Optimal Dynamic Spectrum Management for DSL Interference/Broadcast Channel

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***Abstract*—In this paper, we consider optimal dynamic spectrum management (DSM) for a downstream (DS) DSL scenario in which users are divided into a few separate groups, where vector encoding based signal coordination can be applied in each group and spectrum coordination is possible for all users. This can be seen as a mixed interference/broadcast channel (IF/BC) scenario. We investigate several candidates for vector encoding the signals inside the groups, including the linear zero-forcing (ZF) pre-compensator, ZF Tomlinson-Harashima pre-coder (THP), optimal linear pre-compensator (OLP), and THP with optimal transmit filters. The calculation of the optimal transmit filters (for both the linear pre-compensator and THP) is a non-convex problem. To resolve this problem, we develop a generalized duality between the IF/BC and IF/multiple-access channel (MAC). In order to achieve the highest data rates, optimal spectrum balancing (OSB) is applied to all users on top of the vector encoding inside the groups. Simulation results show that the grouped THP with optimal transmit filters achieves considerably higher bit rates than the other schemes. The resulting algorithm is referred to as the IF/BC-OSB algorithm, and encompasses the OSB (i.e., IF-OSB) and the earlier developed BC-OSB algorithm as special cases.**

***Index Terms*—Digital subscriber line (DSL), dynamic spectrum management (DSM), broadcast channel (BC), interference channel, BC-MAC duality, dirty paper coding (DPC), Tomlinson-Harashima pre-coder (THP), resource allocation, vectoring.**

I. Introduction

Far-end crosstalk (FEXT) is one of the dominant impairments in very-high-speed digital subscriber line (VDSL) networks. Promising crosstalk mitigation and canceling techniques have been proposed in the last decade to counteract crosstalk based on spectrum coordination and signal coordination. These techniques are jointly referred to as dynamic spectrum management (DSM) techniques. When the bit rate of the users in the network are configured centrally but the computation of the transmit spectrum of the users is left uncoordinated, we obtain a level 1 DSM scheme, also known as DSM 1. When the transmit spectrum of the users is controlled by a spectrum management center (SMC), we obtain a DSM 2 scheme. DSM 2 schemes may achieve higher rates for the users compared to DSM 1 schemes. Finally, when the signals of the users are modulated and/or demodulated jointly using so-called vectoring schemes, we obtain DSM 3. DSM 3 is capable of achieving considerably higher bit rates than DSM 2. In state-of-the-art DSM 3 schemes, also the transmit spectra and bit rate of the users may be managed by an SMC.

DSM 3 requires spectrum coordination as well as signal coordination among all users. However, sometimes due to the excessive complexity or physical limitations (e.g. as in joint CO/RT deployments), DSM 3 is not possible for all users. Hence, the users may be divided into separate groups, where signal and spectrum coordination is possible inside each group but only spectrum coordination is possible for the users belonging to different groups. For these cases, so-called joint DSM 2/3 schemes can be used to maximize the achievable bit rates.

In this paper, we investigate joint DSM 2/3 for downstream (DS) VDSL. The joint DSM 2/3 problem for upstream (US) VDSL has been studied in [1]. The results show a considerable bit rate boost for the users when optimal joint DSM 2/3 schemes are used. Although, the joint DSM 2/3 in US and DS are closely related in practice, in general the vectoring schemes for them are totally different because in US the vectoring is applied at the receiver side while in DS it is applied at the transmitter side. Moreover, the DS problem is more difficult because, unlike in the US problem, the calculation of the optimal filters is a non-convex problem.

The Joint DSM 2/3 scenario can be viewed as a generalization of the so-called interference (IF) channel and vector broadcast channel (BC) and so will be referred to as an IF/BC. It has been shown that the capacity region of the vector BC is achievable using Costa's dirty paper coding (DPC) scheme. First, it was shown that DPC achieves the sum capacity of

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the BC with two [2] or multiple [3], [4] transmitters under a sum transmit power constraint. The solution in [3] is based on a duality theory between the BC and multiple-access channel (MAC), which states that the same rates can be achieved under equal total power constraints over a BC and a MAC with their channel matrices being the transpose of each other. Later it was shown that the entire capacity region can be achieved by DPC as well [5].

All these results consider a constraint on the sum transmit power, summed over all transmitters (i.e., lines in the DSL context). However, for VDSL systems, we are more interested in a per-line power constrained solution, because the transmit power on each line is bounded by the nominal power spectral density (PSD) mask constraints. Moreover, the total transmit power on each line over all tones can be bounded by analog front-end limitations or standard regulations. This problem has been addressed in [6] for single-carrier vector BCs, where the BC-MAC duality is generalized using the Lagrange dual optimization technique. The outcome is that a vector BC problem with per-transmitter power constraint can be solved in a dual MAC with uncertain noise powers. The uncertain noise powers are in fact the Lagrange multipliers corresponding to the per-transmitter power constraints. The solution is found by selecting appropriate values for the noise powers which satisfy the constraints. In [7], optimal spectrum and signal coordination for DS DSL has been considered and a BC-OSB algorithm has been proposed. The BC-OSB algorithm uses the Lagrange dual optimization approach to enforce the regulatory PSD mask and per-line total power constraints. The Lagrange multipliers are used to scale the dual MAC. The optimality of this technique can be shown using the results in [6].

