Next Cancellation and DSM for Bonded DSL

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NEXT CANCELLATION

We are interested in VDSL systems operating in a bonded fashion. A set of N lines is bonded through co-ordination at both ends of the link. Such a system could be employed for high bandwidth access to a switch within a large apartment complex, for replacement of existing fiber to the home/business services, or for replacement of fiber connections between the CO and ONU/RT.

Symbol transmission is synchronized such that all symbols arrive simultaneously at the receivers on either end of the link. Such synchronized reception ensures that FEXT occurs only on the same tones ie. this prevents inter-carrier FEXT. Simultaneous reception is achieved by applying appropriate delays to the lines based on their electrical length.

In the example in Fig. 1, modem B1 must transmit its block $\triangle_B \triangleq (L_B - L_A)/c_{line}$ seconds earlier than modem A1, where c_{line} is the propagation speed in a line. In the other direction, modem B2 must also transmit $\triangle_B$ seconds early.

Since all lines are of similar length in a bonded system, these delays will typically be very small.

The received signal of interest and NEXT will be out of synch by δ. For line A

$$\begin{aligned}\delta_A &= \frac{1}{c_{line}} L_A - \delta_{NEXT} + \triangle_B \\ &= \frac{1}{c_{line}} L_B - \delta_{NEXT}\end{aligned}$$

where δ_{NEXT} is the delay of the NEXT channel in seconds. Similarly $\delta_B = \frac{1}{c_{line}} L_A - \delta_{NEXT}$. Provided that transmitted blocks are appended with a cyclic suffix of sufficient length, this asynchronism will not give rise to inter-block interference (IBI)[1]. The required CS length to prevent IBI is

$$\begin{aligned}CS/\mu_{sampling} &\geq \max(\delta_A, \delta_B) \\ &\geq \frac{1}{c_{line}} \max(L_A, L_B) \end{aligned} \quad (1)$$

To prevent IBI it is also necessary that NEXT from previously transmitted blocks does not leak into the received block. The required CP length to prevent IBI is

$$CP/\mu_{sampling} \geq l_{NEXT} - \min(\delta_A, \delta_B) \quad (2)$$

where l_{NEXT} is the length of the NEXT impulse response in seconds. Conditions (1) and (2) are always satisfied in practice.

