Robustness and Stability Optimization of Power Generating Kite Systems in a Periodic Pumping Mode

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***Abstract*— In this paper we formulate and solve optimal control problems for power generating kite systems. Here, the kite generates energy by periodically pulling a generator on the ground while flying fast in a crosswind direction. We are searching for an intrinsically open-loop stable trajectory such that the kite generates as much power as possible without needing feedback, while neither the kite nor the cable should touch the ground in the presence of wind turbulence. As the wind turbulences are unknown, robustness aspects need to be taken into account. The formulation of the associated optimal control problem makes use of periodic Lyapunov differential equations in order to guarantee local open-loop stability while robustness aspects are regarded in a linear approximation. The main result of this paper is that open-loop stable kite orbits exist and that open-loop stability only costs approximately 23% compared to the power-optimal unstable orbit.**

I. INTRODUCTION

The idea of using kites for wind power generation was previously motivated by Loyd [23] and is often discussed in the literature [6], [20], [24], [28], [30], [34]. For a system with a single kite and one fixed generator on the ground the principle is very simple: the kite pulls as strong as possible on its cable slowly driving the generator while flying fast in a crosswind direction. To achieve a periodic power-generating cycle the kite is depowered periodically by changing the angle of attack and retracted easily while the tension in the cable is low.

Note that in [12] power optimal orbits for power generating kites are discussed. In this paper, we point out that these power optimal kite orbits are typically open-loop unstable. In the present paper, we concentrate on more challenging robust optimal control formulations with the additional requirements that the kite should not only produce as much power as possible but also fly on an open-loop stable trajectory without any feedback still respecting path constraints in a robust way for the case that random wind turbulences are present. Here, a main contribution of this paper is to show that open-loop stable kite orbits exist.

Note that during the last decades robust optimal control problems have received a lot of attention. Especially robust optimal control for linear systems is a well-developed field (cf. e.g. [1], [2], [36]). But also for nonlinear systems we can find approaches in the literature. For example in [8], [13], [15], [26] we can find techniques to optimize the robustness of nonlinear systems in a linear approximation. If we think about the robustness of open-loop controlled periodic systems, it is usually a necessary requirement that these systems are stable. Most existing stability optimization techniques are either based on the optimization of the asymptotical decay rate of the system [25], the optimization of the so called pseudo-spectral abscissa [5], [32] or on the smoothed spectral abscissa [33] or radius [9].

The paper is organized as follows: We start in Section II with a brief review of existing concepts for wind power generation with kites in a periodic pumping mode discussing periodic and power optimal but unstable kite trajectories. In Section III we introduce periodic Lyapunov differential equations for linear uncertain systems and discuss their importance for robust optimal control problems. These considerations are in Section IV transferred to nonlinear kite-trajectory optimization problems where the numerical solution is presented. Finally, we conclude the paper in Section V.

II. POWER GENERATING KITES

In this section we review the state of the art concepts for power generating kite system in a pumping mode with a focus on power optimization. The main concept for the energy production with kites in pumping mode is that these kites periodically pull their cables to drive a generator on the ground, as it is shown in Figure 1 for a single kite. Note that prototypes of such kite systems are being built at the University of Torino [6] as well as at the University of Delft [20], [24], [29], [30] while the SkySails company [31], [35] is already using wind power to pull large cargo ships [11], [28]. In this paper we only consider the case that the generator is fixed at the ground. Of course, every kite has to be pulled back at some point in time to achieve a periodic power generating cycle. As suggested in [10], [11], [12] this can be achieved by using a lift or drag control to reduce the kite's pulling force. But there are also other methods [6], [21], [27].

The motivations to use kites instead of conventional windmills are multifaceted: First, as the power is increasing with the third power of the wind speed, it is an important fact that kites can use winds at higher altitudes. Additionally, kites avoid the statical problems with the masts and basements of windmills and they can coat a larger wind area. The start and landing of kites can be achieved by rotating devices which accelarate the wings to flying speed.