In [8], the linear zero-forcing (ZF) pre-compensator (ZFP) has been proposed to cancel crosstalk in DS DSL and it is has been shown that it is near-optimal in DSM 3 due to the row-wise diagonal dominance (RWDD) property [9] of the DS DSL channel. By using the linear ZFP in each group and applying OSB over all users, the grouped ZFP-OSB algorithm has been derived for the joint DSM 2/3 scenarios [10]. As we will show in this paper, grouped ZFP-OSB does not necessarily achieve the highest possible rates, because the ZF solution is selfish and does not take into account the interests of the users in other groups.

In this paper, we develop a generalized duality theory between the IF/MAC and IF/BC with per-line total power and nominal PSD mask constraints[1]. This solution can be considered as a generalization of the BC-MAC duality theory with per-transmitter power constraints in [6]. Using this theory, the optimal transmit filters for each group can be calculated. Our simulation results show that by using the THP with optimal transmit filters for each group together with OSB over all users, considerably higher rates are achieved compared to the other available schemes. We refer to the resulting algorithm as the IF/BC-OSB algorithm. The IF/BC-OSB algorithm encompasses the OSB (i.e. IF-OSB) [11] and the BC-OSB algorithm [7] as special cases.

The paper is organized as follows. In Sec. II, we describe the grouped DS DSL transmission model. Then, we develop our generalized IF/BC-IF/MAC duality theory and we propose an optimal joint DSM 2/3 algorithm for DS DSL accordingly in Sec. III. Simulation results are provided in Sec. IV. Finally the paper is concluded in Sec. V.

II. Grouped DS DSL Transmission Model

We consider discrete multi-tone (DMT) transmission with K tones. We assume that N managed users form G groups with N_g users in group g ($g = 1 \ldots G$). We indicate the n-th user in group g by (g,n) and we refer to it as user (g,n). Similarly, we refer to the corresponding line as line (g,n). We assume that the users in each group are coordinated at the transmit signal level and all managed users are coordinated with each other at the transmit spectrum level. All of the users are assumed to be DMT frame synchronized at the receiver side. Therefore, the transmission over tone k can be modeled as

$$\boldsymbol{y}^k = \left[\boldsymbol{H}^k\right]^{\mathrm{H}} \boldsymbol{x}^k + \boldsymbol{z}^k, \tag{1}$$

where $[\cdot]^{\mathrm{H}}$ denotes the transpose conjugate operation[2], $\boldsymbol{x}^k \equiv \left(\left[\boldsymbol{x}_1^k\right]^{\mathrm{T}}, \ldots, \left[\boldsymbol{x}_G^k\right]^{\mathrm{T}}\right)^{\mathrm{T}}$, $\boldsymbol{y}^k \equiv \left(\left[\boldsymbol{y}_1^k\right]^{\mathrm{T}}, \ \ldots, \left[\boldsymbol{y}_G^k\right]^{\mathrm{T}}\right)^{\mathrm{T}}$, and $\boldsymbol{z}^k \equiv \left(\left[\boldsymbol{z}_1^k\right]^{\mathrm{T}}, \ldots, \left[\boldsymbol{z}_G^k\right]^{\mathrm{T}}\right)^{\mathrm{T}}$ are transmitted, received, and noise vector, respectively, and

$$\boldsymbol{H}^k \equiv \begin{bmatrix} \boldsymbol{H}_{11}^k & \cdots & \boldsymbol{H}_{1G}^k \\ \vdots & \ddots & \vdots \\ \boldsymbol{H}_{G1}^k & \cdots & \boldsymbol{H}_{GG}^k \end{bmatrix}. \tag{2}$$

The sub-vectors $\boldsymbol{x}_g^k \equiv \left(x_{(g,1)}^k, \ldots, x_{(g,N_g)}^k\right)^{\mathrm{T}} = \sum_{n=1}^{N_g} u_{(g,n)}^k \boldsymbol{q}_{(g,n)}^k$, $\boldsymbol{y}_g^k \equiv \left(y_{(g,1)}^k, \ldots, y_{(g,N_g)}^k\right)^{\mathrm{T}}$, and $\boldsymbol{z}_g^k \equiv \left(z_{(g,1)}^k, \ldots, z_{(g,N_g)}^k\right)^{\mathrm{T}}$ are the transmitted, received, and noise vector for group g, respectively, the sub-matrix $\boldsymbol{H}_{gg'}^k$ is the crosstalk channel matrix from group g to group g' on tone k with the n-th column denoted by $\boldsymbol{h}_{(gg',n)}^k$, and $u_{(g,n)}^k$ and $\boldsymbol{q}_{(g,n)}^k$ are the data symbol and the transmit filter for user (g,n) on tone k. We assume that the channel is normalized such that $\mathrm{E}\left\{\left|z_{(g,n)}^k\right|^2\right\} = 1$, where $\mathrm{E}\{\cdot\}$ denotes the expectation operator. The correlation between the elements of $\boldsymbol{z}^k$ can be nonzero (e.g., in the presence of unmanaged users) but is ignored as it cannot be exploited [7]. Finally, the transmit power for user (g,n) on tone k is $s_{(g,n)}^k \left|\left|\boldsymbol{q}_{(g,n)}^k\right|\right|^2$, where $s_{(g,n)}^k \equiv \mathrm{E}\left\{\left|u_{(g,n)}^k\right|^2\right\}$ and $||\cdot||$ denotes the ℓ_2 norm. From (2), we have

$$\boldsymbol{y}^k = \left[\hat{\boldsymbol{H}}^k\right]^{\mathrm{H}} \boldsymbol{u}^k + \boldsymbol{z}^k, \tag{3}$$

[1]Note that the current BC-MAC duality theory is applicable only when full signal coordination is available among the users.