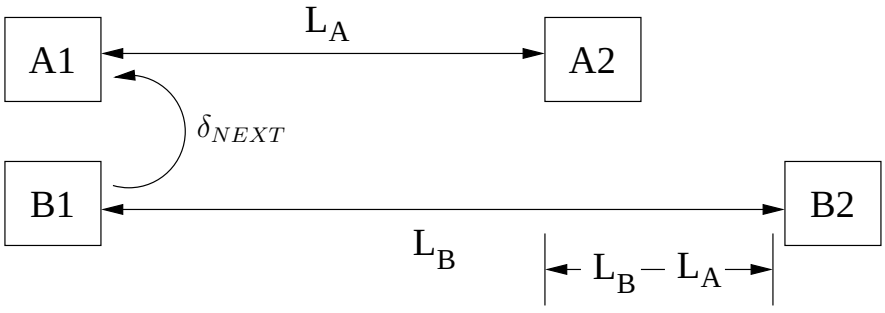


Fig. 1. Synchronized Transmission

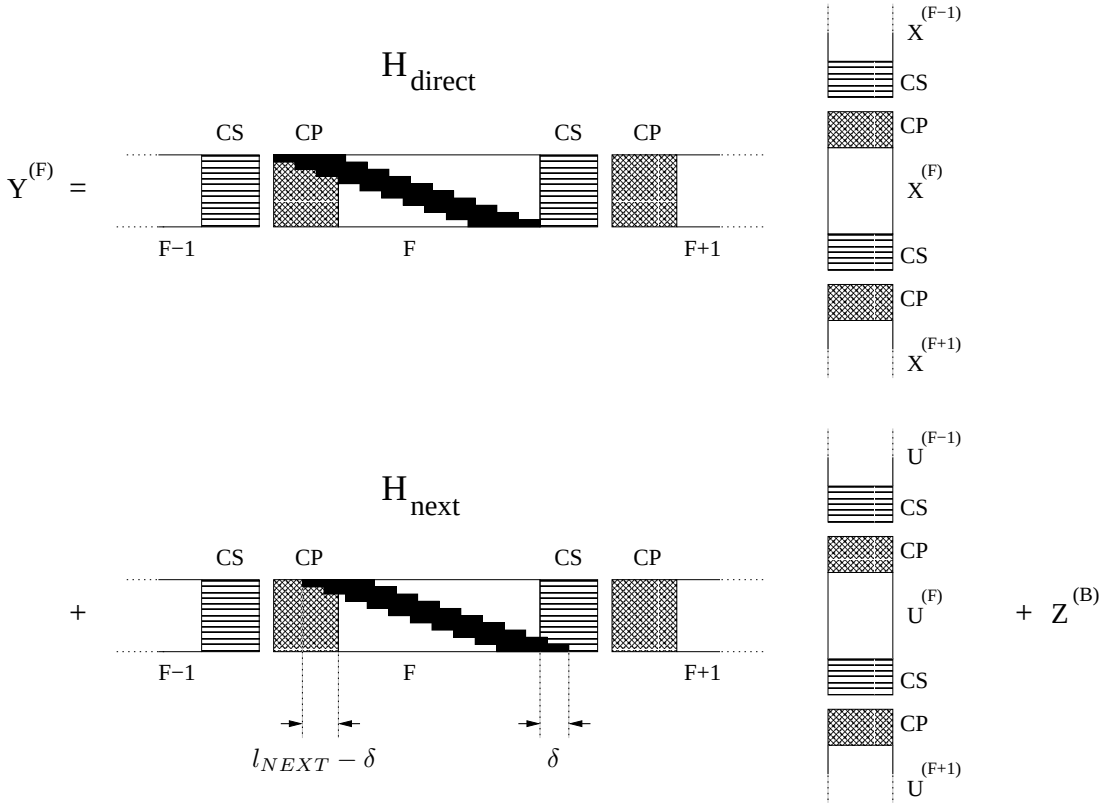


Fig. 2. NEXT and Cyclic Extensions (F - block index)

So the use of cyclic extensions, and synchronized transmission prevents inter-carrier FEXT, NEXT and echo. This allows us to perform FEXT, NEXT and echo cancellation per-tone, which requires $2NK$ mults/user/DMT-block, in contrast to the NK mults/user/DMT-block required by FEXT cancellation alone.

Model transmission on one tone as

$$\mathbf{y}_k = \mathbf{H}_k\mathbf{D}_k\mathbf{s}_k + \mathbf{H}_k^{next}\mathbf{D}_k^u\mathbf{u}_k + \mathbf{z}_k$$

where the diagonal matrices $\mathbf{D}_k \triangleq \operatorname{diag}\left\{d_k^1, \ldots, d_k^N\right\}$ and $\mathbf{D}_k^u$ shape the spectrum of all users at tone k.

NEXT Cancellation: NEXT and echo cancellation is performed by simple subtraction.

$$\mathbf{y}_k - \mathbf{H}_k^{next}\mathbf{D}_k^u\mathbf{u}_k$$

Noise Whitening: Let $\mathcal{E}\left\{\mathbf{z}_k\mathbf{z}_k^H\right\} = \mathbf{Z}_k$. We decompose the noise spectrum using a Cholesky decomposition

$$\mathbf{Z}_k \stackrel{chol}{=} \mathbf{G}_k\mathbf{G}_k^H$$

which is then used to whiten the noise at the receiver

$$\begin{aligned}\widetilde{\mathbf{y}}_k &= \mathbf{G}_k^{-1}\left(\mathbf{y}_k - \mathbf{H}_k^{next}\mathbf{D}_k^u\mathbf{u}_k\right)\\ &= \mathbf{G}_k^{-1}\mathbf{H}_k\mathbf{D}_k\mathbf{s}_k + \widetilde{\mathbf{z}}_k\end{aligned}$$

such that $\mathcal{E}\left\{\widetilde{\mathbf{z}}_k\widetilde{\mathbf{z}}_k^H\right\} = \mathbf{G}_k^{-1}\mathbf{Z}_k\mathbf{G}_k^{-H} = \mathbf{I}_N$.

