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Data-based Subsystem Identification for Dynamic Model Updating

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Abstract— **The accuracy of control and estimation tasks can strongly depend on the accuracy of the underlying model. In this paper, we consider a linear physical state space model subject to unmodeled dynamics in the state equation. In a first contribution, an optimal filter is outlined which yields estimates of the model error and the system states from measurements of the true system. In a second contribution, a technique is developed to update the model in case the unmodeled dynamics are arising from an unknown linear subsystem. In a third contribution, techniques are extended to nonlinear systems. Two illustrative simulation examples are included, a linear tape-drive modeling example and a nonlinear motor-pump example.**

I. Introduction

Models induced from physical laws and models identified from data are both approximate. In physical models, inaccuracies can be due to unmodeled dynamics or to incorrect parameter values. In empirical models, inaccuracies can be due to an inappropriate model class or to bad data quality. In this paper, we propose a *grey-box* modeling technique which combines physical and empirical modeling. More precisely, we consider the case where the known dynamics of the system have been converted into a linear physical state space model. However, due to unknown system-dynamics, the physical model is subject to unmodeled dynamics arising from an unknown subsystem. The aim of this paper is to correct or *update* the physical model using empirical modeling techniques, while the states keep their physical meaning.

Physical models with errors are also considered in [10]. A method is outlined for adaptively updating a nonlinear neural state observer in presence of unmodeled dynamics and incorrect parameter values. However, the aim of [10] is neither to estimate the model error, nor to correct the model, but rather to design the observer such that it compensates model errors.

In [4], a nonlinear model representation consisting of an interpolation of several first principles models, which are valid within certain operation regimes, is considered. In operation points where the physical models are not satisfactory accurate, they are integrated with empirical models to compensate for unmodeled dynamics.

Furthermore, in [9] linear state space models are updated by adding a *delta* model in parallel, cascade or feedback with the initial model. However, this method yields delta models which are generally of higher order than the initial model.

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This paper is outlined as follows. In section II, we show that the problem of model error estimation in physical models is closely related to the problem of unknown input estimation, for which optimal linear methods have been developed [6], [1], [2]. In section III, a technique is developed to update an incorrect physical model in case the error is due to unmodeled dynamics arising from a linear subsystem. In section IV, techniques are extended to nonlinear systems. Finally, in section V, two simulation examples are considered. The first example is linear and deals with updating a tape drive model. The second example addresses the problem of state estimation for a nonlinear motor-pump system in presence of unknown input.

II. Linear model error estimation

In this section, we first introduce the type of model errors that will be considered. Next, we show that the problem of estimating this type of errors is closely related to unknown input estimation, for which optimal methods have been developed.

A. Problem formulation

Consider a set of linear ordinary differential equations (ODE's) representing a physical model. Introducing measurements which are linearly dependent on the physical variables, this model can be written in state space form

$$\dot{x}(t) = A(t)x(t) + B(t)u(t), \tag{1}$$
$$y(t) = C(t)x(t), \tag{2}$$

where the state vector $x(t) \in \mathbb{R}^n$, the input vector $u(t) \in \mathbb{R}^m$ and the vector of measurements $y(t) \in \mathbb{R}^p$ all have a physical meaning. For simulation on a computer, the continuous-time model (1-2) is usually discretized in time, resulting in

$$x_{k+1} = A_k x_k + B_k u_k + w_k, \tag{3}$$
$$y_k = C_k x_k + v_k, \tag{4}$$

where $x_k \in \mathbb{R}^n$ is the state vector, $u_k \in \mathbb{R}^m$ is the input vector, $y_k \in \mathbb{R}^p$ is the vector of measurements and where the process noise $w_k \in \mathbb{R}^n$ and the measurement noise $v_k \in \mathbb{R}^p$ have been introduced to represent stochastic uncertainties in the state equation (e.g. due to discretization) and in the measurements, respectively. Both stochastic sequences are assumed to be mutually uncorrelated, zero-mean white with covariance matrices $Q_k = \mathbb{E}[w_k w_k^\mathsf{T}]$ and $R_k = \mathbb{E}[v_k v_k^\mathsf{T}]$, respectively. For linear ODE's, a substantial amount of stable and accurate discretization methods are available. While some methods preserve the physical meaning of the state, other methods lack this property. We consider discretization

methods which preserve the physical meaning of the state, such that $x_k \simeq x(kT_s)$, with T_s the sampling time.