[2]Following the literature, we define the IF/BC matrix as $\left[\boldsymbol{H}^k\right]^{\mathrm{H}}$. As we will see later, the dual IF/MAC matrix will then be $\boldsymbol{H}^k$ which simplifies the notation.

where $\boldsymbol{u}^k \equiv \left(u^k_{(1,1)}, \ldots, u^k_{(G,N_G)}\right)^{\mathrm{T}}$, $\hat{\boldsymbol{H}}^k \equiv \left[\boldsymbol{Q}^k\right]^{\mathrm{H}} \boldsymbol{H}^k$,

$$\boldsymbol{Q}^k \equiv \begin{bmatrix} \boldsymbol{Q}^k_1 & 0 & \cdots & 0 \\ 0 & \boldsymbol{Q}^k_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \boldsymbol{Q}^k_G \end{bmatrix}, \tag{4}$$

and $\boldsymbol{Q}^k_g \equiv \left[\boldsymbol{q}^k_{(g,1)} \middle| \boldsymbol{q}^k_{(g,2)} \middle| \cdots \middle| \boldsymbol{q}^k_{(g,N_g)}\right]$.

The number of bits that can be loaded on tone k of user (g,n) is given by

$$b^k_{(g,n)} = \min\left\{b_{\max}, \left\lfloor \log_2\left(1 + \tfrac{1}{\Gamma}\mathrm{SNR}^k_{(g,n)}\right)\right\rfloor\right\}, \tag{5}$$

where $b_{\max} = 15$ is the maximum number of bits that can be loaded on a tone, $\lfloor\cdot\rfloor$ denotes the floor function, $\Gamma = 9.45$ dB is the SNR gap, and $\mathrm{SNR}^k_{(g,n)}$ is the SNR of user (g,n) on tone k. The SNR depends on the structure of the vector encoders for each BC group. For linear encoders the SNR is given by

$$\mathrm{SNR}^k_{(g,n)} = \frac{s^k_{(g,n)}\left|\hat{h}^k_{(g,n)(g,n)}\right|^2}{1 + \sum_{(g',n')\neq(g,n)} s^k_{(g',n')}\left|\hat{h}^k_{(g',n')(g,n)}\right|^2}, \tag{6}$$

where by definition $\hat{h}^k_{(g',n')(g,n)}$ is the element of $\hat{\boldsymbol{H}}^k$ located on the row and the column corresponding to the lines (g',n') and (g,n), respectively. For non-linear vector DPC encoders (e.g., the vector THP [9]), the SNR depends on the transmit filters (i.e., the matrix $\boldsymbol{Q}^k$) as well as the encoding order. A vector DPC encodes the users one by one, such that the crosstalk originating from the previously encoded users is canceled. Assuming the encoding order is from the first user to the last user in each group, the SNR is given by

$$\mathrm{SNR}^k_{(g,n)} = s^k_{(g,n)}\left|\hat{h}^k_{(g,n)(g,n)}\right|^2 \Bigg/ \Bigg[1 + \sum_{n'=n+1}^{N_g} s^k_{(g,n')} \times \left|\hat{h}^k_{(g,n')(g,n)}\right|^2 + \sum_{g'=1;g'\neq g}^{G}\sum_{n'=1}^{N_{g'}} s^k_{(g',n')}\left|\hat{h}^k_{(g',n')(g,n)}\right|^2\Bigg]. \tag{7}$$

The total bit rate of user (g,n) and transmit power on line (g,n) are respectively given by $R_{(g,n)} = f_{\mathrm{s}}\sum_{k=1}^{K} b^k_{(g,n)}$, and $P_{(g,n)} = \Delta_{\mathrm{f}}\sum_{k=1}^{K} p^k_{(g,n)}$, where f_{s} is the DMT symbol rate, Δ_{f} is the tone spacing,

$$p^k_{(g,n)} \equiv \left[\boldsymbol{Q}^k_g \boldsymbol{S}^k_g \left[\boldsymbol{Q}^k_g\right]^{\mathrm{H}}\right]_{n,n} \tag{8}$$

is the transmit power on tone k of line (g,n), $\boldsymbol{S}^k_g \equiv \mathrm{diag}\left\{\boldsymbol{s}^k_g\right\}$, $\boldsymbol{s}^k_g \equiv \left(s^k_{(g,1)}, \ldots, s^k_{(g,N_g)}\right)^{\mathrm{T}}$, and $\mathrm{diag}\{\boldsymbol{a}\}$ denotes a diagonal matrix with diagonal elements equal to the elements of vector $\boldsymbol{a}$. Note that the transmit power on line (g,n) is not equal to the transmit power of user (g,n) in the general case.

