Katholieke Universiteit Leuven

Departement Elektrotechniek

ESAT-SISTA/TR 06-166

Linear Recursive Filtering with Noisy Input and Output Measurements[1]

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October 2006

Internal Report
Submitted for publication

[1]This report is available by anonymous ftp from *ftp.esat.kuleuven.ac.be* in the directory *pub/sista/gillijns/reports/TR-06-166.pdf*

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Abstract

This paper considers an extension of the errors-in-variables filtering problem to the case where a linear combination of the input vector is measured instead of the entire input vector. Based on an optimal filter for system with unknown inputs, a recursive filter is developed where the estimation of the system state and the unknown input are interconnected. The filter provides a new solution to the errors-in-variables filtering problem which is shown to be algebraically equivalent to existing results.

1 Introduction

Since the introduction of the Kalman filter in 1960 [12], the optimal filtering problem has received considerable attention. The number of applications involving the Kalman filter is still growing day by day. This gain in popularity is probably due to the recursive structure of the Kalman filter which allows for real-time state estimation.

However, the derivation of the Kalman filter is based on several assumptions. One of these assumptions is that the system input is known exactly. This assumption is realistic in control applications where the input to the system is generated by a known control law, but can be too restrictive in other applications.

The errors-in-variables filtering problem, an extension of the Kalman filtering problem to the case where the input is known up to an additive noise term, was first addressed in [8, 3]. They derived filter equations for the linear time-invariant SISO case based on a transfer function approach. The MIMO case is first addressed in [14], where it is shown that the errors-in-variables problem is equivalent to the Kalman filtering problem for a system with correlated process and measurement noise. A similar result can be found in [4].

The problem of optimal filtering in the presence of inputs which are completely unknown has also received a lot of attention. Numerous approaches to deal with input disturbances can be found in literature, such as methods based on state augmentation [1], moving horizon approaches [15], sliding mode observers [5] and estimators which minimize the variance of the estimation error under an unbiasedness condition [13, 2, 9]. Also, the problem of joint input and state estimation has been intensively studied [10, 7, 6].

In this paper, we consider an extension of the errors-in-variables filtering problem to the case where a linear combination of the input vector is measured instead of the entire input vector. We solve that this problem can be reformulated as the unknown input filtering problem considered in [6]. Based on the results of [6], we derive a recursive filter where the estimation of the system state and the input vector are interconnected. As a special case, the filter provides a new solution to the errors-in-variables filtering problem which is shown to be algebraically equivalent to the filters of [14, 4].

This paper is outlined as follows. In section 2, we state the filtering problem considered in the paper. Next, in section 3, we give the relation to the unknown input filter considered in [6]. In section 4, we derive specialize to the case where the measurement noises are uncorrelated. Section 5 deals with errors-in-variables filtering. And finally, in section 6, we summarize the filter equations.

2 Problem statement

Consider the linear discrete-time system

$$x_{k+1} = A_k x_k + G_k d_k + w_k, \tag{1}$$

together with the measurements

$$\underbrace{\begin{bmatrix} y_{1,k} \\ y_{2,k} \end{bmatrix}}_{y_k} = \underbrace{\begin{bmatrix} C_{1,k} \\ 0 \end{bmatrix}}_{C_k} x_k + \underbrace{\begin{bmatrix} H_{1,k} \\ H_{2,k} \end{bmatrix}}_{H_k} d_k + \underbrace{\begin{bmatrix} v_{1,k} \\ v_{2,k} \end{bmatrix}}_{v_k}, \tag{2}$$

where $x_k \in \mathbb{R}^n$ is the state vector, $d_k \in \mathbb{R}^m$ is an unknown input vector, $y_{1,k} \in \mathbb{R}^p$ is a measurement at the output of the system and $y_{2,k} \in \mathbb{R}^l$ is a measurement at the input of the system. We assume that the measurement noises $v_{1,k}$ and $v_{2,k}$ are zero-mean white random with

$$R_k := \mathbb{E}\left\{ \begin{bmatrix} v_{1,k} \\ v_{2,k} \end{bmatrix} \begin{bmatrix} v_{1,k}^\mathsf{T} & v_{2,k}^\mathsf{T} \end{bmatrix} \right\} = \begin{bmatrix} R_{1,k} & R_{12,k} \\ R_{12,k}^\mathsf{T} & R_{2,k} \end{bmatrix}. \tag{3}$$