In this paper we can not reprint all model equations that are needed to simulate a realistic kite system but we refer to [10], [12] for a complete overview over the model we

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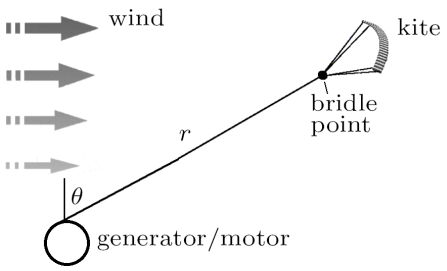

Fig. 1. Energy production with a single kite

are using here. To explain the following results and figures independently we only summarize the main aspects of this model: We assume that the average wind is blowing in e_x - direction, where e_x is a unit vector. The vector e_z is an unit vector pointing to the sky and $e_y := e_z \times e_x$, such that $\{e_x, e_y, e_z\}$ is an orthonormal right-handed basis of the 3-dimensional Euclidean space. The generator has the fixed position $0 \in \mathbb{R}^3$ in this earth system while the kite's position $p \in \mathbb{R}^3$ is given by $p = re_r$. Here r is the distance between the generator and the kite that is coinciding with the cable length for the case that the cable is tight. The unit vector e_r is in the earth system defined by $e_r := (\sin(\theta)\cos(\phi), \sin(\theta)\sin(\phi), \cos(\theta))^T$ where $q := (r, \phi, \theta)^T$ are the standard spherical coordinates describing the kite's position. Note that the cable is not necessarily a straight line. Especially in the kite's retraction phase when the tension in the cable is low we might have a recognizable sagging due the gravitation of the cable. Let α be the angle between the ground and the tangent to the cable close to the generator. If we have $\alpha < 0$ the cable touches the ground. Please note that we compute α in our model only in a quasi-statical approximation. In particular we may assume that α is not explicitly depending on high-frequent wind turbulences but only on the reference position of the kite and the corresponding tension in the cable [10].

To steer the kite system we have three controls: the second derivative of the cable length with respect to the time (indirectly depending on the force at the generator), the kite's roll angle Ψ and the kite's lift coefficient C_{L}. We collect these three control inputs in a vector valued function $u : \mathbb{R} \to \mathbb{R}^3$ that will later be optimized. Note that the roll angle Ψ is defined by $\sin(\Psi) := e_{\mathrm{t}} \cdot e_{\mathrm{r}}$, where e_{t} is a unit vector pointing from the left to the right wing tip[1]. It can be controlled by varying the difference between the lengths of the cables leading to the kite's right and left wing tip respectively. The kite's lift coefficient C_{L} can be controlled by an elevator at the tail of the kite.

If F_{c} is the tension in the cable, we can compute the average power $\overline{P}$ at the generator by

$$\overline{P} := \frac{1}{T}\int_0^T P dt = \frac{1}{T}\int_0^T F_{\mathrm{c}}\dot{r} dt \ . \tag{1}$$

[1]Note that it is also possible to steer the kite without roll. In this case an additional control airfoil orthogonal to the main wing is needed. However, a discussion of such control airfoils is beyond the scope of this paper.

Here, T is the time the kite needs for one loop. Now, we investigate the solution of an optimal control problem which maximizes this average power $\overline{P}$ subject to the kite's (nonlinear) equations of motion [12], periodic boundary conditions for the states, a few control bounds as well as the important constraint that the cable does not touch the ground during the retraction phase (i.e. we require $\alpha(t) \geq 0$ for all $t \in \mathbb{R}$).

Note that this optimal control problem can be summarized in the standard form for periodic optimal control problems:

$$\begin{aligned} &\underset{x(\cdot),u(\cdot),p,T}{\text{maximize}} && J[x(\cdot), u(\cdot), p, T] \\ &\text{subject to:} && \\ &\forall t \in [0,T]: && \dot{x}(t) = f(x(t), u(t), p, 0) \\ &\forall t \in [0,T]: && 0 \geq h(x(t), u(t), p) \\ & && x(0) = x(T) \end{aligned} \tag{2}$$

In our case, the objective $J[x(\cdot), u(\cdot), p, T] := \overline{P}$ is the average power as defined in Equation 1. The dynamic model f for the state x has not been introduced above, but we refer once more to [10] for the details. The controls u and the parameters p also influence f while the fourth argument of f is 0 indicating that we consider no disturbance w in this section. The inequality state and control constraints can be imposed via the function h, while the constraint of the form $x(0) = x(T)$ implies the periodicity.