III. Joint DSM 2/3 Problem and Optimization

The joint DSM 2/3 problem is stated as follows:

$$\text{maximize} \quad \sum_{g=1}^{G}\sum_{n=1}^{N_G} \omega_{(g,n)} R_{(g,n)}/f_{\mathrm{s}}, \tag{9a}$$

$$\text{subject to} \quad P_{(g,n)} \leq P^{\max}_{(g,n)}, \forall g,n, \tag{9b}$$

$$\text{and} \quad p^k_{(g,n)} \leq p^{k,\mathrm{mask}}_{(g,n)}, \forall g,n,k, \tag{9c}$$

where the variable $\omega_{(g,n)} \geq 0$ is the (bit rate) weight factor for user (g,n), $P^{\max}_{(g,n)}$ is the total transmit power budget for line (g,n), and $p^{k,\mathrm{mask}}_{(g,n)}$ is the nominal PSD mask for line (g,n) on tone k.

The joint DSM 2/3 design consists of the encoder design inside the groups (the DSM 3 part) and spectrum balancing among all of the users in all groups (the DSM 2 part). In this paper we use dual optimization techniques to solve the problem optimally[3]. By applying dual optimization techniques, we obtain the following K per-tone optimization problems

$$\text{maximize } \mathcal{L}_k, \quad \text{for} \quad k = 1, \ldots, K, \tag{10}$$

where $\mathcal{L}_k$ is the Lagrangian on tone k defined by

$$\mathcal{L}_k \equiv \sum_{(g,n)} \omega_{(g,n)} b^k_{(g,n)} - \sum_{(g,n)} \left(\theta_{(g,n)} + \lambda^k_{(g,n)}\right) p^k_{(g,n)}, \tag{11}$$

where $\theta_{(g,n)} \geq 0$ and $\lambda^k_{(g,n)} \geq 0$ are the dual variables associated with the constraints in (9b) and (9c), respectively. These variables will be referred to as the Lagrange multiplier, and the per-tone Lagrange multiplier associated with line (g,n), respectively. The maximization in (10) is carried out over the structure of the encoders and the transmit powers of the users. For a linear encoder, the structure is described by the transmit filters $\boldsymbol{q}^k_{(g,n)}$. For a non-linear THP, it is described by the transmit filters as well as the order of encoding in each group.

A. IF/BC-IF/MAC Duality

In this section, we generalize the well-known duality between (fully coordinated) BC and MAC scenarios to IF/BC and IF/MAC scenarios with per-line power constraints. The original BC-MAC duality states that with the same sum transmit power, the same set of multiuser SNRs (or bit rate) is achievable in a (flat single-carrier) BC and in its dual MAC, where the dual MAC matrix is simply the conjugate transpose of the primal BC matrix [3]. This result was later generalized in [6], to the case where per-transmitter power constraints are defined in the BC rather than a sum transmit power constraint. This form of duality is of a much higher interest in the DSL context as usually the relevant constraints in DSL are indeed the per-line total transmit power and nominal PSD mask constraints, as described by (9b) and (9c).

First we consider linear vector encoders (see Sec. III-B for the case of non-linear encoders). Our goal here is to calculate the optimal transmit filters $\boldsymbol{q}^k_{(g,n)}$ and the transmit power of

[3]The proof of optimality is omitted here for the sake of brevity.

the users (or alternatively $s^k_{(g,n)}$) maximizing $\mathcal{L}_k$ in (11). For ease of notation, we drop the tone index k and ignore the group index g and use a single index (e.g., i ($1 \le i \le N$)) to refer to the users. With these simplifications, (6) is written as

$$\mathrm{SNR}_i^{\mathrm{BC}} = \frac{s_i \left|\hat{h}_{ii}\right|^2}{1+\sum_{j\neq i} s_j \left|\hat{h}_{ji}\right|^2}, \tag{12}$$

where $\hat{h}_{ji} \equiv \left[\hat{\boldsymbol{H}}\right]_{j,i}$, and $\hat{\boldsymbol{H}} \equiv \boldsymbol{Q}^{\mathrm{H}}\boldsymbol{H}$. From this, the required transmit powers $\boldsymbol{s} \equiv (s_1, s_2, \ldots, \ s_N)^{\mathrm{T}}$ to reach the SNR vector $(\gamma_1, \gamma_2, \ldots, \gamma_N)$ is obtained by [11]

$$\boldsymbol{s} = \boldsymbol{X}^{-\mathrm{T}}\mathbf{1}, \tag{13}$$

where $\mathbf{1}$ is the all one column vector of size N, which is the noise power vector in the normalized channel and

$$\boldsymbol{X} \equiv \begin{bmatrix} \gamma_1^{-1}\left|\hat{h}_{11}\right|^2 & -\left|\hat{h}_{12}\right|^2 & \cdots & -\left|\hat{h}_{1N}\right|^2 \\ -\left|\hat{h}_{21}\right|^2 & \gamma_2^{-1}\left|\hat{h}_{22}\right|^2 & & \vdots \\ \vdots & & \ddots & -\left|\hat{h}_{(N-1)N}\right|^2 \\ -\left|\hat{h}_{N1}\right|^2 & \cdots & -\left|\hat{h}_{N(N-1)}\right|^2 & \gamma_N^{-1}\left|\hat{h}_{NN}\right|^2 \end{bmatrix}.$$