The process noise w_k is assumed to be a zero-mean white random signals, uncorrelated with $v_{1,k}$ and $v_{2,k}$, with covariance matrix $Q_k := \mathbb{E}[w_k w_k^\mathsf{T}]$. Results are easily generalized to the case where w_k is correlated to $v_{1,k}$ and / or $v_{2,k}$ by transforming (1)-(2) into a system where process and measurement noise are uncorrelated [1]. Finally, we assume that the pair (C_k, A_k) is observable and that an unbiased estimate $\hat{x}_0$ of the initial state x_0 is available with covariance matrix P_0^x.

The objective of this paper is to design an optimal recursive filter which estimates both the system state x_k and the input d_k based on the initial estimate $\hat{x}_0$ and the sequences of measurements $\{y_{1,0}, y_{1,1}, \ldots, y_{1,k}\}$ and $\{y_{2,0}, y_{2,1}, \ldots, y_{2,k}\}$. No other information about d_k is assumed to be available and no assumption about its evolution is made. Therefore, it is not possible to use the well-known technique of augmenting the state vector with the input vector.

Note that in case $H_{2,k} = I$, the problem reduces to the errors-in-variables filtering problem [8, 14, 4].

3 Relation to unknown input filtering

Note that the filtering problem considered in the previous section is equivalent to the problem addressed in [6]. In this section, we summarize the equations of the optimal filter developed in [6].

The optimal filter takes the form of the Kalman filter, except that the true value of the input is replaced by an optimal estimate. The filter equations consist of three steps: 1) the estimation of the unknown input, 2) the measurement update and 3) the time update. These steps are given by:

1. Input estimation:

$$\hat{d}_k = M_k(y_k - C_k\hat{x}_{k|k-1}), \tag{4}$$
$$M_k = (H_k^\mathsf{T}\tilde{R}_k^{-1}H_k)^{-1}H_k^\mathsf{T}\tilde{R}_k^{-1}, \tag{5}$$
$$\tilde{R}_k = C_kP_{k|k-1}^xC_k^\mathsf{T} + R_k, \tag{6}$$
$$P_k^d = (H_k^\mathsf{T}\tilde{R}_k^{-1}H_k)^{-1}, \tag{7}$$

2. Measurement update:

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k(y_k - C_k\hat{x}_{k|k-1} - H_k\hat{d}_k), \tag{8}$$
$$K_k = P_{k|k-1}^xC_k^\mathsf{T}\tilde{R}_k^{-1}, \tag{9}$$
$$P_{k|k}^x = P_{k|k-1}^x - K_k(\tilde{R}_k - H_kP_k^dH_k^\mathsf{T})K_k^\mathsf{T}, \tag{10}$$
$$P_{k|k}^{xd} = (P_{k|k}^{dx})^\mathsf{T} = -K_kH_kP_k^d, \tag{11}$$

3. Time update:

$$\hat{x}_{k+1|k} = A_k\hat{x}_{k|k} + G_k\hat{d}_k, \tag{12}$$
$$P_{k+1|k}^x = [A_k\ G_k]\begin{bmatrix} P_{k|k}^x & P_k^{xd} \\ P_k^{dx} & P_k^d \end{bmatrix}\begin{bmatrix} A_k^\mathsf{T} \\ G_k^\mathsf{T} \end{bmatrix} + Q_k, \tag{13}$$

where $\hat{x}_{k|l}$ denotes the optimal estimate of x_k given measurements up to time l, and where the covariance matrices $P_{k|k}^x, P_{k|k-1}^x, P_k^d, P_{k|k}^{xd}$ and $P_{k|k}^{dx}$ are defined by

$$P_{k|k}^x := \mathbb{E}[(x_k - \hat{x}_{k|k})(x_k - \hat{x}_{k|k})^\mathsf{T}],$$
$$P_{k|k-1}^x := \mathbb{E}[(x_k - \hat{x}_{k|k-1})(x_k - \hat{x}_{k|k-1})^\mathsf{T}],$$
$$P_k^d := \mathbb{E}[(d_k - \hat{d}_k)(d_k - \hat{d}_k)^\mathsf{T}],$$
$$P_{k|k}^{xd} = (P_{k|k}^{dx})^\mathsf{T} := \mathbb{E}[(x_k - \hat{x}_{k|k})(d_k - \hat{d}_k)^\mathsf{T}].$$