For a kite with a wing area of $500\,\mathrm{m}^2$ and a nominal reference wind velocity of $10\,\frac{\mathrm{m}}{\mathrm{s}}$ a locally optimal solution is shown in Figure 2. The result for the average power is

$$\overline{P} = 5.37\,\mathrm{MW} \ . \tag{3}$$

Theoretical upper bounds for the power that can be produced with kites [10] show that this result either is the global optimum or can at least not be far from the global optimum. Besides the control function u and the periodic trajectory the cycle duration T has also been optimized and we found $T = 16.82\,\mathrm{s}$ in the optimal solution. Note that the optimal cable length r is between $1.4\,\mathrm{km}$ and $1.5\,\mathrm{km}$. Due to the red dotted retraction phase it is possible to achieve that the cable length r is periodic. The optimal solution for the lift coefficient has a clear structure: during the pulling phase the lift is switched to its upper bound 1.5 such that the kite pulls as strong as possible on the generator creating a large amount of energy while it is optimal to switch to the lower bound 0.3 reducing the tension in the cable during the retraction phase such that we only have to invest a very small amount of energy to retract the kite.

But now let us state the problem: the above locally power optimal trajectory is unstable. In order to demonstrate this we have simulated the kite by applying small wind disturbances. The corresponding system reaction is shown as the blue dotted line: without even considering the angle α between the cable and the ground the altitude of the kite becomes negative after less than three cycle durations. Mathematically, the instability of the locally power optimal solution, which

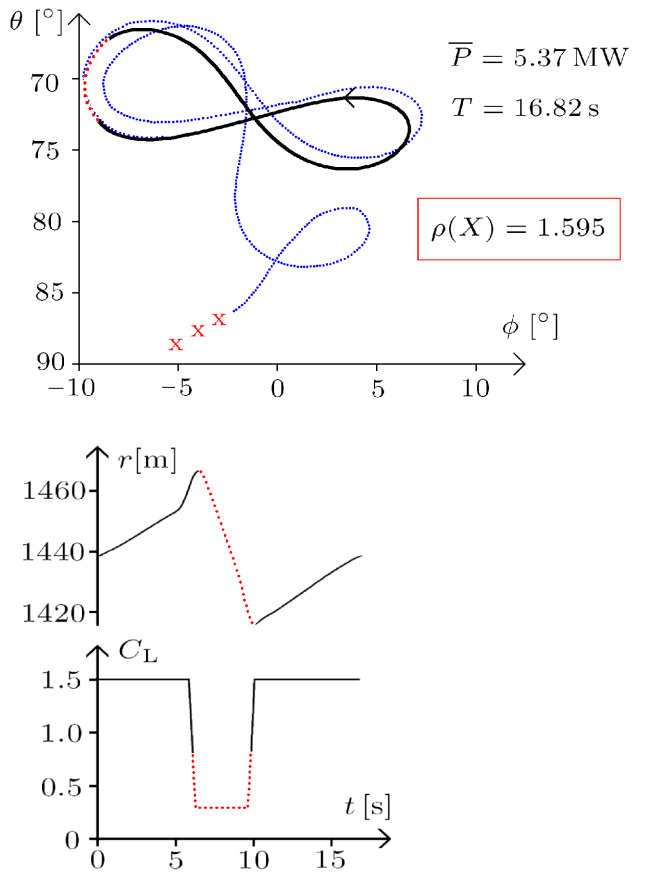

Fig. 2. The power-optimal but unstable kite-trajectory on the $\phi - \theta$-plane as well as the cable length R and the lift coefficient C_L. The reel-in phase is red colored and dotted.

is shown in Figure 2, can be confirmed by computing the associated monodromy matrix X, which has a spectral radius of $\rho(X) = 1.595 > 1$, i.e. the system is unstable. Note that we will give a formal definition of the monodromy matrix within the following section.

The first idea on how we can fix the instability problem is to introduce a feedback controller such that the kite can be stabilized on the power optimal trajectory. For example in [16] and also in [7], it is shown that this is e.g. possible by using a nonlinear model predictive controller (NMPC). However, in this paper we ask the question whether it is also possible to find open-loop stable trajectories that allow us to fly the kite without needing any sensors in combination with feedback strategies. Such open loop stable solutions would have certain advantages in practice: first it is cheaper and easier to have as little sensors as possible but even if we like to implement a feedback controller for a real-world kite this is a much easier task if the kite is already flying stable on a suitably robustified trajectory.