Now consider the following dual MAC

$$\tilde{\boldsymbol{y}} = \boldsymbol{H}\tilde{\boldsymbol{x}} + \tilde{\boldsymbol{z}}, \tag{14}$$

with noise covariance matrix

$$\mathrm{E}\left\{\tilde{\boldsymbol{z}}\tilde{\boldsymbol{z}}^{\mathrm{H}}\right\} = \boldsymbol{\Lambda} \equiv \mathrm{diag}\left\{\lambda_1, \lambda_2, \ldots, \lambda_N\right\}, \tag{15}$$

for given λ_i's and assume that the reception filters are $\boldsymbol{Q}^{\mathrm{H}}$. That is, the decision variables are obtained by

$$\hat{\boldsymbol{y}} = \boldsymbol{Q}^{\mathrm{H}}\boldsymbol{H}\tilde{\boldsymbol{x}} + \boldsymbol{Q}^{\mathrm{H}}\tilde{\boldsymbol{z}}. \tag{16}$$

The SNR for user i is calculated by

$$\mathrm{SNR}_i^{\mathrm{MAC}} = \frac{\tilde{s}_i \left|\hat{h}_{ii}\right|^2}{\sigma_i^2 + \sum_{j\neq i} \tilde{s}_j \left|\hat{h}_{ij}\right|^2}, \tag{17}$$

where $\tilde{s}_i$ is the transmit power for user i, and $\sigma_i^2 \equiv \left[\mathrm{E}\left\{\boldsymbol{Q}^{\mathrm{H}}\tilde{\boldsymbol{z}}\tilde{\boldsymbol{z}}^{\mathrm{H}}\boldsymbol{Q}\right\}\right]_{i,i} = \left[\boldsymbol{Q}^{\mathrm{H}}\boldsymbol{\Lambda}\,\boldsymbol{Q}\right]_{i,i}$. Then the required transmit powers $\tilde{\boldsymbol{s}} \equiv (\tilde{s}_1, \tilde{s}_2, \ldots, \tilde{s}_N)^{\mathrm{T}}$ to reach the SNR vector $(\gamma_1, \gamma_2, \ldots, \gamma_N)$ is obtained by

$$\tilde{\boldsymbol{s}} = \boldsymbol{X}^{-1}\boldsymbol{\sigma}, \tag{18}$$

where $\boldsymbol{\sigma} \equiv \left(\sigma_1^2, \sigma_2^2, \ldots, \sigma_N^2\right)^{\mathrm{T}}$. It can be seen that the following relationship exists between the transmit powers in the dual MAC and the per-line transmit powers in the primal BC

$$\boxed{\sum_i \tilde{s}_i = \sum_i \lambda_i p_i}. \tag{19}$$

The proof is straightforward using (13) and (18) as follows

$$\begin{aligned} \textstyle\sum_i \tilde{s}_i &= \mathbf{1}^{\mathrm{T}}\tilde{\boldsymbol{s}} \overset{(18)}{=} \mathbf{1}^{\mathrm{T}}\boldsymbol{X}^{-1}\boldsymbol{\sigma} \overset{(13)}{=} \boldsymbol{s}^{\mathrm{T}}\boldsymbol{\sigma} = \mathrm{trace}\left\{\boldsymbol{S}\boldsymbol{Q}^{\mathrm{H}}\boldsymbol{\Lambda}\boldsymbol{Q}\right\} \\ &= \mathrm{trace}\left\{\boldsymbol{Q}\boldsymbol{S}\boldsymbol{Q}^{\mathrm{H}}\boldsymbol{\Lambda}\right\} = \textstyle\sum_i \lambda_i p_i, \end{aligned}$$

where $\boldsymbol{S} \equiv \mathrm{diag}\left\{\boldsymbol{s}\right\}$ and $p_n = \left[\boldsymbol{Q}\boldsymbol{S}\boldsymbol{Q}^{\mathrm{H}}\right]_{n,n}$.

Now consider the maximization problem in (10). Consider the dual MAC defined by $\boldsymbol{H}^k$ and assume that its noise power for line (g,n) is $\theta_{(g,n)} + \lambda^k_{(g,n)}$. It is noteworthy that there is a one to one relationship between the primal BC bit loadings $b^k_{(g,n)}$ and the minimum SNRs needed to load them, i.e., $\gamma^k_{(g,n)} = \Gamma\left(2^{b^k_{(g,n)}} - 1\right)$. A similar formula holds for the dual MAC bit loadings $\tilde{b}^k_{(g,n)}$. Therefore, if use the same SNRs $\gamma^k_{(g,n)}$ in the primal BC and dual MAC, we can load the same number of bits in the primal BC and dual MAC, i.e., $b^k_{(g,n)} = \tilde{b}^k_{(g,n)}$. Using (19), the Lagrangian in (11) can then be written as

$$\mathcal{L}_k = \tilde{\mathcal{L}}_k \equiv \sum_{(g,n)} \omega_{(g,n)}\tilde{b}^k_{(g,n)} - \sum_{(g,n)} \tilde{s}^k_{(g,n)}. \tag{20}$$

Thus, instead of solving (10), which is a non-convex problem, we can maximize (20). Maximizing (20) is not a convex problem in the dual IF/MAC channel either. However, for each set of bit loadings, the required transmit powers in the dual IF/MAC and the optimal reception filters can be calculated easily (Alg. 2 in [1]). Therefore, by running an exhaustive search over the space of bit loadings or the space of transmit powers, the optimal solution in the dual channel can be found.