For physical models, the measurements are usually direct observations of a state variable or well-known linear combinations of only a few state variables. Consequently, the output equations (2) and (4) are assumed to be very accurate. The state equation (1), on the other hand, can be subject to incorrect parameter values or unmodeled dynamics. To compensate for these *model errors*, we add a correction term $d_k \in \mathbb{R}^{m_d}$ to the state equation (3), resulting in

$$x_{k+1} = A_k x_k + B_k u_k + G_k d_k + w_k, \quad (5)$$

$$y_k = C_k x_k + v_k, \quad (6)$$

where the matrix G_k is assumed to be known, or chosen appropriately. Without loss of generality, we assume that G_k is chosen such that there exists an initial state x_0 and a particular set $\{d_0, d_1, \ldots\}$, for which (5-6) has the same behavior as the true system, in the sense that starting from x_0, x_k equals the system state and y_k equals the measurement at time-instant kT_s. In the remainder of this section, we are concerned with obtaining optimal estimates of the model error d_k and the system state x_k from measurements y_k of the true system.

B. Optimal state and model error estimation

Notice that d_k enters (5) like an unknown input. Optimal recursive state filters for linear systems with unknown inputs have been developed in [6], [1] and extended to joint input and state estimation in [2]. The filter developed in [2], takes the recursive form

$$\hat{x}_{k|k-1} = A_{k-1}\hat{x}_{k-1|k-1} + B_{k-1}u_{k-1}, \quad (7)$$

$$\hat{d}_{k-1} = M_k(y_k - C_k\hat{x}_{k|k-1}), \quad (8)$$

$$\hat{x}^*_{k|k} = \hat{x}_{k|k-1} + G_{k-1}\hat{d}_{k-1}, \quad (9)$$

$$\hat{x}_{k|k} = \hat{x}^*_{k|k} + K_k(y_k - C_k\hat{x}^*_{k|k}), \quad (10)$$

where $x_{k|l}$ denotes the estimate of the system state at the discrete time instant k, given measurements up to time l. Notice that due to the model error, (7) is not an unbiased estimate of the system state x_k. Therefore, in the second step, M_k is determined such that (8) is a minimum-variance unbiased (MVU) estimate of d_{k-1}. This estimate is then used for compensation, such that (9) is unbiased. In the final step, K_k is determined such that (10) is a MVU estimate of the system state x_k.

First, we calculate M_k such that (8) is a MVU estimate of d_{k-1}. Defining the *innovation* $\tilde{y}_k$ by $\tilde{y}_k = y_k - C_k\hat{x}_{k|k-1}$, it follows from (5-7) that

$$\tilde{y}_k = C_k G_{k-1} d_{k-1} + e_k, \quad (11)$$

where

$$e_k = C_k(A_{k-1}\tilde{x}_{k-1} + w_{k-1}) + v_k, \quad (12)$$

with $\tilde{x}_k = x_k - \hat{x}_{k|k}$. Let $\hat{x}_{k-1|k-1}$ be unbiased, then it follows from (12) that $\mathbb{E}[e_k] = 0$. Consequently, it follows from (11) that a MVU estimate of d_{k-1} can be obtained from the innovation $\tilde{y}_k$ by weighted least-squares estimation with weighting matrix $\tilde{R}_k = \mathbb{E}[e_k e_k^\mathsf{T}]$. Using (12), yields

$$\tilde{R}_k = C_k(A_{k-1}P_{k-1|k-1}A_{k-1}^\mathsf{T} + Q_{k-1})C_k^\mathsf{T} + R_k, \quad (13)$$

where the error covariance matrix $P_{k|k}$ is defined by $P_{k|k} = \mathbb{E}[\tilde{x}_k\tilde{x}_k^\mathsf{T}]$. Furthermore, defining the matrix $P_{k|k-1}$ by

$$P_{k|k-1} = A_{k-1}P_{k-1|k-1}A_{k-1}^\mathsf{T} + Q_{k-1}, \quad (14)$$

it follows from (13) that $\tilde{R}_k$ can be rewritten as

$$\tilde{R}_k = C_k P_{k|k-1} C_k^\mathsf{T} + R_k. \quad (15)$$

A MVU input estimate of d_{k-1} is then given in the following theorem.