How can we include stability aspects into the formulation of an optimal control problem? In order to give an answer to this question we have to consider some theoretical concepts first, which will be outlined in the following section.

III. PERIODIC LYAPUNOV DIFFERENTIAL EQUATIONS

Let us consider a linear time-periodic system with a differential state vector $x : \mathbb{R} \to \mathbb{R}^{n_x}$ that is excited by a disturbance function $w : \mathbb{R} \to \mathbb{R}^{n_w}$:

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + B(t)w(t) \\ y(t) &= C(t)x(t) \\ \lim_{t_0 \to -\infty} x(t_0) &= 0 \end{aligned} \tag{4}$$

for all $t \in \mathbb{R}$. We assume that the coefficients $A \in \mathbb{R}^{n_x \times n_x}$, $B \in \mathbb{R}^{n_x \times n_w}$, and $C \in \mathbb{R}^{n_y \times n_x}$ are integrable periodic matrix functions with a constant period T on the whole time interval. In addition $y : \mathbb{R} \to \mathbb{R}^{n_y}$ is called the output function. The fundamental solution $G : \mathbb{R} \times \mathbb{R} \to \mathbb{R}^{n_x \times n_x}$ of (4) is the unique solution of the initial value problem:

$$\frac{\partial G(t,\tau)}{\partial t} = A(t)G(t,\tau) \quad \text{with} \quad G(\tau,\tau) = 1 \tag{5}$$

for all $t, \tau \in \mathbb{R}$. In the following we assume that the system is asymptotically stable, i.e. all eigenvalues of the monodromy matrix $X \in \mathbb{R}^{n_x \times n_x}$ with $X := G(T, 0)$ are contained in the open unit disc such that x and y are well defined [25] by the linear differential system (4). Using this notation we can write the output function y in the form [36]

$$y(t) = \hat{H}_t w := \int_{-\infty}^{\infty} H_t(\tau) w(\tau) \mathrm{d}\tau \tag{6}$$

with the Green's or impulse response function $H_t : \mathbb{R} \to \mathbb{R}^{n_y \times n_w}$ being defined by

$$H_t(\tau) := \begin{cases} C(t)G(t,\tau)B(\tau) & \text{if } \tau \le t \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

for all $t, \tau \in \mathbb{R}$. Obviously, the differential equation (5) for the fundamental solution G is completely independent of the matrix functions B and C. Thus, the computation of the monodromy matrix X is useful to discuss the stability of the system but not enough to study any robustness aspects in dependence on the disturbance function w. To overcome this limitation, Periodic Lyapunov Differential Equations (PLDE's) with periodic boundary conditions,

$$\begin{aligned} \dot{P}(t) &= A(t)P(t) + P(t)A(t)^T + B(t)B(t)^T \\ P(0) &= P(T) \end{aligned} \tag{8}$$

are a well-known tool in the linear control theory for periodic systems. As we assume that X is asymptotically stable there exists a unique and symmetric matrix function $P : \mathbb{R} \to \mathbb{R}^{n_x \times n_x}$ that satisfies (8). With the additional assumption that the system (4) is reachable, i.e. the reachability Grammian matrix

$$Q := \int_0^T G(t,\tau)B(\tau)B(\tau)^T G(t,\tau)^T \mathrm{d}\tau \tag{9}$$

is positive definite[2] it can be guaranteed that also the unique solution P of (8) is positive definite [4]. Note that

[2]If Q is not positive definite we can still guarantee the positive definiteness of P in the reachable subspace (cf. [18] how to compute Kalman's canonical decomposition with respect to the reachability of the system).

this existence statement, known under the name Lyapunov Lemma [4], is also reversible: under the assumption that Q is positive definite the existence of a T-periodic and positive definite solution P guarantees that X is asymptotically stable.

