Time-Frequency Splitting Algorithms for Neuromorphic Circuits

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1 Motivation

Spiking neural networks, neuromorphic computing, and computational neuroscience, create new demand and interest in the simulation, analysis, and design of large-scale electrical circuits. The current methodology is dominated by the *forward simulation in time* of nonlinear ordinary differential equations. However, this approach has several limitations.

First, the differential equations of any spiking behavior is inherently stiff. Second, numerical simulations of differential equations rarely scale with the dimension of large-scale networks. Finally, and most importantly, the difficulty of capturing the *variability and uncertainty* of system components via the parameters of the nonlinear differential equation hinders the robustness analysis.

A promising approach to resolve these drawbacks is to model the nonlinear circuits as interconnections of basic elements, write the governing dynamics as the zero of an operator sum expression, and compute the spiking behaviors of the circuits as fixed point solutions of iterative schemes in the *space of trajectories* rather than forward in time. See e.g. [1].

2 Monotone and Mixed-Feedback Circuits

The recent paper [2] proposes that the trajectories of monotone circuits, that is, port interconnections of monotone elements, can be characterized as zeros of monotone operators, for which scalable splitting algorithms exist [3]. This approach was recently extended to arbitrary circuit topologies through an elegant decomposition of the zero inclusion problem to a sum of a monotone operator and a skew-symmetric linear operator [2].

Spiking circuits cannot be modeled as monotone circuits because they require elements with negative conductance. However, their solutions can be expressed as differences of monotone circuits [2]. The challenge is to extend the splitting algorithms from monotone to the difference of monotone circuits. A first step in that direction was proposed in [4].

3 Time-Frequency Splitting

Nonlinear circuit models frequently include two main types of elements: the classical dynamical elements of LTI circuit theory, on the one hand, and static nonlinearities, on the other hand. We propose to exploit this dichotomy in the splitting algorithms designed to simulate the circuit.

Splitting algorithms rely on the computation of the proximal operator (or correspondingly, the resolvent operator). Nonlinear resistive elements are best modeled in the time domain whereas the LTI elements are best modeled in the frequency domain. This talk will exploit the simple idea of mixing time-domain and frequency-domain representations of the operators in the iterative algorithm (and thus, computing the proximal algorithms in different domains) that computes the circuit solutions. We will detail how conventional splitting algorithms enable an efficient implementation of this simple idea. As an example, the Peaceman-Rachford algorithm will run each iteration in two distinct domains (where A is LTI dynamic and B static nonlinear) as

$$z^{k+1} = \underbrace{R_{\alpha A}}_{\text{Freq. Domain}} \quad \underbrace{R_{\alpha B}(z^k)}_{\text{Time Domain}} . \tag{1}$$

The results will be illustrated on circuits decomposed as interconnections of FitzHugh-Nagumo circuits. We will discuss the improvement when contrasted to algorithms that work exclusively in the time domain, the potential of time-frequency splitting for large-scale simulations of spiking neural networks, and the current research challenges to be overcome.

References

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