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Scroll Maps from n-scroll Attractors

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Abstract—*In this paper, we report n-scroll maps raleted to n-scroll attractors. n-scroll maps are constructed through the projection of the Poincaré cross section in the n-scroll attractor. Explicit equations for the 1- and 2-scroll maps are given. Also a 1-scroll map has been verified with its simple circuit realization.*

I. Introduction

Recently, chaotic piecewise-linear one dimensional (PL1D) maps have received considerable attention because of their potential application in communication [1], [2], cryptography [3], [4] and also neural networks [5]. Furthermore, theory of one dimensional maps [6], [7] is well developed. It would be useful if an appropriate 1-D map can be constructed from the system under the study. An elegant way for this purpose has been given by Poincaré. Wu and Rul'kov [8] demonstrate that 1-D maps can be used to analyze the dynamic and bifurcation phenomena of the Chua's circuit [9], [10] using the Poincaré section. In this paper, we will report our recent experimental observation on the n-scroll attractors. First observation of n-scroll attractor [11] has been reported by Suykens and Vandewalle as n-double scroll attractors in 1993. Later a complete family of n-scroll attractor has been obtained from generalized Chua's circuit [12]. In [13] an alternative way for generating double scroll like behavior was proposed by means of a simple circuit implementation. Starting from this basic circuit model, a generalization to n-scroll attractors has been obtained in [14]. A further extension has been recently obtained by Yalcin *et al.* [15], with a new family of scroll grid attractors. Within this new family scrolls can be located in a plane or a 3D body instead of on a line [15], [16]. Here, we present the scroll maps which are imitated behavior of the two-side Poincaré map. Furthermore, a simple current feedback opamps (CFOA)-based circuit has been used to realize the 1-scroll map. This paper is organized as follows. In Section II, we present the scroll maps on the projection of the Poincaré section in n-scroll attractor. In Section III, 1-scroll and 2-scroll maps are presented. A circuit realization is discussed in Section IV.

II. Generating Maps from N-scroll Attractors

Based upon the simple model for chaos generator [13] , a generalization to n -scrolls is given by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\Phi(\mathbf{x}) \tag{1}$$

with $\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a & -a & -a \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & a \end{bmatrix}$, $\Phi = \begin{bmatrix} 0 & 0 & f_1(x) \end{bmatrix}^T$ where,

$$f_1(x) = \sum_{i=1}^{M_x} g_{\frac{(-2i+1)}{2}}(x) + \sum_{i=1}^{N_x} g_{\frac{(2i-1)}{2}}(x) \tag{2}$$

and

$$g_\theta(\zeta) = \begin{cases} 1, & \zeta \geq \theta \quad \theta > 0 \\ 0, & \zeta < \theta \quad \theta > 0 \\ 0, & \zeta \geq \theta \quad \theta < 0 \\ -1, & \zeta < \theta \quad \theta < 0. \end{cases} \tag{3}$$

[15], [16]. The number of scrolls generated from the generalized nonlinearity is equal to $M_x + N_x + 1$. A map simply constructed through the projection of the Poincaré cross section in the state space of the system. Poincaré section is a hyperplane in $\mathbb{R}^{n-1}$ which cuts the phase space trajectory of a dynamical system in $\mathbb{R}^n$ [17]. Furthermore, one-side Poincaré map maps given points on this hyperplane to the next points on the trajectory through this point which intersects the hyperplane in the positive or negative direction [17]. Also, two-side Poincaré map [17] might be defined taking both directional intersections on the hyperplane. In this paper, we define a Poincaré section as $U = \{\mathbf{x} : [0\ 1\ 0]\mathbf{x} = 0\}$ hence a two-side Poincaré

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map can be obtained from $\mathbb{R}^2$ into $\mathbb{R}^2$. Figure 1. (a) shows a 2-double scroll attractor, where $a = 0.8$ and $M_x = 0$, $N_x = 1$, intersecting the plane U transversally. The corresponding Poincaré map is shown in Figure 1. (b) shows a 2-D map. 1-D maps which might be constructed form this 2-D map by means of a projection. In the Figures 2. (a) and (b) two constructed map from two-side Poincaré map are shown by plotting the trajectory the coordinates $(x_3, f(x_3))$ and $(x_1, f(x_1))$ respectively, where f is the Poincaré map and x_i are the successive points of the trajectory on the 1-D map. Graphical iteration of the maps has been also shown on the figure 2. (a) and (b) which lead us to call the appearing maps as 1- and 2-scroll map, respectively. Similar observation might be made on n-scroll attractor using same Poincaré section and n-scroll maps can be observed.