We are now interested in the worst-case excitation of an output component y_i of the linear-time periodic system under the assumption that the disturbance w is bounded by an L_2-norm. For this aim, we consider the Hilbert space L_2 of all square-integrable functions from $\mathbb{R}$ to $\mathbb{R}^{n_w}$ with the L_2-scalar product $\langle \cdot \mid \cdot \rangle_{L_2} : L_2 \times L_2 \to \mathbb{R}$ and the corresponding L_2-norm $\| \cdot \|_{L_2} : L_2 \to \mathbb{R}$

$$\langle w_1 \mid w_2 \rangle_{L_2} := \int_{-\infty}^{\infty} w_1(\tau)^T w_2(\tau)\, \mathrm{d}\tau\,, \qquad (10)$$

$$\|w_1\|_{L_2} := \sqrt{\langle w_1, w_1 \rangle_{L_2}} \qquad (11)$$

for all $w_1, w_2 \in L_2$. Writing equation (6) in the form

$$y_i(t) = \langle H_{t,i}^T \mid w \rangle_{L_2}$$

and defining the γ-ball $\mathcal{B} \subseteq L_2$ by

$$\mathcal{B} := \{ w \in L_2 \mid \|w\|_{L_2} \leq \gamma \}$$

we find the worst case excitation of the output component $y_i(t)$:

$$\max_{w \in \mathcal{B}} y_i(t) = \gamma \|H_{t,i}^T\|_{L_2} = \gamma \sqrt{C_i(t) P(t) C_i(t)^T} \qquad (12)$$

for all $t \in \mathbb{R}$ and all $i \in \{1, ..., n_y\}$. The first equation in (12) follows immediately from Cauchy's inequality and the fact that $\gamma \frac{H_{t,i}}{\|H_{t,i}\|_{L_2}} \in \mathcal{B}$. The second equation in (12) is a standard relation [36] for the T-periodic solution of the PLDE (8): the function

$$P(t) := \int_{-\infty}^{t} G(t,\tau) B(\tau) B(\tau)^T G(t,\tau)^T \mathrm{d}\tau$$

is T-periodic and satisfies the PLDE. Thus, providing that X is asymptotically stable it must be the unique solution of the system (8). In a direct consequence we have $\|H_{t,i}^T\|_{L_2}^2 = C(t)P(t)C(t)^T$.

We call the variable γ in the above consideration the confindence level. This notation is motivated by the stochastic interpretation of Lyapunov differential equations [3], [17], [19]: if the disturbance w entering the asymptotically stable system (4) is a stationary Gaussian white noise process with

$$\begin{array}{l} \forall t \in \mathbb{R}:\ \mathbb{E}\{w(t)\} \ = \ 0 \quad \text{and} \\ \forall t_1, t_2 \in \mathbb{R}, i, j \in \{1, .., n_w\}: \\ \mathbb{E}\{w_i(t_1) w_j(t_2)^T\} \ = \ \delta(t_2 - t_1)\delta_{i,j} \end{array}$$

then P is the variance-covariance matrix of the state vector x. Here, $\delta(\cdot)$ denotes the Dirac-distribution while $\delta_{i,j}$ is defined to be 1 for equal indices i and j and 0 otherwise. Consequently, the variance covariance matrix of the output y is at each time $t \in \mathbb{R}$ given by $C(t)P(t)C(t)^T$. Thus, if we have

$$\forall t \in \mathbb{R}\,,\, i \in \{1, ..., n_y\}: \quad \gamma^2\, C_i(t) P(t) C_i(t)^T \leq 1 \qquad (13)$$

the probability for a violation of the constraint $y_i(t) \leq 1$ for a given time t and a given component i is less than $\int_\gamma^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}\, \mathrm{d}z$.

IV. ROBUSTNESS AND STABILITY OPTIMIZATION FOR POWER GENERATING KITES

In this section, we transfer the considerations from the previous section to our kite optimization problem in order to take robustness and stability aspects in a linear approximation into account. In principle, this transfer is straightforward: we linearize the system with respect to the disturbance around the nominal trajectory x_r and use a Lyapunov differential equation to compute approximate robustness margins with respect to the inequality state constraints:

$$\begin{array}{lrcl} \underset{x_r(\cdot), P(\cdot), u(\cdot), p, T}{\text{minimize}} & J(x_r(\cdot), u(t), p, T) & & \\ \text{subject to:} & & & \\ \forall t \in [0,T]: & \dot{x}_r(t) & = & f(x_r(t), u(t), p, 0) \\ \forall t \in [0,T]: & \dot{P}(t) & = & A(t)P(t) + P(t)A(t)^T \\ & & & +B(t)B(t)^T \\ & x_r(0) & = & x_r(T) \\ & P(0) & = & P(T) \ \succeq \ 0 \\ \forall t \in [0,T] & & & \\ \forall i \in I: & 0 & \geq & h_i(x_r(t), u(t), p) \\ & & & +\gamma \sqrt{C_i(t) P(t) C_i(t)^T} \end{array} \qquad (14)$$

