PARAFAC WITH ORTHOGONALITY IN ONE MODE AND APPLICATIONS IN DS-CDMA SYSTEMS

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ABSTRACT

Blind deterministic receivers for DS-CDMA systems based on the PARAFAC model have been proposed in several papers since their conception in [1]. In many cases, the transmitted signals can be considered uncorrelated. Hence, we develop PARAFAC receivers for uncorrelated signals. We introduce several numerical algorithms for orthogonality constrained PARAFAC on which receivers for uncorrelated signals can be based. Simulation results show an increase in performance when the PARAFAC receiver takes the uncorrelatedness of the transmitted signals into account.

Index Terms— PARAFAC, DS-CDMA, blind signal separation.

1. INTRODUCTION

A blind receiver for DS-CDMA systems was proposed in [1]. The receiver takes a deterministic approach based on the multilinear PARAFAC model [2]. By exploiting the uniqueness properties of the PARAFAC model blind equalization of DS-CDMA systems was made possible.

Later on, a blind PARAFAC receiver that allows for more users was proposed in [3]. It was based on a link between the PARAFAC model and simultaneous matrix diagonalization, as explained in [4].

In many cases the symbol data will be uncorrelated. Hence, the first purpose of this paper is to incorporate the data uncorrelatedness assumption into existing blind PARAFAC receivers for DS-CDMA signals.

Second, in order to implement blind PARAFAC receivers for uncorrelated signals we will also propose orthogonality constrained versions of the Simultaneous matrix Diagonalization PARAFAC (SD-PARAFAC) method [4] and the Simultaneous Schur Decomposition PARAFAC (SSD-PARAFAC) method [5]. We will also discuss an orthogonality constrained Alternating Least Squares PARAFAC (ALS-PARAFAC) method [2, 1].

The paper is organized as follows. In the rest of the introduction the notations will be presented, followed by a quick review of the DS-CDMA signal model and PARAFAC receiver. Section 2 will propose orthogonality constrained PARAFAC methods capable of incorporating the uncorrelatedness assumption of the data. In section 3 some computer results will be reported and we will conclude in section 4.

1.1. Notations

Vectors, matrices and tensors are denoted by lower case boldface, upper case boldface and upper case calligraphic letters, respectively. Let $\circ$, $\otimes$ and $\odot$ denote the outer, Kronecker and Khatri-Rao product, respectively, and let $(\cdot)^T$, $(\cdot)^*$, $(\cdot)^H$, $\mathrm{Re}\{\cdot\}$, $\|\cdot\|_F$, $\mathrm{Col}(\cdot)$ denote the transpose, conjugate, conjugate-transpose, real part, Frobenius norm and the column space of a matrix, respectively. Let $\mathrm{triu}(\cdot)$ and $\mathrm{diag}(\cdot)$ denote the operator that sets the strictly lower triangular elements and off-diagonal elements of a matrix equal to zero, respectively. Moreover, let $\mathbf{A} \in \mathbb{C}^{I\times J}$, then $\mathrm{Vec}(\mathbf{A}) \in \mathbb{C}^{IJ}$ will denote a column vector with the property $(\mathrm{Vec}(\mathbf{A}))_{i+(j-1)I} = (\mathbf{A})_{ij}$. Let $\mathbf{a} \in \mathbb{C}^{IJ}$, then the reverse operation is $\mathrm{Unvec}(\mathbf{a}) = \mathbf{A} \in \mathbb{C}^{I\times J}$ such that $(\mathbf{a})_{i+(j-1)I} = (\mathbf{A})_{ij}$. Let $\mathbf{A} \in \mathbb{C}^{I\times J}$, then $D_k(\mathbf{A}) \in \mathbb{C}^{J\times J}$ denotes the diagonal matrix holding row k of $\mathbf{A}$ on its diagonal. Given a matrix $\mathbf{A} \in \mathbb{C}^{I\times J}$, then $\mathbf{a}_j$ denotes the jth column vector of $\mathbf{A}$. Finally, $\mathbf{I}_R \in \mathbb{C}^{R\times R}$ denotes the identity matrix.

1.2. DS-CDMA signal model and PARAFAC receiver

In DS-CDMA systems the transmitted encoded symbol s_{kr} from user r at the kth symbol period is given by $y_{jkr} = h_{jr}s_{kr}$, where h_{jr} denotes the jth chip of the applied spreading code. Assume there are I receive antennas and R users transmitting over a flat fading channel with a gain a_{ir} between the ith receiver and the rth transmitter.