Theorem 1 ([2]): Let $\hat{x}_{k-1|k-1}$ be unbiased, let $\tilde{R}_k$ be positive definite and let M_k be given by

$$M_k = \left(F_k^\mathsf{T}\tilde{R}_k^{-1}F_k\right)^{-1} F_k^\mathsf{T}\tilde{R}_k^{-1}, \quad (16)$$

where $F_k = C_k G_{k-1}$, then (7-8) is the MVU estimator of d_{k-1} given the innovation $\tilde{y}_k$. The variance of the corresponding input estimate is given by $(F_k^\mathsf{T}\tilde{R}_k^{-1}F_k)^{-1}$.

Notice that the inverse in (16) exists under the assumption that

$$\text{rank } C_k G_{k-1} = \text{rank } G_{k-1} = m_d. \quad (17)$$

Throughout the paper, we assume that this assumption holds. Notice that it implies $n \geq m_d$ and $p \geq m_d$.

As shown in [2], the optimal gain matrix K_k is not unique. One of the optimal gain matrices equals the expression for the Kalman gain,

$$K_k = P_{k|k-1}C_k^\mathsf{T}\tilde{R}_k^{-1}. \quad (18)$$

III. Subsystem identification and dynamic model updating

The objective of this section is to *correct* or *update* an inaccurate model in case the error is arising from an unknown linear *subsystem*.

A. Problem formulation

Consider the case where the initial model (3-4) is subject to unmodeled dynamics, such that true system is given by (5-6) with d_k arising from an *unknown* linear time-invariant *subsystem* driven by u_k *and* x_k,

$$z_{k+1} = A_d z_k + B_{d_u}u_k + B_{d_x}x_k + \omega_k, \quad (19)$$

$$d_k = C_d z_k + \nu_k, \quad (20)$$

where $z_k \in \mathbb{R}^{n_d}$ is the state vector and where the process noise $\omega_k \in \mathbb{R}^{n_d}$ and the measurement noise $\nu_k \in \mathbb{R}^{m_d}$ are unknown, mutually uncorrelated, zero-mean white signals with unknown covariance matrices, $Q_\omega = \mathbb{E}[\omega_k\omega_k^\mathsf{T}]$ and $R_\nu = \mathbb{E}[\nu_k\nu_k^\mathsf{T}]$, respectively. The aim of this section is to update the initial model (3-4) in case of unmodeled dynamics of the form (19-20).

B. Existing methods

The *parallel delta-augmentation* method in [9] updates an inaccurate initial model by identifying a *delta-model* with input u_k and with output an additive correction term for the output of the initial model. Let the initial model be given by (3-4) and let the identified delta-model be given by

$$z_{k+1} = \hat{A}_\Delta z_k + \hat{B}_\Delta u_k + \varpi_k, \tag{21}$$
$$\Delta y_k = \hat{C}_\Delta z_k + \upsilon_k, \tag{22}$$

then the updated model with corrected output $\breve{y}_k$ is given by

$$\begin{bmatrix} x_{k+1} \\ z_{k+1} \end{bmatrix} = \begin{bmatrix} A_k & 0 \\ 0 & \hat{A}_\Delta \end{bmatrix} \begin{bmatrix} x_k \\ z_k \end{bmatrix} + \begin{bmatrix} B_k \\ \hat{B}_\Delta \end{bmatrix} u_k + \begin{bmatrix} w_k \\ \varpi_k \end{bmatrix}, \tag{23}$$
$$\breve{y}_k = \begin{bmatrix} C_k & \hat{C}_\Delta \end{bmatrix} \begin{bmatrix} x_k \\ z_k \end{bmatrix} + \upsilon_k + \upsilon_k. \tag{24}$$