FEXT Cancellation: Using the SVD, the channel $\widetilde{\mathbf{H}}_k \triangleq \mathbf{G}_k^{-1}\mathbf{H}_k\mathbf{D}_k$ can be transformed into a set of independent, parallel sub-channels[2]. Define

$$\mathbf{U}_k\Lambda_k\mathbf{V}_k^H \stackrel{svd}{=} \widetilde{\mathbf{H}}_k$$

where $\Lambda_k \triangleq \operatorname{diag}\left\{\sigma_k^1, \ldots, \sigma_k^N\right\}$. We now pre-code the transmitted symbols $\mathbf{s}_k = \mathbf{V}_k\mathbf{x}_k$, such that $\mathbf{x}_k$ contains the true QAM symbols and $\mathcal{E}\left\{\mathbf{x}\mathbf{x}_k^H\right\} = \mathbf{I}_N$. Since $\mathbf{V}_k$ is unitary, it has no effect on the power of $\mathbf{s}_k$ and hence $\mathcal{E}\left\{\mathbf{s}_k\mathbf{s}_k^H\right\} = \mathbf{I}_N$.

Algorithm 1 Iterative Vector Waterfilling

init $d_k^n = 0 \; \forall n, k$
repeat
  for $n = 1 \dots N$
$$d_k^n = \left(\left[\frac{1}{\lambda_n} - \frac{1}{\underline{\mathbf{h}}_k^{n\,H} \left[\mathbf{I}_N + \sum_{m \neq n} \underline{\mathbf{h}}_k^m \, \underline{\mathbf{h}}_k^{m\,H} \, d_k^{m\,2} \right]^{-1} \underline{\mathbf{h}}_k^n} \right]^+ \right)^{1/2} \quad \forall k$$
  end
until convergence

An estimate of the transmitted symbols is formed by equalization with $\Lambda_k^{-1}\mathbf{U}_k^H$. Hence

$$\begin{aligned} \widehat{\mathbf{x}}_k &= \Lambda_k^{-1}\mathbf{U}_k^H\mathbf{G}_k^{-1}\left(\mathbf{y}_k - \mathbf{H}_k^{next}\mathbf{D}_k^u\mathbf{u}_k\right) \\ &= \mathbf{x}_k + \Lambda_k^{-1}\mathbf{U}_k^H\widetilde{\mathbf{z}}_k \end{aligned}$$

where the power of the noise term $\overline{\mathbf{z}}_k \triangleq \Lambda_k^{-1}\mathbf{U}_k^H\widetilde{\mathbf{z}}_k$ is $\mathcal{E}\left\{\overline{\mathbf{z}}_k\overline{\mathbf{z}}_k^H\right\} = \Lambda_k^{-2}$.

Hence the SNRs seen by each of the sub-channels is

$$SINR_k^n = \sigma_k^{n\,2}$$

The total capacity of the system if we employ a standard slicer after equalization is thus

$$\begin{aligned} \sum_k \log_2\left|\mathbf{I}_N + \Lambda_k^2\right| &= \sum_k \log_2\left|\mathbf{I}_N + \mathbf{U}_k^H\mathbf{G}_k^{-1}\mathbf{H}_k\mathbf{D}_k^2\mathbf{H}_k^H\mathbf{G}_k^{-H}\mathbf{U}_k\right| \\ &= \sum_k \log_2\left|\mathbf{I}_N + \mathbf{Z}_k^{-1}\left(\mathbf{H}_k\mathbf{D}_k^2\mathbf{H}_k^H\right)\right| \end{aligned}$$

which is the theoretical capacity of the channel using ML detection. Thus we achieve full capacity using simple single user detection (standard slicer) plus linear pre-compensation and equalization on a per-tone basis.