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Then the output of the ith antenna is

$$x_{ijk} = \sum_{r=1}^{R} a_{ir} y_{jkr} = \sum_{r=1}^{R} a_{ir} h_{jr} s_{kr}.$$

Let us stack the channel gains, spreading codes and the transmitted symbols in the matrices $\mathbf{A} \in \mathbb{C}^{I\times R}$, $\mathbf{H} \in \mathbb{C}^{J\times R}$ and $\mathbf{S} \in \mathbb{C}^{K\times R}$. The received data can then be stored in the tensor $\mathcal{X} \in \mathbb{C}^{I\times J\times K}$ such that

$$\mathcal{X} = \sum_{r=1}^{R} \mathbf{a}_r \circ \mathbf{h}_r \circ \mathbf{s}_r, \tag{1}$$

where $\mathbf{a}_r$, $\mathbf{h}_r$ and $\mathbf{s}_r$ denote the rth column vector of $\mathbf{A}$, $\mathbf{H}$ and $\mathbf{S}$, respectively. Using guard chips, the model still holds when reflections take place in the far field of the array; $\mathbf{h}_r$ then consists of the convolution of the rth spreading sequence and the rth channel impulse response [1]. The decomposition (1) of $\mathcal{X}$ is sometimes referred to as the PARAFAC decomposition. If R is the minimal value for which (1) holds, then it is called the rank of $\mathcal{X}$. It can can be shown that under certain conditions the decomposition (1) is unique up to a permutation and scaling of the columns of the matrices $\mathbf{A}$, $\mathbf{H}$ and $\mathbf{S}$ [4, 6]. We say that the decomposition (1) is essentially unique.

Due to the essential uniqueness properties of the PARAFAC model it is possible to blindly estimate the model parameters, i.e., solely based on the observation data, up to the mentioned ambiguities. This is the basis of the PARAFAC receiver proposed in [1].

2. PARAFAC WITH ORTHOGONALITY CONSTRAINT IN ONE MODE

The transmitted signals can usually be considered uncorrelated and for sufficiently large K we have $\mathbf{S}^H\mathbf{S} \approx \alpha\mathbf{I}_R$, $\alpha \in \mathbb{R}$. Hence, in this paper we will incorporate this knowledge in the PARAFAC receiver.

When $R \leq \min(IJ, K)$ and $R(R-1) \leq \frac{1}{2}I(I-1)J(J-1)$, then in subsection 2.1 we will show how to incorporate an orthogonality constraint in one mode in the SD-PARAFAC method. Next, when $I, K \geq R$ or $J, K \geq R$, then in subsection 2.2 we will show how to incorporate an orthogonality constraint in one mode in the SSD-PARAFAC method. The orthogonality constrained ALS procedure will be discussed in subsection 2.3. The latter can be used to refine the results obtained with the SD-PARAFAC or SSD-PARAFAC procedures.

2.1. SD-PARAFAC with orthogonality constraint in one mode

Consider $\mathcal{X} \in \mathbb{C}^{I\times J\times K}$ in (1). Let $\mathbf{X}^{(i)} \in \mathbb{C}^{J\times K}$ denote the matrix such that $(\mathbf{X}^{(i)})_{jk} = (\mathcal{X})_{ijk}$, then $\mathbf{X}^{(i)} = \mathbf{H}D_i(\mathbf{A})\mathbf{S}^T$. Stack the matrices $\{\mathbf{X}^{(i)}\}_{i=1}^{I}$ into the matrix

$$\mathbb{C}^{IJ\times K} \ni \mathbf{X}_{(1)} \triangleq \begin{bmatrix} \mathbf{X}^{(1)} \\ \vdots \\ \mathbf{X}^{(I)} \end{bmatrix} = \begin{bmatrix} \mathbf{H}D_1(\mathbf{A}) \\ \vdots \\ \mathbf{H}D_I(\mathbf{A}) \end{bmatrix} \mathbf{S}^T = (\mathbf{A} \odot \mathbf{H})\mathbf{S}^T.$$