Note from (5) and (7) that a necessary condition for the existence of the estimator is rank $H_k = m$. Furthermore, note that this condition is always satisfied in case of errors-in-variables filtering.

The filter equations derived above provide a general solution to the filtering problem formulated in section 2 and thus also to the errors-in-variables problem. In contrast to the results of [14, 4], this solution is not limited to $R_{12,k} = 0$.

4 Uncorrelated measurement noise

In this section, we specialize to the case where the noise on the input measurements is uncorrelated to the noise on the output measurements. That is, we derive specific equations for the case $R_{12,k} = 0$.

Defining $\tilde{y}_k := y_k - C_k\hat{x}_{k|k-1}$, it follows from (2) that

$$\tilde{y}_k = \begin{bmatrix} y_{1,k} - C_{1,k}\hat{x}_{k|k-1} \\ y_{2,k} \end{bmatrix}. \tag{14}$$

Furthermore, for $R_{12,k} = 0$, it follows from (6) and (3) that

$$\tilde{R}_k = \begin{bmatrix} \tilde{R}_{1,k} & 0 \\ 0 & R_{2,k} \end{bmatrix}, \tag{15}$$

with

$$\tilde{R}_{1,k} = C_{1,k}P^x_{k|k-1}C^{\mathsf{T}}_{1,k} + R_{1,k}.$$

Now, we derive equations for the estimation of the input and for the measurement update.

1. Input estimation:

   By substituting (14) and (15) in (4), (5) and (7), we obtain the following equations for the estimation of the input,

   $$\hat{d}_k = M_k\tilde{y}_k, \tag{16}$$
   $$M_k = P^d_k \begin{bmatrix} H^{\mathsf{T}}_{1,k}\tilde{R}^{-1}_{1,k} & H^{\mathsf{T}}_{2,k}R^{-1}_{2,k} \end{bmatrix}, \tag{17}$$
   $$P^d_k = (H^{\mathsf{T}}_{1,k}\tilde{R}^{-1}_{1,k}H_{1,k} + H^{\mathsf{T}}_{2,k}R^{-1}_{2,k}H_{2,k})^{-1}. \tag{18}$$

2. Measurement update:

   Furthermore, by substituting (15) in (9), it follows that

   $$K_k = [K_{1,k} \;\; 0], \tag{19}$$

   where $K_{1,k} \in \mathbb{R}^{n\times p}$ is given by

   $$K_{1,k} = P^x_{k|k-1}C^{\mathsf{T}}_{1,k}\tilde{R}^{-1}_{1,k}. \tag{20}$$

   Finally, substituting (19) in (8), (10) and (11), yields

   $$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_{1,k}(y_{1,k} - C_{1,k}\hat{x}_{k|k-1} - H_{1,k}\hat{d}_k), \tag{21}$$

   and

   $$P^x_{k|k} = P^x_{k|k-1} - K_{1,k}(\tilde{R}_{1,k} - H_{1,k}P^d_kH^{\mathsf{T}}_{1,k})K_{1,k}, \tag{22}$$
   $$P^{xd}_k = (P^{dx}_k)^{\mathsf{T}} = -K_{1,k}H_{1,k}P^d_k, \tag{23}$$

   respectively.

We conclude from (16) and (21) that the input measurement $y_{2,k}$ is only needed during the estimation of the input and not during the measurement update.