The optimal transmit filters in the dual MAC are the minimum mean squared error (MMSE) filters. The MMSE filter for user (g,n) is calculated by

$$\tilde{\boldsymbol{q}}^k_{(g,n)} = \left(\boldsymbol{H}^k_g\tilde{\boldsymbol{S}}^k_{(g,n)}\left[\boldsymbol{H}^k_g\right]^{\mathrm{H}} + \boldsymbol{\Theta}_g + \boldsymbol{\Lambda}^k_g\right)^{-1}\boldsymbol{h}^k_{(gg,n)} \tag{21}$$

and the SNR is

$$\mathrm{SNR}^k_{(g,n)} = \tilde{s}^k_{(g,n)}\left[\tilde{\boldsymbol{q}}^k_{(g,n)}\right]^{\mathrm{H}}\boldsymbol{h}^k_{(gg,n)}, \tag{22}$$

where $\boldsymbol{H}^k_g \equiv \left[\boldsymbol{H}^k_{g1}\middle|\,\boldsymbol{H}^k_{g2}\middle|\cdots\middle|\boldsymbol{H}^k_{gG}\right]$, $\boldsymbol{h}^k_{(gg,n)}$ is the n-th column of $\boldsymbol{H}^k_{gg}$, $\tilde{\boldsymbol{S}}^k \equiv \mathrm{diag}\left\{\tilde{s}^k_{(1,1)}, \ldots, \ \tilde{s}^k_{(g,n-1)}, 0, \tilde{s}^k_{(g,n+1)}, \ldots, \tilde{s}^k_{(G,N_G)}\right\}$, $\boldsymbol{\Theta}_g \equiv \mathrm{diag}\left\{\theta_{(g,1)}, \ldots, \theta_{(g,N_g)}\right\}$, and $\boldsymbol{\Lambda}^k_g \equiv \mathrm{diag}\left\{\lambda^k_{(g,1)}, \ldots, \lambda^k_{(g,N_g)}\right\}$. At the termination of the exhaustive search, the optimal solution found is used to calculate the required per-user and per-line transmit powers in the primal BC using (13) and (8).

B. Non-Linear Encoder and Decoder Structures

So far we have studied the general duality between the BC and MAC assuming linear encoders and decoders. By using the non-linear vector DPC encoder, the contribution of the crosstalk for the users that are already encoded in each group is removed in the SNR (compare (7) to (6)), the effect of which is that the corresponding entries in matrix $\boldsymbol{X}$ will be zero. Consequently

$$\boldsymbol{s} = \overline{\boldsymbol{X}}^{-\mathrm{T}}\mathbf{1}, \tag{23}$$

where

$$\left[\overline{\boldsymbol{X}}\right]_{i,j} \equiv \begin{cases} 0, & \text{if } i \text{ and } j \text{ are coordinated at the signal level and } i \text{ is encoded before } j \\ [\boldsymbol{X}]_{i,j}, & \text{otherwise.} \end{cases} \tag{24}$$

The BC-MAC duality will not hold unless we use the *dual* decoder structure of the vector DPC encoder in the MAC. The dual of the vector DPC encoder is the generalized decision feedback equalizer (GDFE) [3]. Assuming the same combination of signal coordination among users, by using a GDFE structure in the dual MAC and setting the decoding order as the reverse of the encoding order in the primal BC, we will be able to restore the IF/BC-IF/MAC duality. The GDFE removes the crosstalk originating from the already decoded users, resulting in exactly the same zeros in $\boldsymbol{X}$ as those resulting from the DPC encoder operation in the primal BC. Therefore,

$$\tilde{\boldsymbol{s}} = \overline{\boldsymbol{X}}^{-1}\boldsymbol{\sigma}. \tag{25}$$

Equations (23) and (25) portray the duality between the BC using the DPC encoder and the MAC using the GDFE with partial signal coordination. Assuming that the decoding order is from the first user to the last user in each group, the MMSE-GDFE filter for user (g,n) is calculated by (21) where

$$\tilde{\boldsymbol{S}}^{k} \equiv \operatorname{diag}\left\{\tilde{s}^{k}_{(1,1)},\ldots,\tilde{s}^{k}_{(g-1,N_{g-1})},\overbrace{0,\ldots,0}^{n \text{ times}},\tilde{s}^{k}_{(g,n+1)},\ldots,\tilde{s}^{k}_{(G,N_G)}\right\}.$$

C. IF/BC-OSB Algorithm

A pseudocode representation of the optimal joint DSM 2/3 algorithm in DS direction, referred to as the IF/BC-OSB algorithm, is given in Alg. 1. The IF/BC-OSB algorithm finds the optimal transmit powers based on OSB over all users as well as the optimal encoding order and transmit filters for vector THPs per group. The algorithm consists of five nested loops. In the first loop, the Lagrange multipliers corresponding to the per-line total power constraints ($\theta_{(g,n)}$) are optimized. In the second loop, the tone index is selected in order to solve the corresponding per-tone optimization problem. In the third loop, the per-tone Lagrange multipliers corresponding to the nominal PSD mask constraints ($\lambda^{k}_{(g,n)}$) are optimized. In the fourth loop, an exhaustive search is carried out over the space of multi-user bit loadings. The inner-most loop tests all decoding orders in the dual IF/MAC channel to find those resulting in the largest per-tone Lagrangian. The optimal reception filters ($\tilde{\boldsymbol{q}}^{k}_{(g,n)}$) and the corresponding transmit powers ($\tilde{s}^{k}_{(g,n)}$) in the dual channel are calculated using Alg. 2 in [1].