Here, we still use the same equations of motion for the kite, i.e. the same right-hand-side function f, as for the presented power-optimal solution. Note that the matrix valued functions A, B, C denote the partial derivatives

$$A := \frac{\partial f}{\partial x} \quad B := \frac{\partial f}{\partial w} \Sigma^{-\frac{1}{2}} \quad C := \frac{\partial h}{\partial x}\,, \qquad (15)$$

which are needed to compute the linear approximation. Here, we should explain that the unknown wind turbulence w enters the right-hand side function f in a nonlinear way. The function B denotes the corresponding sensitivity but scaled with a matrix Σ which is the variance covariance matrix associated with the statistical properties of the wind turbulences. In our example, these statiscal properties are given by

$$\mathbb{E}\{w(t)w(\tau)^T\} \ = \ \Sigma\, \delta(t - \tau) \quad \text{with} \qquad (16)$$

$$\Sigma := \begin{pmatrix} 0.5 & 0 & 0 \\ 0 & 0.25 & 0 \\ 0 & 0 & 0.25 \end{pmatrix} \left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2 \mathrm{s}\,. \qquad (17)$$

Futhermore, we choose the constant confidence level $\gamma = 2$ maximizing the average power at the generator in a robust counterpart formulation.

Note that the robustness with respect to inequality constraints is regarded via the approximate margin terms $\gamma \sqrt{C_i(t) P(t) C_i(t)^T}$ in the path-inequalities in the above

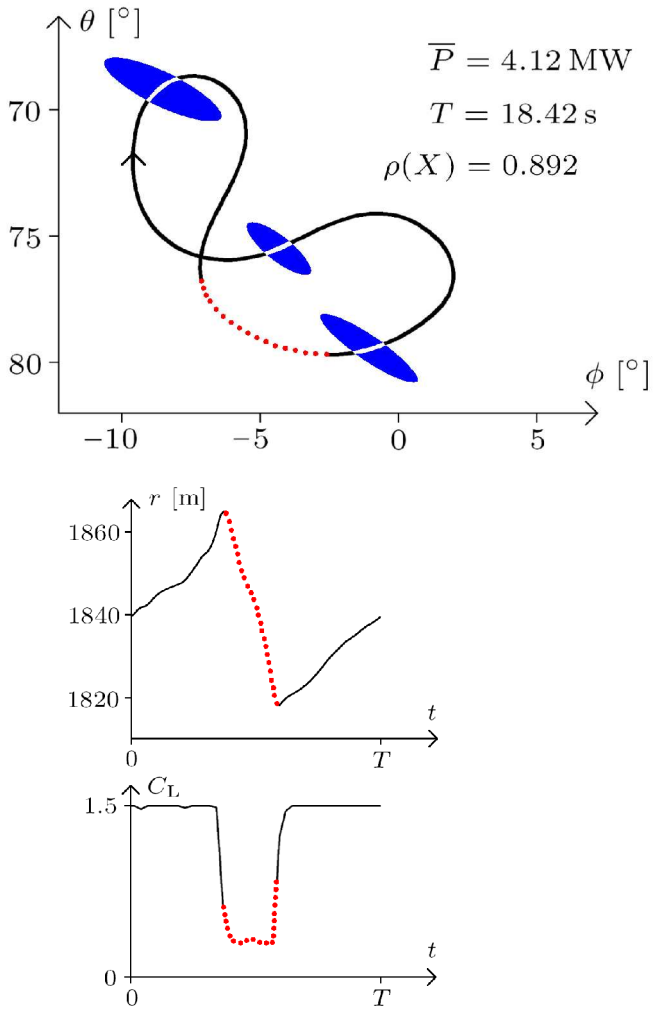

Fig. 3. An open-loop stable kite-trajectory on the $\phi-\theta$-plane as well as the cable length and the lift coefficient. The reel-in phase is dotted. The ellipsoids are projections of the confidence ellipsoid $\mathcal{E}$ onto the $\phi-\theta$-plane at different time points.

formulation which amount to linearized single-chance constraints. The local open-loop stability of the kite-system can now under mild regularity assumptions (i.e. reachibility of the linearized system) be guaranteed as long as we find a T-periodic and positive definite solution P of the Lyapunov differential equation.