DSM

We can convert our channel with coloured, (possibly) correlated noise into an equivalent AWGN channel $\underline{\mathbf{H}}_k \triangleq \mathbf{G}_k^{-1}\mathbf{H}_k$. Our channel is effectively a single user, vector (MIMO) channel. The theoretical capacity is thus

$$C = \sum_k \log\left|\mathbf{I}_N + \underline{\mathbf{H}}_k\mathbf{D}_k^2\underline{\mathbf{H}}_k^H\right|$$

We desire to maximize C as a function of $\{\mathbf{D}_k\}_{k=1,\dots,K}$ under a total power constraint on each line

$$\sum_k d_k^{n\,2} \leq P \; \forall n \tag{3}$$

Simultaneous Vector Waterfilling: This problem was addressed in [3] where it was shown that the optimal power allocation is a form of simultaneous vector waterfilling. The optimal power allocation is thus

$$d_k^n = \left(\left[\frac{1}{\lambda_n} - \frac{1}{\underline{\mathbf{h}}_k^{n\,H} \left[\mathbf{I}_N + \sum_{m \neq n} \underline{\mathbf{h}}_k^m \, \underline{\mathbf{h}}_k^{m\,H} \, d_k^{m\,2} \right]^{-1} \underline{\mathbf{h}}_k^n} \right]^+ \right)^{1/2} \quad \forall n, k \tag{4}$$

where $\underline{\mathbf{h}}_k^n \triangleq [\underline{\mathbf{H}}_k]_{col\,n}$. The Lagrange multipliers $\{\lambda_1, \dots, \lambda_N\}$ are chosen such that the set of constraints in (3) are all met with equality.

Iterative Vector Waterfilling: No closed form solution is known for solving the set of equations in (3) and (4). Fortunately, the optimal solution can be found using a cheap iterative algorithm[4].

λ_n is chosen in each inner iteration of Alg. 1 to meet the power constraint on line n (see Eq. (3)) with equality.

Noise Only Waterfilling: While iterative vector waterfilling allows us to find the optimal power allocation in an efficient way, we can exploit certain properties of the DSL channel to reduce the complexity of power allocation even more. Define

$$\begin{aligned}\mathbf{Q}_k &\triangleq \mathbf{I}_N + \sum_{m\neq n} \underline{\mathbf{h}}_k^m \, \underline{\mathbf{h}}_k^{m\,H} \, d_k^{m\,2} \\ &= \mathbf{I}_N + \begin{bmatrix} \overline{\mathbf{h}}_k^1 \\ \vdots \\ \overline{\mathbf{h}}_k^N \end{bmatrix} \begin{bmatrix} \overline{\mathbf{h}}_k^{1\,H} & \cdots & \overline{\mathbf{h}}_k^{N\,H} \end{bmatrix}\end{aligned}$$

where $\overline{\mathbf{h}}_k^m \triangleq \begin{bmatrix} \underline{h}_k^{m,1} d_k^1 & \cdots & \underline{h}_k^{m,n-1} d_k^{n-1} & \underline{h}_k^{m,n+1} d_k^{n+1} & \cdots & \underline{h}_k^{m,N} d_k^N \end{bmatrix}$ and $\underline{h}_k^{i,j} \triangleq [\underline{\mathbf{H}}_k]_{i,j}$. In DSL the direct channel from a given transmitter to it's receiver is always much stronger than the channel from that transmitter to another receiver. We call this column dominance since it ensures that

$$\underline{h}_k^{n,n} \gg \underline{h}_k^{m,n}$$

Using this observaton we can approximate

$$d_k^m \simeq \left(\left[\frac{1}{\lambda_n} - \frac{1}{|\underline{h}_k^{n,n}|^2 \left[\mathbf{Q}_k^{-1}\right]_{n,n}} \right]^+ \right)^{1/2}$$