Let $\mathbf{X}_{(1)} = \mathbf{U}\Sigma\mathbf{V}^H$ denote the Singular Value Decomposition (SVD), where $\mathbf{U} \in \mathbb{C}^{IJ\times R}$, $\mathbf{V} \in \mathbb{C}^{K\times R}$ are column-wise orthonormal matrices and $\Sigma \in \mathbb{R}^{R\times R}$ is a positive diagonal matrix. Assume that the tensor $\mathcal{X} \in \mathbb{C}^{I\times J\times K}$ of rank R is constructed from the column-wise orthonormal symbol matrix $\mathbf{S}$, and the two other matrices $\mathbf{A}$ and $\mathbf{H}$. Assume also that $(\mathbf{A} \odot \mathbf{H})$ has full column rank, which is a necessary condition for the essential uniqueness of the PARAFAC decomposition [7]. This assumption is true under mild conditions when $R \leq \min(IJ, K)$ [4].

Since $\mathbf{S}$ is of full column rank by definition, we have that $\mathrm{Col}(\mathbf{U}\Sigma) = \mathrm{Col}\left((\mathbf{A} \odot \mathbf{H})\mathbf{S}^T\right) = \mathrm{Col}(\mathbf{A} \odot \mathbf{H})$. Hence, there exists a nonsingular matrix $\mathbf{F} \in \mathbb{C}^{R\times R}$ such that $\mathbf{A} \odot \mathbf{H} = \mathbf{U}\Sigma\mathbf{F}$. This result together with the relation $\mathbf{X}_{(1)} = (\mathbf{A} \odot \mathbf{H})\mathbf{S}^T = \mathbf{U}\Sigma\mathbf{V}^H$ implies that $\mathbf{S}^T = \mathbf{F}^{-1}\mathbf{V}^H$. Since $\mathbf{S}$ and $\mathbf{V}$ are column-wise orthonormal, we get $\mathbf{V}^H\mathbf{V} = \mathbf{F}\mathbf{S}^T\mathbf{S}^*\mathbf{F}^H = \mathbf{I}_R \Leftrightarrow \mathbf{F}\mathbf{F}^H = \mathbf{I}_R$ and hence $\mathbf{F}$ must be a unitary matrix.

In order to estimate $\mathbf{F}$, the SD-PARAFAC method was proposed in [4]. The problem amounts to simultaneously diagonalizing a set matrices by congruence. In the case that $\mathbf{F}$ is unitary, the problem consists of maximizing the cost function

$$f(\mathbf{F}) = \sum_{k=1}^{R} \|\mathrm{diag}\left(\mathbf{F}^H\mathbf{M}^{(k)}\mathbf{F}\right)\|_F^2, \quad \mathbf{M}^{(k)T} = \mathbf{M}^{(k)},$$

see [4] for the construction of $\{\mathbf{M}^{(k)}\} \subset \mathbb{C}^{R\times R}$. A complex symmetric variant of JADE [8], known as simultaneous Takagi factorization [9], will be applied to find $\mathbf{F}$.

Once $\mathbf{F}$ has been estimated, the unitary matrix $\mathbf{S}$ follows immediately from $\mathbf{S} = \mathbf{V}^*\mathbf{F}^*$. The matrices $\mathbf{A}$ and $\mathbf{H}$ follow from a set of decoupled SVD problems since $\mathrm{Unvec}((\mathbf{U}\Sigma\mathbf{F})_r) = \mathrm{Unvec}(\mathbf{a}_r \otimes \mathbf{h}_r) = \mathbf{h}_r\mathbf{a}_r^T$ is a rank-1 matrix. In other words, $\mathbf{h}_r$ and $\mathbf{a}_r$ can be estimated from the best rank-1 approximation of $\mathrm{Unvec}((\mathbf{U}\Sigma\mathbf{F})_r)$, $r \in [1, R]$.

2.2. SSD-PARAFAC with Orthogonality Constraint in One Mode

When $I, K \geq R$ or $J, K \geq R$, we can consider the orthogonality constrained PARAFAC estimation problem as a Simultaneous Schur Decomposition (SSD) problem [5]. We will restrict the discussion to the case $I, K \geq R$, but similar results can be obtained when $J, K \geq R$.