The optimal control problem (14) can be solved with standard optimal control software [14], [22]. A locally optimal solution is shown in Figure 3. The result

$$\overline{P} = 4.12 \pm 0.44\,\mathrm{MW} \tag{18}$$

for the average power is clearly smaller than the result $\overline{P} = 5.37\,\mathrm{MW}$ without robustification, but a main observation of our case study is that we loose on average only 23% of power to be paid for robustness and stability.

The result for the spectral radius of the monodromy matrix is $\rho(X) = 0.892$. Thus we succeeded in finding an open-loop stable solution. To visualize the result for the variance-covariance function P on the $\phi-\theta$-plane the projection of

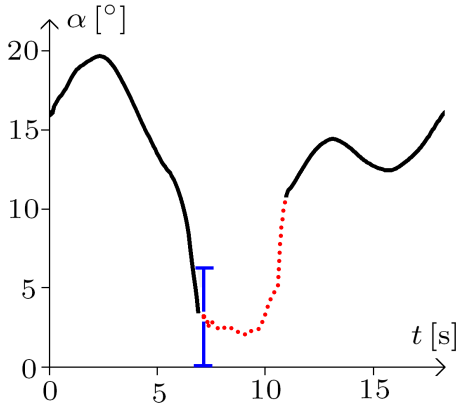

Fig. 4. The angle α between the ground and the cable close to the generator. The reel-in phase is dotted. The error bar shows γ times the standard deviation of $\alpha(t^*)$ for the time $t^* = 7.2\,\mathrm{s}$ and $\gamma = 2$. At this time the confidence constraint for α is active.

the confidence ellipsoids $\mathcal{E}_{t_1}, \mathcal{E}_{t_2}$ and $\mathcal{E}_{t_3}$ onto the $\phi-\theta$-plane is plotted at three times $t_1, t_2, t_3 \in [0, T]$ with $t_1 := 1.8\,\mathrm{s}$, $t_2 := 9.2\,\mathrm{s}$ and $t_3 := 14.7\,\mathrm{s}$. Here, the confidence ellipsoid $\mathcal{E}_t$ is for each $t \in \mathbb{R}$ defined as

$$\mathcal{E}_t \;:=\; \left\{ v \mid v^T P(t)^{-1} v \right\} .$$

Finally, the result for the angle α is shown in Figure 4 At the time $t^* = 7.2\,\mathrm{s}$ the confidence constraint for α was active. Thus, as we use $\gamma = 2$ in our example the probability $p \in [0, 1]$ that the cable touches the ground at this time is in the linear approximation given by:

$$p = \int_2^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}\,\mathrm{d}z \approx 0.023\,. \tag{19}$$

However at this point it arises of course the question whether the linear approximation is sufficiently accurate for our purposes. This question can unfortunately only be answered by a costly long time Monte-Carlo simulation which yielded

$$p_{\text{simulation}} \;\approx\; 0.018$$

for the probability of a constraint violation. Thus, the linear approximation was in this case quite accurate and fortunately conservative.

V. CONCLUSIONS

In this paper we have presented optimal control problem formulations with the aim to optimize power generating kite system in a periodic pumping mode. Here, we have discussed within Section II that a kite with $500\,\mathrm{m}^2$ wing area can generate more than $5\,\mathrm{MW}$ under nominal wind velocities of approximately $10\,\frac{\mathrm{m}}{\mathrm{s}}$. However, these power optimal results have turned out to be non-robust and open-loop unstable. After reviewing concepts from linear control theory we have used a periodic Lyapunov differential equation to compute worst case excitations for uncertain linear systems. Furthermore, we have transferred these considerations to our nonlinear kite model which is assumed to be excited by

small disturbances on an infinite time horizon in the past. The open-loop stability can in this context be guaranteed by making use of the Lyapunov Lemma. Our formulation of robustified optimal control problems has turned out to be a powerful way to increase the robustness and stability of power generating kite systems. We discussed that we can find stable trajectories using the presented formulation such that the kite does not touch the ground even if small wind disturbances arise. Here, the loss of power to be paid for stability and robustness is compared to the nominal solution approximately $23\,\%$. Our robust and open-loop stable solution was successfully tested with long time simulations.

VI. ACKNOWLEDGMENTS

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