Now $\left[\mathbf{Q}_k^{-1}\right]_{n,n} = \left|\overline{\mathbf{Q}}_k^{n,n}\right| |\mathbf{Q}_k|^{-1}$ where $\overline{\mathbf{Q}}_k^{n,n} \triangleq [\mathbf{Q}_k]_{col\ 1:n-1\ n+1:N,\, row\ 1:n-1\ n+1:N}$, ie. $\mathbf{Q}_k$ with row n and column n removed.

$$\begin{aligned}|\mathbf{Q}_k| &= \left| \mathbf{I}_N + \begin{bmatrix} \overline{\mathbf{h}}_k^1 \\ \vdots \\ \overline{\mathbf{h}}_k^N \end{bmatrix} \begin{bmatrix} \overline{\mathbf{h}}_k^{1\,H} & \cdots & \overline{\mathbf{h}}_k^{N\,H} \end{bmatrix} \right| \\ &= \left| \mathbf{I}_N + \begin{bmatrix} \overline{\mathbf{h}}_k^n \\ \overline{\mathbf{h}}_k^1 \\ \vdots \\ \overline{\mathbf{h}}_k^{n-1} \\ \overline{\mathbf{h}}_k^{n+1} \\ \vdots \\ \overline{\mathbf{h}}_k^N \end{bmatrix} \begin{bmatrix} \overline{\mathbf{h}}_k^{n\,H} & \overline{\mathbf{h}}_k^{1\,H} & \cdots & \overline{\mathbf{h}}_k^{n-1\,H} & \overline{\mathbf{h}}_k^{n+1\,H} & \cdots & \overline{\mathbf{h}}_k^{N\,H} \end{bmatrix} \right|\end{aligned}$$

Dividing this into sub-matrices

$$|\mathbf{Q}_k| = \left| \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \overline{\mathbf{Q}}_k^{n,n} \end{bmatrix} \right|$$

where $\mathbf{A} \triangleq 1 + \left\|\overline{\mathbf{h}}_k^n\right\|$, $\mathbf{B} \triangleq \overline{\mathbf{h}}_k^n \mathbf{M}_k$, $\mathbf{C} \triangleq \mathbf{M}_k^H \, \overline{\mathbf{h}}_k^{n\,H}$. Here $\mathbf{M} \triangleq \begin{bmatrix} \overline{\mathbf{h}}_k^1 & \cdots & \overline{\mathbf{h}}_k^{n-1} & \overline{\mathbf{h}}_k^{n+1} & \cdots & \overline{\mathbf{h}}_k^N \end{bmatrix}$. Notice that $\overline{\mathbf{Q}}_k^{n,n} = \mathbf{M}^H \mathbf{M} + \mathbf{I}_{N-1}$. Using the Schur decomposition

$$|\mathbf{Q}_k| = \left|\overline{\mathbf{Q}}_k^{n,n}\right| \left| \mathbf{A} - \mathbf{B} \left(\overline{\mathbf{Q}}_k^{n,n}\right)^{-1} \mathbf{C} \right|$$

hence

$$\left[\mathbf{Q}_k^{-1}\right]_{n,n} = \left| 1 + \left\|\overline{\mathbf{h}}_k^n\right\| - \overline{\mathbf{h}}_k^n \mathbf{M} \left(\mathbf{M}^H \mathbf{M} + \mathbf{I}_{N-1}\right)^{-1} \mathbf{M}^H \, \overline{\mathbf{h}}_k^{n\,H} \right|$$