In case of unmodeled dynamics of the form (19-20), it is easy to show that the *optimal* delta-model has order $n+n_d$, which equals the order of the true system. In cases where n is large, while n_d is relatively small, this may yield a delta-model which is of much higher order than the unknown subsystem (19-20). Furthermore, this method does only correct the output, but not the errorneous state equations of the initial model. In cases where the states have a physical meaning which is of importance for e.g. control, this is a major disadvantage.

C. Subsystem identification and dynamic model updating

To overcome the problems encountered with the method of [9], we consider the problem of identifying a correction model of the form (19-20). Notice that the optimal filter proposed in section II yields estimates $\hat{x}_{k|k}$ of the input x_k and estimates $\hat{d}_k$ of the output d_k of the unknown subsystem (19-20). More precisely, after running the filter from time $k=0$ to $k=N+1$, the following three data-sets are obtained,

$$\mathcal{U}_N = \{u_j \mid j=0 \dots N\}, \tag{25}$$
$$\mathcal{X}_N = \{\hat{x}_{j|j} \mid j=0 \dots N\}, \tag{26}$$
$$\mathcal{D}_N = \{\hat{d}_j \mid j=0 \dots N\}. \tag{27}$$

Hence, one deterministic data-set $\mathcal{U}_N$ and one noisy data-set $\mathcal{X}_N$ of inputs of the unknown subsystem (19-20) are available, as well as one noisy data-set $\mathcal{D}_N$ of outputs of the subsystem. A LTI correction model approximating the dynamics of the subsystem, can be identified from these data-sets by a combined deterministic-stochastic *subspace identification* algorithm [8]. Subspace identification is an empirical identification technique which yields a linear stochastic state space model from a set of input-output data only. The major advantage of subspace identification algorithms over the classical prediction error methods is the absence of non-linear parametric optimization problems. The initial and current state are estimated, as well as the system-matrices and the covariance matrices of the noise. Hence, the identified *correction model* takes the form

$$z_{k+1} = \hat{A}_d z_k + \hat{B}_{d_u} u_k + \hat{B}_{d_x} \hat{x}_k + \hat{\omega}_k, \tag{28}$$
$$\hat{d}_k = \hat{C}_d z_k + \hat{\nu}_k, \tag{29}$$

where $z_k \in \mathbb{R}^{\hat{n}_d}$ is the state vector. The process noise $\hat{\omega}_k \in \mathbb{R}^{\hat{n}_d}$ and measurement noise $\hat{\nu}_k \in \mathbb{R}^{m_d}$ are unknown, mutually uncorrelated, zero-mean white signals with estimated covariance matrices $\hat{Q}_{\hat{\omega}} = \mathbb{E}[\hat{\omega}_k \hat{\omega}_k^\mathsf{T}]$ and $\hat{R}_{\hat{\nu}} = \mathbb{E}[\hat{\nu}_k \hat{\nu}_k^\mathsf{T}]$, respectively. In practical applications where a lot of data is available, it is to be expected that $\hat{n}_d$ is close to n_d. In cases where the true system is high order, while the error is relatively low order, this may yield a considerable storage and computational saving over the parallel delta-augmentation method.

The initial model (3-4) is then *augmented* with the identified correction model (28-29) to the single, linear stochastic state space model

$$\begin{bmatrix} x_{k+1} \\ z_{k+1} \end{bmatrix} = \begin{bmatrix} A_k & G_k \hat{C}_d \\ \hat{B}_{d_x} & \hat{A}_d \end{bmatrix} \begin{bmatrix} x_k \\ z_k \end{bmatrix} + \begin{bmatrix} B_k \\ \hat{B}_{d_u} \end{bmatrix} u_k + \begin{bmatrix} w_k + G_k \hat{\nu}_k \\ \hat{\omega}_k \end{bmatrix}, \tag{30}$$
$$\begin{bmatrix} y_k \\ \hat{d}_k \end{bmatrix} = \begin{bmatrix} C_k & 0 \\ 0 & \hat{C}_d \end{bmatrix} \begin{bmatrix} x_k \\ z_k \end{bmatrix} + \begin{bmatrix} v_k \\ \hat{\nu}_k \end{bmatrix}, \tag{31}$$