Algorithm 1: The IF/BC-OSB Algorithm

repeat
  Set/update $\theta_{(g,n)}, \forall (g,n)$;
  for $k = 1$ **to** K **do**
    repeat
      Set/update $\lambda^{k}_{(g,n)}$, $\forall\ (g,n)$;
      $\mathcal{L}^{\max}_{k} \leftarrow 0$;
      repeat
        Set/update $b^{k}_{(g,n)}$, $\forall\ (g,n)$;
        for *all decoding order combinations in the* G *groups* **do**
          Calculate $\tilde{\boldsymbol{q}}^{k}_{(g,n)}$ and $\tilde{s}^{k}_{(g,n)}$, $\forall\ (g,n)$ using Alg. 2 in [1];
          if *the bit-loading is achievable* **then**
            $\mathcal{L}_k \leftarrow \sum_{(g,n)} \omega_{(g,n)} \tilde{b}^{k}_{(g,n)} - \sum_{(g,n)} \tilde{s}^{k}_{(g,n)}$;
            if $\mathcal{L}_k > \mathcal{L}^{\max}_{k}$ **then**
              $\mathcal{L}^{\max}_{k} \leftarrow \mathcal{L}_k$;
              $\boldsymbol{q}^{k,\ \text{opt}}_{(g,n)} \leftarrow \tilde{\boldsymbol{q}}^{k}_{(g,n)}$;
              $\tilde{s}^{k,\ \text{opt}}_{(g,n)} \leftarrow \tilde{s}^{k}_{(g,n)}$;
              $b^{k,\ \text{opt}}_{(g,n)} \leftarrow b^{k}_{(g,n)}$;
              Store/update the optimal decoding orders in the dual channel on tone k;
        Encoding order $\leftarrow$ Reverse of the optimal decoding order in the dual channel;
        Calculate $p^{k}_{(g,n)}$, $\forall\ (g,n)$;
      until *the entire bit loading space is searched*;
      $P_{(g,n)} \leftarrow \Delta_{\text{f}} \sum_{k=1}^{K} p^{k,\ \text{opt}}_{(g,n)}$, $\forall\ (g,n)$;
    until $p^{k}_{(g,n)} = p^{k,\text{mask}}_{(g,n)} \vee (p^{k}_{(g,n)} < p^{k,\text{mask}}_{(g,n)} \wedge \lambda^{k}_{(g,n)} = 0)$;
until $P_{(g,n)} = P^{\max}_{(g,n)} \vee (P_{(g,n)} < P^{\max}_{(g,n)} \ \wedge\ \theta_{(g,n)} = 0)$;

IV. Simulation Results

For a three-user VDSL2 scenario depicted in Fig. 1, we have simulated the proposed algorithm as well as several other algorithms or schemes, including grouped ZF-OSB, grouped ZF-THP-OSB, grouped optimal linear pre-coder (OLP)-OSB, OSB, and static spectrum management (SSM). The scenario consists of two groups of VDSL2 E17 [ITU-T G.993.2] loops. The first group consists of an 800 m loop and a 400 m loop and the second group consists of a single 400 m loop. Direct channels are calculated using the 26 AWG cable model and crosstalk couplings are calculated using the standard ANSI 1% worst case crosstalk model. The crosstalk channel phases are assumed to be independent for different users and tones and unifromly distributed over $[0, 2\pi)$.

Figure 2 shows the average bit rate of the lines in the two groups versus each other. As it can be seen, the grouped OLP-OSB and IF/BC-OSB algorithms achieve considerably higher bit rates compared to the grouped ZFP-OSB and grouped ZF-THP-OSB algorithms. This is a paradoxical result as the ZFP is known to be near-optimal in DSL [8], and ZF-THP is obviously superior to ZFP. Moreover, the same spectrum balancing algorithm (i.e., OSB) is used for all these algorithms. The combination of these near-optimal encoding schemes with OSB, however, is sub-optimal in IF/BC scenarios. This can be explained as follows. For any set of transmit powers, the ZFP or ZF-THP in the first group acts (near-) optimally if

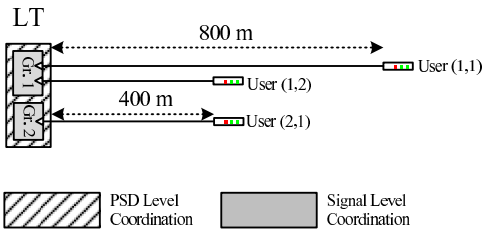

Fig. 1. The simulated scenario.

we consider the benefit of the users in the same group only. However, the resulting crosstalk power to the lines in the second group has not been taken into account and could reduce the bit rate of the line in that group. On the other hand, the grouped OLP-OSB and IF/BC-OSB algorithms optimize the transmit powers and transmit filters by taking into account the benefit of the users in one group as well as the users outside the group. Therefore, by proper design of the transmit filters, the crosstalk could be effectively canceled not only for the users in one group but also for the users belonging to the other groups.