Using the SVD $\mathbf{M} \overset{svd}{=} \mathbf{U}_M \Lambda_M \mathbf{V}_M^H$. If $\overline{\mathbf{h}}_k^i \neq \alpha \overline{\mathbf{h}}_k^j \; \forall i, j$ then $\mathbf{M}$ will have full rank and $\mathbf{U}_M$ and $\mathbf{V}_M$ with be unitary matrices of size $N-1 \times N-1$. This is typically the case in practice. Hence

$$\mathbf{M} \left(\mathbf{M}^H \mathbf{M} + \mathbf{I}_{N-1}\right)^{-1} \mathbf{M}^H = \mathbf{U}_M \Lambda_M^2 \left(\Lambda_M^2 + \mathbf{I}_{N-1}\right)^{-1} \mathbf{U}_M^H$$

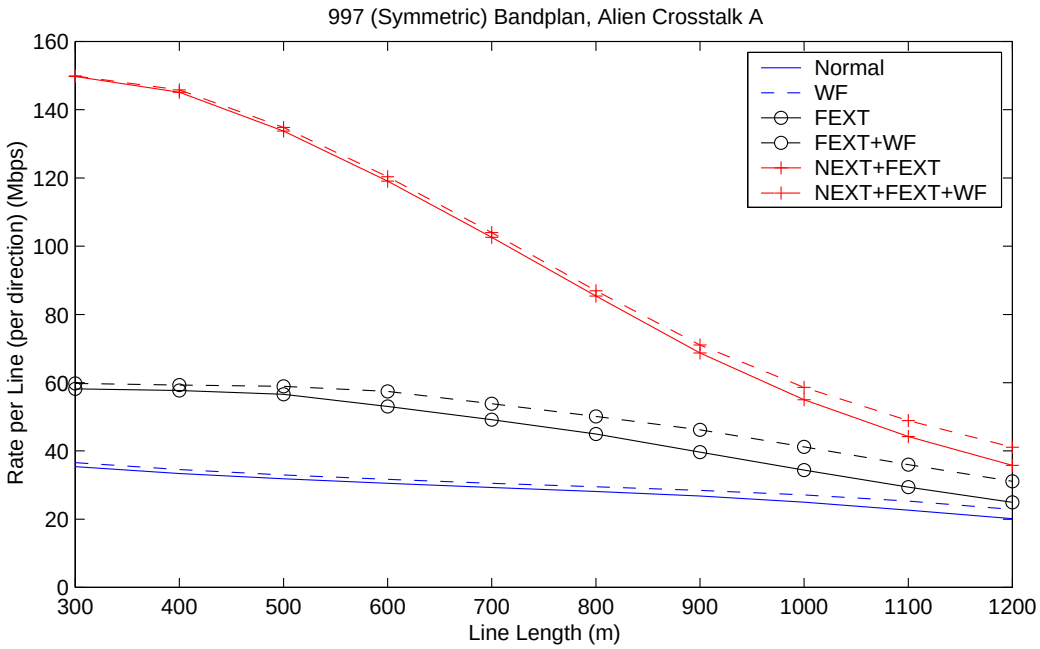


Fig. 3. Average Rate (per line) vs. Line Length (Alien Crosstalk A)

Since the SNR in DSL is high we can approximate $\Lambda_M^2 + \mathbf{I}_{N-1} \simeq \Lambda_M^2$ and

$$\mathbf{M}\left(\mathbf{M}^H\mathbf{M} + \mathbf{I}_{N-1}\right)^{-1}\mathbf{M}^H \simeq \mathbf{I}_{N-1}$$

Hence

$$\left[\mathbf{Q}_k^{-1}\right]_{n,n} = 1$$

and

$$d_k^n \simeq \left(\left[\frac{1}{\lambda_n} - \frac{1}{|\underline{h}_k^{n,n}|^2}\right]^+\right)^{1/2} \tag{5}$$

Using this power allocation strategy, each user's power allocation can be done independently. This considerably reduces power allocation complexity. Each user waterfills against their own direct channel and the background noise as if no interference was present.

The reason that interference is explicitly ignored in (5) is because the high SNR and column dominant nature of the channel allow for near-perfect crosstalk cancellation.