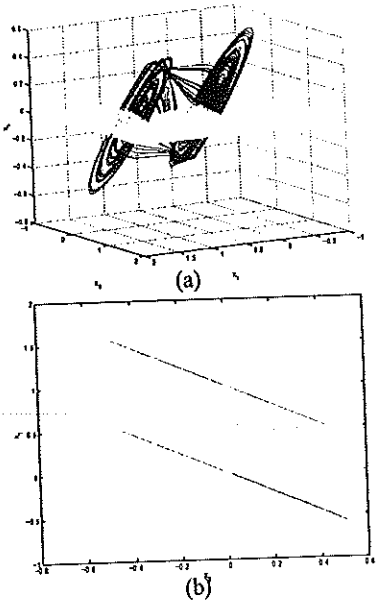


Fig. 1. (a) A Poincaré section for the 2-scroll attractor. (b) Poincaré map from intersection (Figure 1.(a).)

III. 1- AND 2-SCROLL MAPS

The results which are given in previous section draw our attention to constructing a similar a 1-D map which is defined by $x_{n+1} = f(x_n)$ where $f : [0,1] \to [0,1]$. First, we propose a 1-scroll map which is described by

$$x_{n+1} = \begin{cases} 2(x_n - \frac{1}{8}) + \frac{3}{4} + \gamma, & 0 \le x_n < \frac{1}{4} \\ 2(\frac{1}{2} - x_n) + \frac{1}{2}, & \frac{1}{4} \le x_n \le \frac{3}{4} \\ 2(x_n - \frac{7}{8}) + \frac{1}{4} + \gamma, & \frac{3}{4} < x_n \le 1 \end{cases} \quad (4)$$

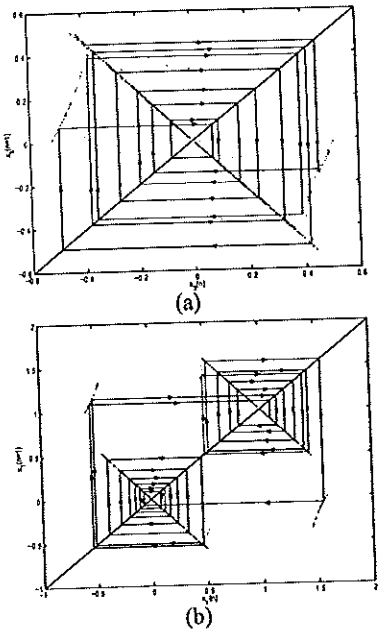


Fig. 2. (a) 1-scroll map and (b) 2-scroll map from the projection of Poincaré map (Figure 1. (b)) are illustrated with some geometrical iteration step

with $\gamma \in (0, \frac{1}{2})$ and shown in Figure 3. It is possi-

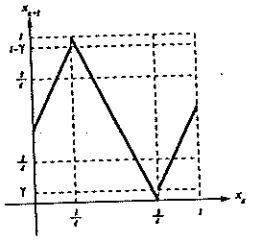
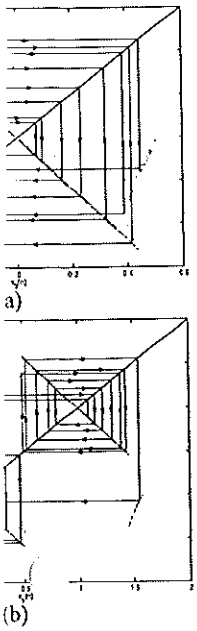


Fig. 3. 1-scroll map.

ble to observe a rich dynamic behavior due to the γ value. Figure 4 shows the iteration result for $\gamma = 0.1$. The quantification of the system is done by estimating the Lyapunov exponent. Theoretically, the Lyapunov exponent of one-dimensional maps described by the map f is defined by

$$\lambda := \lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} \ln \left| \frac{df}{dx}(f^{(n)}(x_0)) \right| \quad (5)$$

for $x_0 \in [0,1]$ [7]. One should note that $|\frac{d}{dx}(f(x_i))|$ is equal to 2 in each linear region. Hence, the Lyapunov exponent will be equal to $\ln(2)$. We decided to calculate the Lyapunov exponent from the time series [18],

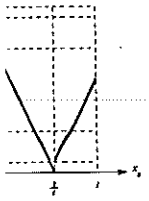


a)

(b)