Let the tensor $\mathcal{X} \in \mathbb{C}^{I\times J\times K}$ of rank R be constructed from a column-wise orthonormal symbol matrix $\mathbf{S}$ with $R \leq K$, and the two other matrices $\mathbf{A}$ and $\mathbf{H}$. Let $\mathbf{X}^{(j)} \in \mathbb{C}^{I\times K}$ denote the matrix such that $(\mathbf{X}^{(j)})_{ik} = (\mathcal{X})_{ijk}$, then $\mathbf{X}^{(j)} = \mathbf{A}D_j(\mathbf{H})\mathbf{S}^T$. Stack the matrices $\{\mathbf{X}^{(j)}\}_{j=1}^{J}$ into the matrix

$$\mathbb{C}^{IJ\times K} \ni \mathbf{X}_{(2)} \triangleq \begin{bmatrix} \mathbf{X}^{(1)} \\ \vdots \\ \mathbf{X}^{(J)} \end{bmatrix} = \begin{bmatrix} \mathbf{A}D_1(\mathbf{H}) \\ \vdots \\ \mathbf{A}D_J(\mathbf{H}) \end{bmatrix} \mathbf{S}^T = (\mathbf{H} \odot \mathbf{A})\mathbf{S}^T.$$

Let $\mathbf{X}_{(2)} = (\mathbf{H} \odot \mathbf{A})\,\mathbf{S}^T = \mathbf{U}\Sigma\mathbf{V}^H$ denote the SVD. Again, assuming PARAFAC essential uniqueness, we can assume that $(\mathbf{H} \odot \mathbf{A})$ has full column rank. Hence $\mathrm{Col}\,(\mathbf{U}\Sigma) = \mathrm{Col}\,(\mathbf{H} \odot \mathbf{A})$ which implies the existence of a nonsingular matrix $\mathbf{F} \in \mathbb{C}^{R\times R}$ such that $\mathbf{H} \odot \mathbf{A} = \mathbf{U}\Sigma\mathbf{F}$. Due to the relations $\mathbf{S}^H\mathbf{S} = \mathbf{I}_R, \mathbf{X}_{(2)} = (\mathbf{H} \odot \mathbf{A})\,\mathbf{S}^T = \mathbf{U}\Sigma\mathbf{V}^H$ and $\mathbf{H} \odot \mathbf{A} = \mathbf{U}\Sigma\mathbf{F}$ we obtain the factorization $\mathbf{S} = \mathbf{V}^*\mathbf{F}^*$ with $\mathbf{F}$ being a unitary matrix.

The first step of the SSD-PARAFAC method is to find the matrix $\mathbf{V}$ from the SVD of $\mathbf{X}_{(2)}$. Let $\widetilde{\mathbf{X}}_{(2)} = \mathbf{X}_{(2)}\mathbf{V} = (\mathbf{H} \odot \mathbf{A})\,\mathbf{F}^H$ and let $\mathbf{A} = \mathbf{Q}\mathbf{R}$ be the QR factorization of $\mathbf{A}$, where $\mathbf{Q} \in \mathbb{C}^{I\times I}$ is a unitary matrix and $\mathbf{R} \in \mathbb{C}^{I\times R}$ is an upper triangular matrix. Then an estimate of the unknowns will be obtained from the cost function

$$\begin{aligned} f\,(\mathbf{Q},\mathbf{R},\mathbf{H},\mathbf{F}) &= \sum_{j=1}^{J} \|\widetilde{\mathbf{X}}^{(j)} - \mathbf{Q}\mathbf{R}D_j\,(\mathbf{H})\,\mathbf{F}^H\|_F^2 \\ &= \sum_{j=1}^{J} \|\mathbf{Q}^H\widetilde{\mathbf{X}}^{(j)}\mathbf{F} - \mathbf{R}D_j\,(\mathbf{H})\,\|_F^2. \end{aligned}$$

To simplify the problem, we will in the first step of the SSD-PARAFAC method estimate the unitary matrices $\mathbf{Q}$ and $\mathbf{F}$ that make the matrices $\{\widetilde{\mathbf{X}}^{(j)}\}$ as upper triangular as possible. Formally, we maximize

$$g\,(\mathbf{Q},\mathbf{F}) = \sum_{j=1}^{J} \|\mathrm{triu}\left(\mathbf{Q}^H\widetilde{\mathbf{X}}^{(j)}\mathbf{F}\right)\|_F^2.$$

This approach can be seen as an orthogonality constrained version of the SSD-PARAFAC method presented in [5]. For the estimation of $\mathbf{Q}$ and $\mathbf{F}$ we will apply the extended QZ method [10].