of order $n+\hat{n}_d$. In contrast to the parallel delta-augmentation method, this kind of model updating corrects the errorneous state equations of the initial model. The difference between both methods is also noticeable in the interconnection between the dynamics of the initial model and the correction model. There is no interconnection in the parallel delta-augmentation method, while in our approach the state equations of both models are interconnected by x_k and $\hat{d}_k$. This interconnection can also be seen in the $A-$matrix of the augmented model (30), which is dense, in contrast to (23), where it is block-diagonal.

IV. Extension to nonlinear systems

In this section, we extend the techniques of sections II and III to nonlinear systems. Consider the nonlinear discrete-time stochastic model

$$x_{k+1} = f_k(x_k, u_k) + w_k, \tag{32}$$
$$y_k = C_k x_k + v_k, \tag{33}$$

where $x_k \in \mathbb{R}^n$ is the state vector, $u_k \in \mathbb{R}^m$ is the input vector and $y_k \in \mathbb{R}^p$ is the vector of outputs. Again, we assume that a non-negligible error is present in the state equation (32). Without loss of generality, we assume that for a particular set $\{d_0, d_1, \dots\}$ the model

$$x_{k+1} = f_k(x_k, u_k) + G_k d_k + w_k, \tag{34}$$
$$y_k = C_k x_k + v_k, \tag{35}$$

has the same behavior as the true system.

We extend the model error estimation technique of section II to the nonlinear case by using Monte Carlo filtering techniques. By using an approach similar to the particle filter,

an ensemble of state estimates and model error estimates may be obtained. This approach is advised when non-smooth and strong nonlinearities are present in the model operator $f_k(\cdot)$. Alternatively, the Unscented Kalman filter (UKF) [5] can be used. Unlike the particle filter which requires lots of sample point, the UKF uses only $n+1$ deterministically chosen sample points which completely capture the mean $\hat{x}_{k-1|k-1}$ and covariance $P_{k-1|k-1}$. When propagated through the nonlinear model $f_{k-1}(\cdot)$, the forecasted set of sample points captures the posterior mean and covariance accurately to the second order (Taylor series expansion) for any nonlinearity. This means that $\hat{x}_{k|k-1}$ and $P_{k|k-1}$ are accurate to the second order. Next, the model error is estimated from the innovations and the state is updated using the same formulas as in the linear case, that is, using (8-10).

The subsystem identification and model updating technique developed in section III can in principle also be extended to the nonlinear case by applying nonlinear identification methods. Nonlinear extensions of subspace identification comprise for example identification of Wiener and Hammerstein systems [3], [11].

V. Simulation examples

We consider two simulation examples. In the first example, an inaccurate linear tape drive model is updated. The second and nonlinear example, deals with state estimation of a motor-pump in presence of model errors due to an unknown input.

A. Tape drive modeling

The tape drive considered in this example comprises one tape, two drive wheels and two DC-motors which are independently controllable by voltage sources V_1 and V_2 and which drive the take-up wheels. The armature circuit of DC-motor j $(j=1,2)$ is expressed as

$$L_j \frac{dI_j(t)}{dt} + R_j I_j(t) + K_{e_j}\omega_j(t) = V_j(t), \qquad (36)$$

where L is the armature inductance, R is the armature resistance, K_e is the electrical constant of the motor, I is the current and ω is the rotational speed of the drive wheel. The position p of the drive wheels is related to ω by the radius r,

$$\frac{dp_j(t)}{dt} = r_j\omega_j(t). \qquad (37)$$

The dynamic equation for the rotational speed of the drive wheels is described by

$$J_j \frac{d\omega_j(t)}{dt} = -T(t)r_j - \beta\omega_j(t) + K_{t_j} I_j(t), \qquad (38)$$

where J is the inertia of the drive wheel and motor, β is the rotational friction of the drive wheel and motor, K_t is the torque constant of the motor and T is the tape tension. This tension is given by

$$T(t) = \frac{K}{2}\Delta p(t) + \frac{D}{2}\left(\frac{d\Delta p(t)}{dt}\right), \qquad (39)$$

