tfrac{\Omega_{\mathrm{c}}}{2\pi}\left[t-u\right]\right)\exp\left(j\left(n-m\right)\Omega_{\mathrm{c}}u\right)\mathrm{d}u, & -T_{\mathrm{w}}\le t<0 \\ 0, & |t|>T_{\mathrm{w}},\end{cases}
\end{aligned} \tag{46}$$

[7] M. G. Shayesteh and M. Nasiri-Kenari, "Multiple-access performance analysis of combined time-hopping and spread-time CDMA system in the presence of narrowband interference," *IEEE Trans. Veh. Technol.*, vol. 58, no. 3, pp. 1315-1328, Mar. 2009.

[8] L. Lu and V. K. Dubey, "Performance of a complete complementary code-based spread-time CDMA system in a fading channel," *IEEE Trans. Veh. Technol.*, vol. 57, no. 1, pp. 250-259, Jan. 2008.

[9] M. G. Shayesteh, M. Nasiri-Kenari, and J. A. Salehi, "Spread-time CDMA performance with and without windowing," in *Proc. IEEE Int'l. Symp. Personal, Indoor, Mobile Radio Commun.*, vol. 3, Sep. 2002, pp. 1166-1170.

[10] J. A. Salehi, A. M. Weiner, and J. P. Heritage, "Coherent ultrashort light pulse code-division multiple access communication systems," *J. Lightw. Technol.*, vol. 8, no. 3, pp. 478-491, Mar. 1990.

[11] N. Suehiro, "Complete complementary code composed of n-multipleshift orthogonal sequences," *Trans. IECE Jpn.*, vol. J65-A, pp. 1247-1253, Dec. 1982.

[12] J. G. Proakis, *Digital Communications*. McGraw-Hill, 2001.

[13] E. W. Weisstein, "Set partition," Mathword-A Wolfram web resource. Available: http://mathworld.wolfram.com/SetPartition.html

[14] B. Alidaee, F. Glover, G. A. Kochenberger, and C. Rego, "A new modeling and solution approach for the number partitioning problem," *J. Applied Math. Decision Sciences*, no. 2, pp. 113-121, 2005.

[15] A. R. Forouzan and L. M. Garth, "Orthogonal code design for spectrally encoded CDMA systems," in *Proc. IEEE Global Telecom. Conf.*, Nov. 2007, pp. 4434-4439.



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