5 Errors-in-variables filtering

If in addition to $R_{12,k} = 0$ also $H_{2,k} = I$, then the problem reduces to the errors-in-variables filtering problem considered in [8, 3, 4, 14]. They showed that the errors-in-variables filtering problem for the system (1)-(2) with $H_{2,k} = I$ is equivalent to the Kalman filtering problem for the system

$$\begin{aligned} x_{k+1} &= A_k x_k + G_k y_{2,k} + \tilde{w}_k, \\ y_{1,k} &= C_{1,k} x_k + H_{1,k} y_{2,k} + \tilde{v}_k, \end{aligned}$$

where the process noise $\tilde{w}_k := w_k - G_k v_{2,k}$ is correlated to the measurement noise $\tilde{v}_k := v_{1,k} - H_{1,k} v_{2,k}$. The estimation of the unknown input is also considered in [14].

Here, we propose a new solution to the errors-in-variables filtering problem based on the filter derived in the previous section. Substituting $H_{2,k} = I$ in (18) and applying the matrix inversion lemma, yields

$$P_k^d = (I - L_{1,k} H_{1,k}) R_{2,k}, \tag{24}$$

where the gain matrix $L_{1,k}$ is given by

$$L_{1,k} = R_{2,k} H_{1,k}^{\mathsf{T}} (H_{1,k} R_{2,k} H_{1,k}^{\mathsf{T}} + \tilde{R}_{1,k})^{-1}.$$

Substituting (24) in (17) and (16) yields the following estimate of the input,

$$\hat{d}_k = y_{2,k} + L_{1,k} (\tilde{y}_{1,k} - H_{1,k} y_{2,k}). \tag{25}$$

We thus obtain a new solution to the errors-in-variables filtering problem where, in contrast to the results of [8, 3, 4, 14], information between the state estimator and the input estimator is exchanged in tow directions. By eliminating the estimates of the input from the filter equations, it can be show that the new filter is algebraically equivalent to the filters developed in [14, 4]. A proof of the equivalence is given in Appendix A.

6 Summary of filter equations

In this section, we summarize the equations of the filter developed in this paper for $R_{12,k} = 0$. We assume that $\hat{x}_0$, the estimate of the initial state, is unbiased and has known variance P_0^x. The initialization step of the filter is then given by,

Initialization:

$$\begin{aligned} \hat{x}_{0|-1} &= \mathbb{E}[x_0], \\ P_{0|-1}^x &= \mathbb{E}[(x_0 - \hat{x}_{0|-1})(x_0 - \hat{x}_{0|-1})^{\mathsf{T}}]. \end{aligned}$$

The recursive part of the filter consists of three steps: 1) the estimation of the unknown input, 2) the measurement update and 3) the time update. These three steps are given by:

1. Input estimation:

$$\begin{aligned}\tilde{y}_{1,k} &= y_{1,k} - C_{1,k}\hat{x}_{k|k-1},\\ \tilde{R}_{1,k} &= C_{1,k}P^x_{k|k-1}C^{\mathsf{T}}_{1,k} + R_{1,k},\end{aligned}$$

   (a) Noisy input measurements

$$\begin{aligned}\hat{d}_k &= P^d_k(H^{\mathsf{T}}_{2,k}R^{-1}_{2,k}y_{2,k} + H^{\mathsf{T}}_{1,k}\tilde{R}^{-1}_{1,k}\tilde{y}_{1,k}),\\ P^d_k &= (H^{\mathsf{T}}_{2,k}R^{-1}_{2,k}H_{2,k} + H^{\mathsf{T}}_{1,k}\tilde{R}^{-1}_{1,k}H_{1,k})^{-1},\end{aligned}$$

   (b) Errors-in-variables filtering ($H_{2,k} = I$)

$$\begin{aligned}\hat{d}_k &= y_{2,k} + L_{1,k}(\tilde{y}_{1,k} - H_{1,k}y_{2,k}),\\ L_{1,k} &= R_{2,k}H^{\mathsf{T}}_{1,k}(H_{1,k}R_{2,k}H^{\mathsf{T}}_{1,k} + \tilde{R}_{1,k})^{-1},\\ P^d_k &= (I - L_{1,k}H_{1,k})R_{2,k},\end{aligned}$$