TABLE I: Parameter values used in the tape drive example

Parameter	Value	Unit
L	10^{-3}	H
R	1	Ω
K_e	3.10^{-2}	V.s
r	25.10^{-3}	m
J	5.10^{-5}	kg.m^2
β	5.10^{-2}	kg.m^2.s^{-1}
K_t	3.10^{-2}	N.m.A^{-1}
K	2.10^4	N.m^{-1}
D	10	N.m^{-1}.s^{-1}

where $\Delta p(t) = p_2(t) - p_1(t)$, K is the spring constant of the tape and D is the damping in the tape-stretch motion. Equations (36)-(39) constitute the dynamics of the true system. The parameter values used in the example, are given in Table I.

For the initial model, it is assumed that the dynamics and the parameter values of the DC-motors as well as the radius of the drive wheels are known with high precision. Concerning (38), it is assumed that J, β and K_{t_j} are known with high precision. However, the dynamics describing the tension in the tape are not known and thus omitted. Hence, the initial model is given by (36-37) and

$$J_j \frac{d\omega_j(t)}{dt} = -\beta\omega_j(t) + K_{t_j} I_j(t). \qquad (40)$$

A linear continuous-time state space model is obtained by defining the input vector $u(t) = [V_1(t)\ V_2(t)]^\mathsf{T}$ and the state vector $x(t) = [I_1(t)\ I_2(t)\ p_1(t)\ p_2(t)\ \omega_1(t)\ \omega_2(t)]^\mathsf{T}$. This model is discretized in time using the zero order hold method with sampling time $T_s = 10^{-4}s$. To account for the unknown tension, we add a term Gd_k to the discretized model, where

$$G = \begin{bmatrix} 0 & 0 & 0 & 0 & -\frac{r_1}{J_1} & -\frac{r_2}{J_2} \end{bmatrix}^\mathsf{T} \qquad (41)$$

is chosen such that d_k equals the unknown tape tension $T(kT_s)$. It is assumed that measurements of all state variables are available, $C = I_6$. Notice that for these choices of C and G, the existence assumption (17) is satisfied. Process and measurement noise are assumed to be Gaussian white, with covariance matrices $Q = R = \mathrm{diag}([10^{-6}\ 10^{-6}\ 10^{-8}\ 10^{-8}\ 10^{-4}\ 10^{-4}])$.

First, we apply the optimal filter developed in section II to simultaneously estimate the system state and the tape tension from noisy measurements of the true system. The true and estimated value of the tape tension, together with 95% confidence intervals, are shown in Fig. 1. The confidence intervals are calculated from the variance of the estimator (16), given by $(F_k^\mathsf{T}\tilde{R}_k^{-1}F_k)^{-1}$. In Table II, the estimation error (EE), defined by

$$\mathrm{EE} = \frac{100}{p}\sum_{i=1}^{p}\left[\sqrt{\frac{\sum_{k=1}^{N}((y_k)_i - (\hat{y}_{k|k})_i)^2}{\sum_{k=1}^{N}((y_k)_i)^2}}\right]\%, \qquad (42)$$

with $\hat{y}_{k|k} = C_k\hat{x}_{k|k}$, is compared for the unknown input filter and the Kalman filter. Due to the unknown tension, the Kalman filter performs very badly.