Once the matrices $\mathbf{Q}$ and $\mathbf{F}$ have been found, the upper triangular matrix $\mathbf{R}$ and the diagonal matrices $\{D_j\,(\mathbf{H})\}$ can be found from a set of decoupled rank-1 approximation problems as discussed next. Let $\mathbf{R}^{(j)} = \mathrm{triu}\left(\mathbf{Q}^H\widetilde{\mathbf{X}}^{(j)}\mathbf{F}\right)$, then the problem of estimating $\mathbf{R}$ and $\{D_j\,(\mathbf{H})\}$ is equivalent to the minimization of

$$h\,(\mathbf{R},\mathbf{H}) = \sum_{j=1}^{J} \|\mathbf{R}^{(j)} - \mathbf{R}D_j\,(\mathbf{H})\,\|_F^2 = \sum_{r=1}^{R} \|\overline{\mathbf{R}}^{(r)} - \mathbf{r}_r\overline{\mathbf{d}}^{(r)T}\|_F^2,$$

where $\overline{\mathbf{R}}^{(r)} = \left[\mathbf{r}_r^{(1)},\cdots,\mathbf{r}_r^{(J)}\right]$, $\overline{\mathbf{d}}^{(r)} = \left[d_r^{(1)},\cdots,d_r^{(J)}\right]^T$, $\mathbf{r}_r^{(j)}$ and $\mathbf{r}_r$ are the rth column of the upper triangular part of $\mathbf{R}^{(j)}$ and $\mathbf{R}$, respectively, and $d_r^{(j)}$ is the rth entry of the diagonal part of $D_j\,(\mathbf{H})$.

Let $\overline{\mathbf{R}}^{(r)} = \mathbf{U}\Sigma\mathbf{V}^H$ denote the SVD of $\overline{\mathbf{R}}^{(r)}$, then $\mathbf{r}_r = \sigma_1\mathbf{u}_1$ and $\overline{\mathbf{d}}^{(r)} = \mathbf{v}_1^*$, where σ_1 denotes the largest singular value of $\overline{\mathbf{R}}^{(r)}$ and $\mathbf{u}_1$ and $\mathbf{v}_1$ denote its corresponding left and right singular vector, respectively.

2.3. ALS-PARAFAC with Orthogonality Constraint in One Mode

The conventional ALS method attempts to minimize the cost function

$$f\,(\mathbf{A},\mathbf{H},\mathbf{S}) = \|\mathbf{X}_{(1)} - (\mathbf{A} \odot \mathbf{H})\,\mathbf{S}^T\|_F^2.$$

The main idea behind the ALS method is to update a subset of the parameters $\{\mathbf{A},\mathbf{H},\mathbf{S}\}$ conditioned on previously obtained estimates of the remaining parameters. When $\mathbf{H}$ and $\mathbf{S}$ are fixed, then the least squares estimate of $\mathbf{A}$ will be used as the conditional update of $\mathbf{A}$. Due to multilinearity similar conditional updates of $\mathbf{H}$ and $\mathbf{S}$ are applied. When the symbol matrix $\mathbf{S}$ is columnwise orthonormal, the ALS method reduces to a cyclic two-step procedure as explained next. A least squares estimate of $\mathbf{S}$ can be found from the expression

$$\begin{aligned} g\,(\mathbf{S}) &= \|\mathbf{X}_{(1)} - (\mathbf{A} \odot \mathbf{H})\,\mathbf{S}^T\|_F^2 \\ &= \|\mathbf{X}_{(1)}\|_F^2 + \|\mathbf{A} \odot \mathbf{H}\|_F^2 - 2\mathrm{Re}\left\{\mathrm{Tr}\left(\mathbf{S}^*\,(\mathbf{A} \odot \mathbf{H})^H\,\mathbf{X}_{(1)}\right)\right\}. \end{aligned}$$

Let $\mathbf{U}\Sigma\mathbf{V}^H = (\mathbf{A} \odot \mathbf{H})^H\,\mathbf{X}_{(1)}$ denote the SVD, then the optimal $\mathbf{S}$ is $\mathbf{S} = \mathbf{V}^*\mathbf{U}^T$. This is known as the unitary Procrustes problem [11]. Now, let $\mathbf{S}$ be fixed, then an estimate of $\mathbf{A}$ and $\mathbf{H}$ can be obtained from a series of best rank-1 approximations, as explained in subsection 2.1. This two-step conditional updating procedure is repeated until convergence. Similar techniques have been proposed in [12], [13], [14].