   (c) No direct feedthrough ($H_{1,k} = 0$)

$$\begin{aligned}\hat{d}_k &= P^d_k H^{\mathsf{T}}_{2,k}R^{-1}_{2,k}y_{2,k},\\ P^d_k &= (H^{\mathsf{T}}_{2,k}R^{-1}_{2,k}H_{2,k})^{-1},\end{aligned}$$

2. Measurement update:

$$\begin{aligned}\hat{x}_{k|k} &= \hat{x}_{k|k-1} + K_{1,k}(\tilde{y}_{1,k} - H_{1,k}\hat{d}_k),\\ K_{1,k} &= P^x_{k|k-1}C^{\mathsf{T}}_{1,k}\tilde{R}^{-1}_{1,k},\\ P^x_{k|k} &= P^x_{k|k-1} - K_{1,k}(\tilde{R}_{1,k} - H_{1,k}P^d_k H^{\mathsf{T}}_{1,k})K^{\mathsf{T}}_{1,k},\\ P^{xd}_k &= (P^{xd}_k)^{\mathsf{T}} = -K_{1,k}H_{1,k}P^d_k,\end{aligned}$$

3. Time update:

$$\begin{aligned}\hat{x}_{k+1|k} &= A_k\hat{x}_{k|k} + G_k\hat{d}_k,\\ P^x_{k+1|k} &= [A_k\ G_k]\begin{bmatrix} P^x_{k|k} & P^{xd}_k \\ P^{dx}_k & P^d_k \end{bmatrix}\begin{bmatrix} A^{\mathsf{T}}_k \\ G^{\mathsf{T}}_k \end{bmatrix} + Q_k.\end{aligned}$$

7 Conclusion

This paper has considered an extensions of the errors-in-variables to the case where a linear combination of the system inputs is measured instead of the entire input vector. It was shown that this problems can be formulated as the unknown input filtering problem considered in [6]. Based on this result, a new solution to the optimal errors-in-variables filtering problem was derived where, in contrast to the results of [14, 4], the state estimator and the input estimator exchange information in both directions.

A

In this Appendix, we prove that for $H_{2,k} = I$, the errors-in-variables filter summarized in section 6 is algebraically equivalent to the filters developed in [14, 4].

First, we consider the update of the state covariance matrix. Using (22) and (23), we obtain from (13) the following recursion for $P^x_{k|k-1}$,

$$\begin{aligned} P^x_{k+1|k} = \; & A_k P^x_{k|k-1} A_k^\mathsf{T} + G_k P^d_k G_k^\mathsf{T} + Q_k \\ & - A_k K_{1,k} H_{1,k} P^d_k G_k^\mathsf{T} - G_k P^d_k H_{1,k}^\mathsf{T} K_{1,k}^\mathsf{T} A_k^\mathsf{T} \\ & - A_k K_{1,k} (\tilde{R}_{1,k} - H_{1,k} P^d_k H_{1,k}^\mathsf{T}) K_{1,k}^\mathsf{T} A_k^\mathsf{T}. \end{aligned} \tag{26}$$

By substituting the easily verified equalities

$$A_k K_{1,k} H_{1,k} P^d_k G_k^\mathsf{T} = A_k P^x_{k|k-1} C_{1,k}^\mathsf{T} (\tilde{R}_{1,k} + H_{1,k} R_{2,k} H_{1,k}^\mathsf{T})^{-1} H_{1,k} R_{2,k} G_k^\mathsf{T},$$

and

$$\begin{aligned} & G_k P^d_k G_k^\mathsf{T} - A_k K_{1,k} (\tilde{R}_{1,k} - H_{1,k} P^d_k H_{1,k}^\mathsf{T}) K_{1,k}^\mathsf{T} A_k^\mathsf{T} \\ & \quad = \; G_k R_{2,k} G_k^\mathsf{T} - A_k P^x_{k|k-1} C_{1,k}^\mathsf{T} (\tilde{R}_{1,k} + H_{1,k} R_{2,k} H_{1,k}^\mathsf{T})^{-1} C_{1,k} P^x_{k|k-1} A_k^\mathsf{T}, \end{aligned} \tag{27}$$