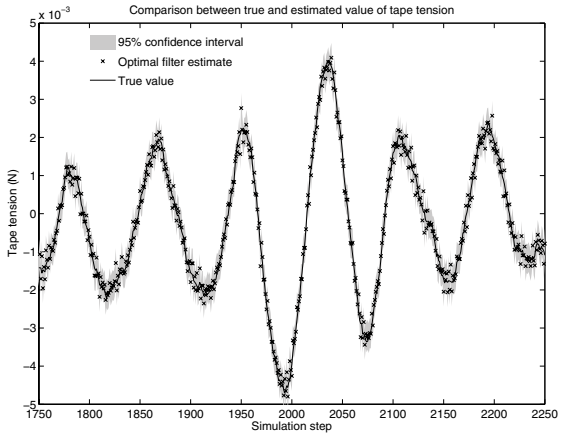


Fig. 1: Tape drive modeling example; comparison between true and estimated value of tape tension.

TABLE II: Comparison between the estimation error (EE) of the Kalman filter and the unknown input filter presented in section II-B.

Filter	EE (%)
Kalman filter	29.5
Unknown input filter (sect. II-B)	0.2

Next, we identify the unknown subsystem with the N4SID subspace identification algorithm [7] using the input data sets $\mathcal{U}_N, \mathcal{X}_N$ and $\mathcal{D}_N$, defined by (25-27). The order of the identified model is determined by the N4SID algorithm and equals 1. Then, the nominal model is augmented with the identified correction model, resulting in an updated model of order 7. For comparison purpose, we also identified a correction model using the parallel delta-augmentation method described in section III-B. The delta model has order 6, resulting in an updated model of order 12.

The updated models are validated is two different ways. Firstly, the initial model and the updated models are simulated using validation inputs and the outputs are compared to the measurements of the true system. Table III compares the simulation error (SE) and the one step ahead prediction error (PE) of all models. The SE is defined by (42) with $\hat{y}_{k|k}$ replaced by the simulated output of the model. The PE is defined by (42) with $\hat{y}_{k|k}$ replaced by $\hat{y}_{k|k-1} = C_k \hat{x}_{k|k-1}$, where $\hat{x}_{k|k-1}$ is obtained with a Kalman filter. Secondly, the dynamically updated model is validated by computing the autocorrelation of the one step ahead prediction residuals $\tilde{y}_k$, which should be uncorrelated in time. Fig. 2 shows the autocorrelation with 99% confidence intervals for the initial model (top) and the updated model (bottom). For the initial model, the correlation between the current and future residuals falls out the confidence region, indicating that the residuals are strongly correlated. The residuals of the updated model are still correlated in time, as can be seen in the lower part of the figure. However, correlation is much smaller than for the initial model.

TABLE III: Comparison between simulation error (SE) and prediction (PE) of initial and updated models.

Model	Order	SE (%)	PE (%)
Initial model	6	38.6	26.4
Dynamic updating	7	5.2	0.3
Parallel updating	12	6.7	4.5

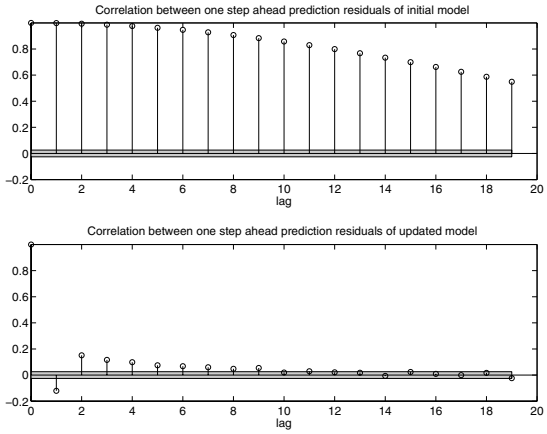


Fig. 2: Tape drive modeling example; autocorrelation of the prediction residuals $\tilde{y}_k$ (with 99% confidence intervals, indicated by the grey rectangles) for the initial model (top) and the updated model (bottom).