3. SIMULATION RESULTS

In order to determine if imposing orthogonality on the symbol matrix will reduce the Bit Error Rate (BER), some simulations have been conducted. In all the simulations the real and imaginary elements of $\mathbf{A} \in \mathbb{C}^{I\times R}$ and $\mathbf{H} \in \mathbb{C}^{J\times R}$ are drawn from a Gaussian distribution with zero mean and unit variance. The symbols are drawn from a QPSK sequence and Additive White Gaussian Noise (AWGN) is added to the transmitted signal.

When the Frobenius norm of the difference between the estimated matrix $\mathbf{F}$ at iteration k and $k+1$ is less than $\epsilon = 10^{-5}$, then we decide that the extended QZ method has converged. Moreover, we decide that the ALS method and the simultaneous Takagi sweeping procedure have converged when the value of their respective cost function at iteration k and $k+1$ has changed less than $\epsilon = 10^{-5}$.

The first simulation will compare the SD-PARAFAC receiver [3], with the extended QZ iteration as its core iteration, against the orthogonality constrained SD-PARAFAC receiver (called SD-PARAFACO). The model parameters were set to $I = 4, J = 4, K = 1000, R = 8$ and the SNR value is varying from 0 to 20 dB with an increment of 5 dB. The mean BER over 100 trials can be seen in figure 1. A gain between 1 and 2 dB is obtained by

imposing column-wise orthonormality on the symbol matrix $\mathbf{S}$.

The second simulation will compare the ALS-PARAFAC receiver [1] with the orthogonality constrained SSD-PARAFAC receiver followed by two orthogonality constrained ALS-PARAFAC refinement iterations (called SSD-ALS-PARAFACO). The model parameters were set to $I = 4, J = 4, K = 120$, $R = 4$ and the SNR value is varying from 0 to 20 dB with an increment of 5 dB. The QPSK sequence is constructed such that $\mathbf{S}^H\mathbf{S} = 120 \cdot \mathbf{I}_4$. The mean BER over 100 trials can be seen in figure 2. A gain between 0.5 and 2 dB is obtained by imposing column-wise orthonormality on the symbol matrix.

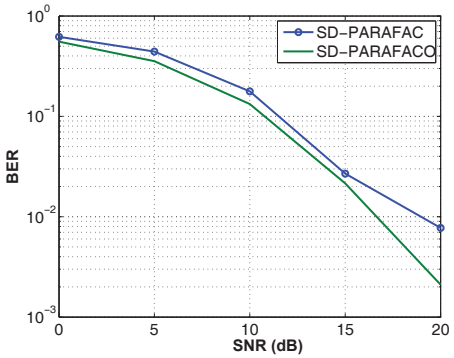


Fig. 1. The mean BER when SNR is varying from 0 to 20 dB.

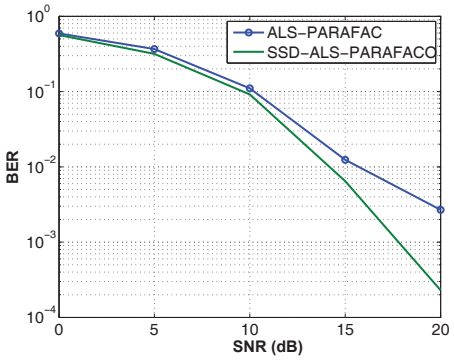


Fig. 2. The mean BER when SNR is varying from 0 to 20 dB.

4. CONCLUSION

We have presented a family of blind PARAFAC receivers for DS-CDMA capable of dealing with uncorrelated transmitted signals. It is based on orthogonality constrained versions of the SD-PARAFAC, SSD-PARAFAC and ALS-PARAFAC methods. Computer simulations showed that a gain between 0.5 and 2 dB was obtained when the uncorrelatedness of the transmitted signals was taken into account by the receiver. The proposed techniques for orthogonality constrained tensor decomposition could also be generalized to more challenging propagation schemes, starting from [15], [16]. A technique that combines the PARAFAC structure with statistical independence, rather than uncorrelatedness, is reported in [17]. Variants of [17] can be based on results derived in the present paper.

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