in (26), it follows that the recursion can be rewritten as

$$\begin{aligned} P^x_{k+1|k} = \; & A_k P^x_{k|k-1} A_k^\mathsf{T} + G_k R_{2,k} G_k^\mathsf{T} + Q_k \\ & - (A_k P^x_{k|k-1} C_{1,k}^\mathsf{T} + S_k)(\tilde{R}_{1,k} + H_{1,k} R_{2,k} H_{1,k}^\mathsf{T})^{-1} (A_k P^x_{k|k-1} C_{1,k}^\mathsf{T} + S_k)^\mathsf{T}, \end{aligned} \tag{28}$$

where $S_k := G_k R_{2,k} H_{1,k}$. Note that for $Q_k = 0$, (28) is equivalent to the update of the state covariance matrix considered in section 4 of [14].

Next, we consider the update of the state estimate. Using (21) and (25), we obtain from (12) the following recursion for $\hat{x}_{k|k-1}$,

$$\hat{x}_{k+1|k} = (A_k - \bar{\bar{K}}_{1,k} C_{1,k}) \hat{x}_{k|k-1} + \bar{\bar{K}}_{1,k} y_{1,k} + (G_k - \bar{\bar{K}}_{1,k} H_{1,k}) y_{2,k}, \tag{29}$$

where $\bar{\bar{K}}_{1,k}$ is given by

$$\bar{\bar{K}}_{1,k} = A_k K_{1,k} - A_k K_{1,k} H_{1,k} L_{1,k} + G_k L_{1,k}. \tag{30}$$

Note that (29) takes the form of the update considered in section 4 of [14]. We now prove that equivalence holds by showing that (30) equals the expression for the gain matrix considered in section 4 of [14]. By substituting the easily verified equality

$$\begin{aligned} & K_{1,k} - K_{1,k} H_{1,k} L_{1,k} \\ & \quad = P^x_{k|k-1} C^{\mathsf{T}}_{1,k} \big[\tilde{R}^{-1}_{1,k} - \tilde{R}^{-1}_{1,k} H_{1,k} R_{2,k} H^{\mathsf{T}}_{1,k} (\tilde{R}_{1,k} + H_{1,k} R_{2,k} H^{\mathsf{T}}_{1,k})^{-1} \big], \\ & \quad = P^x_{k|k-1} C^{\mathsf{T}}_{1,k} (H_{1,k} R_{2,k} H^{\mathsf{T}}_{1,k} + \tilde{R}_{1,k})^{-1}, \end{aligned}$$

in (30), it follows that $\bar{\bar{K}}_{1,k}$ can be rewritten as

$$\bar{\bar{K}}_{1,k} = (A_k P^x_{k|k-1} C^{\mathsf{T}}_{1,k} + S_k)(H_{1,k} R_{2,k} H^{\mathsf{T}}_{1,k} + \tilde{R}_{1,k})^{-1},$$

which is the gain matrix considered in section 4 of [14].

Using similar derivations, it can be shown that the equations for $\hat{x}_{k|k}$ and $\hat{d}_k$ are algebraically equivalent to the corresponding equations in [14, 4].

Acknowledgements

Steven Gillijns is a research assistant and Bart De Moor is a full professor at the Katholieke Universiteit Leuven, Belgium. Our research is supported by Research Council KULeuven: GOA AMBioRICS, several PhD/postdoc & fellow grants; Flemish Government: FWO: PhD/postdoc grants, projects, G.0407.02 (support vector machines), G.0197.02 (power islands), G.0141.03 (Identification and cryptography), G.0491.03 (control for intensive care glycemia), G.0120.03 (QIT), G.0452.04 (new quantum algorithms), G.0499.04 (Statistics), G.0211.05 (Nonlinear), research communities (ICCoS, ANMMM, MLDM); IWT: PhD Grants, GBOU (McKnow); Belgian Federal Science Policy Office: IUAP P5/22 ('Dynamical Systems and Control: Computation, Identification and Modelling', 2002-2006); PODO-II (CP/40: TMS and Sustainability); EU: FP5-Quprodis; ERNSI; Contract Research/agreements: ISMC/IPCOS, Data4s, TML, Elia, LMS, Mastercard

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