As it can be seen, the IF/BC-OSB algorithm achieves higher bit rates than the grouped OLP-OSB. This can be attributed to the use of non-linear THP structure in IF/BC-OSB. In the OLP structure, each line in group 1 receives crosstalk from the other user in group 1 and the user in group 2. The users are cooperative and try to cancel the crosstalk imposed by each user in the group to other users in the group as well as those outside the group. However, the available number of filter taps for each user (the degrees of freedom) is equal to the number of users in the group and hence smaller than the total number of users. This prevents the users from canceling the crosstalk effectively for all users particularly the crosstalk imposed to the users outside the group. However, the optimal THP structure in IF/BC-OSB pre-cancels the crosstalk for previously encoded users in each group, leaving a few degrees of freedom for each user to pre-cancel the crosstalk for the users outside the group. For example, assuming that for the simulated scenario the first user in group 1 (i.e., user (1,1)) is encoded first and the second user in group 1 (i.e., user (1, 2)) is encoded second in the THP structure, user (1, 2) can choose its filter taps to perfectly cancel the crosstalk from the users in group 1 into group 2. Therefore, the user in group 2 (i.e., user (2,1)) requires less power to achieve a desirable bit rate. This in turn reduces the crosstalk received by the users in group 1 from user (2, 1), resulting in an increase in their achievable bit rates as well.

V. Conclusion

In this paper, we have studied optimal joint DSM 2/3 in DS DSL. We have considered THP for signal encoding inside the groups and the OSB algorithm for spectrum balancing among all users. Calculation of the optimal transmit filters for the THP is a non-convex problem. To resolve this problem, we have extended the BC-MAC duality to scenarios with partial signal coordination and per-line power constraints. Based on this, we

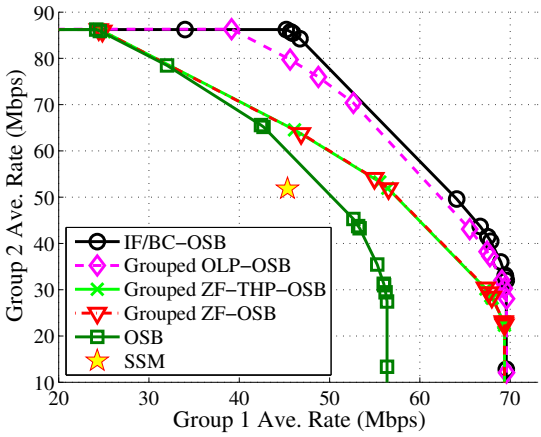

Fig. 2. Achievable bit rate of the user in group 2 vs. average achievable bit rate of the users in group 1 for the scenario in Fig. 1.

have proposed the IF/BC-OSB algorithm. We have compared the performance of our algorithm with several other algorithms. Our simulation results have shown that the proposed algorithm outperforms the other joint DSM 2/3 algorithms with a considerable margin. Our investigation have shown that this can be attributed to the grouped optimal THP structure which is capable of pre-canceling the crosstalk generated by the users in a group to the users inside the same group as well as part of the crosstalk to the users in other groups.

References

[1] A. R. Forouzan, M. Moonen, J. Maes, and M. Guenach, "Joint level 2 and 3 dynamic spectrum management for upstream VDSL," *IEEE Trans. Commun.*, vol. 59, no. 10, pp. 2851–2861, Oct. 2011.

[2] G. Caire and S. Shamai, "On the achievable throughput of a multiantenna Gaussian broadcast channel," *IEEE Trans. Inform. Theory*, vol. 49, no. 7, pp. 1691–1706, Jul. 2003.

[3] P. Viswanath and D. Tse, "Sum capacity of the vector Gaussian broadcast channel and uplink-downlink duality," *IEEE Trans. Inform. Theory*, vol. 49, no. 8, pp. 1912–1921, Aug. 2003.

[4] W. Yu and J. Cioffi, "Sum capacity of Gaussian vector broadcast channels," *IEEE Trans. Inform. Theory*, vol. 50, no. 9, pp. 1875–1892, Sep. 2004.

[5] H. Weingarten, Y. Steinberg, and S. Shamai, "The capacity region of the Gaussian multiple-input multiple-output broadcast channel," *IEEE Trans. Inform. Theory*, vol. 52, no. 9, pp. 3936–3964, Sep. 2006.

[6] W. Yu and T. Lan, "Transmitter optimization for the multi-antenna downlink with per-antenna power constraints," *IEEE Trans. Signal Processing*, vol. 55, no. 6, pp. 2646–2660, Jun. 2007.

[7] V. Le Nir, M. Moonen, J. Verlinden, and M. Guenach, "Optimal power allocation for downstream xDSL with per-modem total power constraints: Broadcast channel optimal spectrum balancing (BC-OSB)," *IEEE Trans. Signal Processing*, vol. 57, no. 2, pp. 690–697, Feb. 2009.

[8] R. Cendrillon, G. Ginis, E. Van den Bogaert, and M. Moonen, "A near-optimal linear crosstalk precoder for downstream VDSL," *IEEE Trans. Commun.*, vol. 55, no. 5, pp. 860–863, may 2007.

[9] G. Ginis and J. M. Cioffi, "Vectored transmission for digital subscriber line systems," *IEEE J. Select. Areas Commun.*, vol. 20, no. 5, pp. 1085–1104, Jun. 2002.

[10] A. R. Forouzan and L. M. Garth, "Generalized iterative spectrum balancing and grouped vectoring for maximal throughput of digital subscriber lines," in *IEEE Global Telecom. Conf., GLOBECOM'05*, vol. 4, Jun. 2005, pp. 2359–2363.

[11] R. Cendrillon, W. Yu, M. Moonen, J. Verlinden, and T. Bostoen, "Optimal multiuser spectrum balancing for digital subscriber lines," *IEEE Trans. Commun.*, vol. 54, no. 5, pp. 922–933, May 2006.