B. Motor-pump modeling

In the second example, we consider the motor-pump system which was also used in [10] for state estimation in presence of model errors. This nonlinear system consists of a motor, a centrifugal pump and piping system. The equation for the DC-motor is given by

$$L\frac{dI(t)}{dt} + R(t)I(t) + K_e\omega(t) = V(t), \tag{43}$$

where the resistance varies with the current according to

$$R(t) = \alpha\frac{dI^2(t)}{dt} + R_0. \tag{44}$$

The equation describing the angular velocity, is given by

$$\theta\frac{d\omega(t)}{dt} = \Psi I(t) - \beta_0 - \beta_1\omega(t) - h_p\dot{M}(t)\omega(t), \tag{45}$$

where θ is the inertia of the motor-pump, Ψ is the motor flux linkage, β_0 is the static friction coefficient, β_1 is the dynamic friction coefficient, h_p is the pump constant and $\dot{M}(t)$ is the mass flow rate. The mass flow rate evolves according to

$$\frac{d\dot{M}(t)}{dt} = h_n\dot{M}^2(t) - h_r(t)\dot{M}^2(t), \tag{46}$$

where h_n is the piping flow resistance and $h_r(t)$ is the time-varying pump load. This load can not be directly measured and thus enters the system like an unknown input. It is assumed that three system states, $I(t), \omega(t)$ and $\dot{M}(t)$ are

TABLE IV: Parameter values used in the motor-pump model

Parameter	Value	Unit
L	2.10^{-3}	H
R	1	Ω
K_e	5.10^{-2}	V.s
α	5.10^{-3}	Ω.s.A^{-2}
R_0	1,5	Ω
θ	5.10^{-4}	kg.m^2
Ψ	5	N.m.A^{-1}
β_0	1.10^{-4}	N.m
β_1	2.10^{-5}	N.m.s^{-1}
h_p	7,5	m^2
h_n	10	kg^{-1}

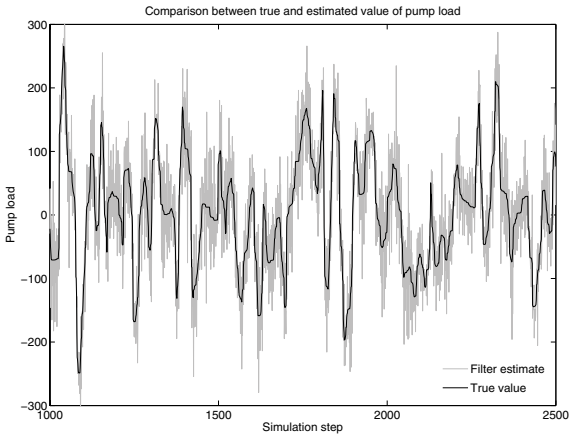


Fig. 3: Motor-pump modeling example; comparison between true and estimated value of the pump load.

measured. The armature resistance $R(t)$, which is an important parameter of the system, can not be directly measured and must be estimated. The parameter values used in this example, are given in Table IV.

The nonlinear model is discretized in time and the nonlinear filtering technique of section IV is used to estimate the system states and the error due to the unknown input in the discretized version of equation (46). Finally, the unknown load $h_r(t)$ is calculated from the estimated state and the estimated model error. Fig. 3 compares the true and estimated value of the pump load. Notice that the variance of the estimated pump load is rather high because the load is nonlinearly computed from estimated values of the model error and the system state. For accurate computation, process and measurement noise variance must be very small. Finally, Table V compares the estimation error of a UKF which neglects the pump-load, to the estimation error of the nonlinear unknown input filter of section IV based on the UKF.

VI. Conclusion

This paper has studied the problem of model error estimation and dynamic model updating for physical state space models. We showed that the problem of model error estimation is closely related to unknown input estimation,

TABLE V: Comparison between EE of a UKF which neglects the pump-load and EE of an unknown input filter based on the UKF (section IV).

Filter	EE (%)
UKF	1.4
Nonlinear unknown input filter based on UKF	0.05

for which optimal linear filters have been developed. The proposed model updating technique consists in augmenting the initial model with an empirical model obtained from the estimates of the model error by subspace identification. This grey-box modeling technique is of particular interest when the states of the initial model have a physical meaning and have to keep this meaning after the update. Simulation results on a linear tape drive model and a nonlinear motor-pump model show the effectiveness of the method.

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