RESULTS

Using these techniques it is possible to support more than double the rates achieved with FEXT cancellation alone. For example, a 20 pair binder of 300m can support 3 Gbps symmetric when used in a bonded fashion with FEXT, NEXT and echo cancellation.

Shown in Fig.s 3 and 4 is the average rate achieved (per line) in a bonded system employing different levels of co-ordination. These results were found using empirical transfer functions. This should be repeated using measured transfer functions.

Note that the crosstalk cancellation gains are largest on short lines whilst the waterfilling gains are largest on long lines. Also note that crosstalk cancellation gains decrease with the severity of background noise, whilst waterfilling gains increase. For example in Fig. 4 long lines experience minimal gain from crosstalk cancellation since the noise (Alien Crosstalk Type F) is so severe. On the other hand the waterfilling gains are significant.

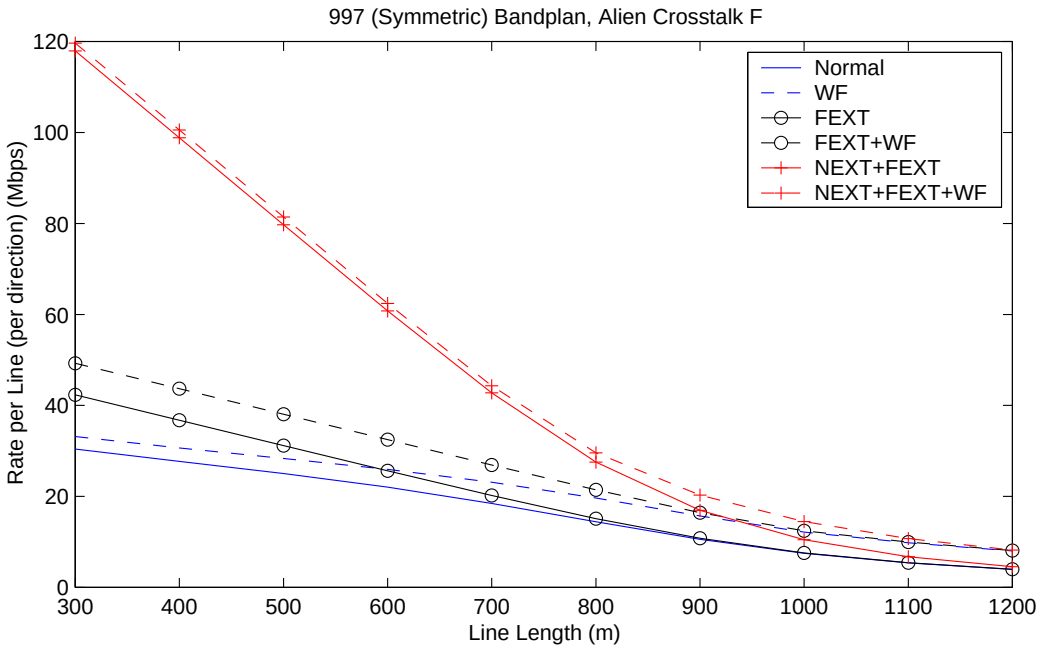


Fig. 4. Average Rate (per line) vs. Line Length (Alien Crosstalk F)

COMMENTS

Due to spectral compatibility concerns, it may be necessary that the transmit powers $d_k^{n\,2}$ fall within a set of spectral masks. This problem is addressed in [5] for the single line power allocation problem. It's extension to multiple (bonded) lines is an open problem.

In unbundled scenarios it becomes more difficult to cancel out-of-domain NEXT. This could be achieved using blind algorithms (blind synchronization) and successive-interference cancellation. If this is not allowed by regulatory bodies it may be possible to protect out-of-domain services, which would employ echo cancellation only, whilst still allowing large performance gains to systems which choose to employ full NEXT cancellation.

This would be done by allowing full-duplex transmission in the low frequency bands where NEXT coupling is small. Optimal power allocation in this case is difficult, however good sub-optimal solutions have been proposed[6].

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