(b) 2-scroll map from the pro-
(Figure 1. (b)) are illustrated
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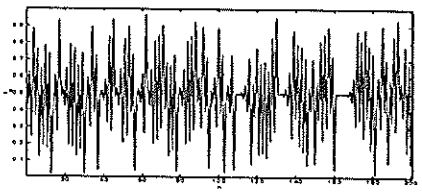


Fig. 4. 200 points of the 1-scroll map for $\gamma = 0.1$

whose first 200 points are given in Figure 4, for testing this theoretical result. TISEAN 2.1 [19], which is a free software project for the analysis of time series with methods based on the theory of nonlinear deterministic dynamical systems, has been used to calculate the maximum Lyapunov exponent. In Figure 7.(a) shows the result which are obtained from this time series. The straight line in Figure 7.(a) indicates that $\lambda = \ln(2)$ [18].

Like the 1-scroll map, 2-scroll map, which is defined in Eq. (6), is imitated the behavior on the Poincaré section (Figure 1.).

$$x_{n+1} = \begin{cases} 2(x_n - \frac{1}{8}) + \frac{3}{4} + \gamma, & 0 \le x_n < \frac{1}{4} \\ 2(\frac{3}{8} - x_n) + \frac{1}{4}, & \frac{1}{4} \le x_n < \frac{1}{2} \\ 2(\frac{5}{8} - x_n) + \frac{3}{4}, & \frac{1}{2} \le x_n < \frac{3}{4} \\ 2(x_n - \frac{7}{8}) + \frac{1}{4} + \gamma, & \frac{3}{4} \le x_n \le 1 \end{cases} \qquad (6)$$

where $\gamma \in (0, \frac{1}{2})$. Again, the Lyapunov exponent of

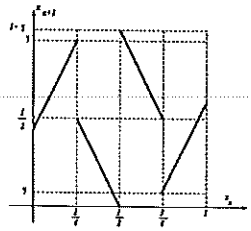


Fig. 5. 2-scroll map.

the 2-scroll map for $\gamma = 0.1$ is estimated from its time series whose first 200 points is shown in Figure 6. Similar results were obtained and shown on Figure 7. (b), which indicate that $\lambda = \ln(2)$.

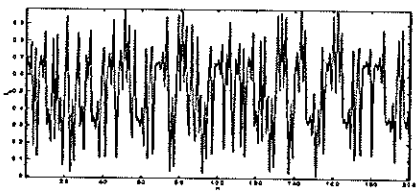


Fig. 6. 200 points of the 2-scroll map for $\gamma = 0.1$

IV. Circuit Realization

In order to verify the proposed maps, the 1-scroll map in Eqn.(4) is implemented using discrete components. The circuit used to this end is shown in Figure 8. The opamps are all current feedback opamps (CFOAs) which are known to offer a number of advantages compared to conventional opamps such as high slew-rate and nonconstant gain-bandwidth product [20]. The voltage follower (VF) is also realized using another CFOA. The voltage V_n corresponds to the value of the state x at the $n-th$ discrete time instance. The subcircuit composed of the resistors $R1$ and $R2$s and the associated opamps is used to implement the required nonlinearity. This circuit follows the approach of the robust Chua diode [10] realisation using voltage negative impedance converters [21], [22]. The subcircuit involving the VF and the associated switches implements the delay, whereas the subcircuit within the dashed lines realises the discontinuity at the breakpoints of the map in Figure 3. The voltages Vbr_+ and Vbr_- are taken as adjustable and they correspond to the breakpoints in the map. The slopes of the piecewise linear function, which determine the dynamic behavior of the system can be adjusted by the ratio $R1/R2$. The values of $R1$ and $R2$s are $15k\Omega$ and $8.2k\Omega$, respectively, which correspond to a slope of 1.83. The capacitors employed in the delay subcircuit are chosen as $50pF$ and all active elements are supplied under $\pm 5V$ DC. The CFOAs are implemented using AD844 IC from Analog Devices, the switches are from 4016 IC chip, the comparators are LM311 IC and the clock frequency is taken as $50kHz$. The measured waveform for the signal V_n is shown in Figure 9. Note from this figure that V_n is return-to-zero type signal. Also, the observed phase-space corresponding to (V_{n+1}, V_n) voltages is given in Figure 10. These observations verify the feasibility of the proposed system.

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References

[1] Rovatti, R., Setti, G., and Mazzini, G., Chaotic complex spreading sequences for asynchronous DS-CDMA. Part II. Some theoretical performance bounds *IEEE Trans. Circuits and Systems-I*, vol 45 (4), pp. 496-506, 1998.

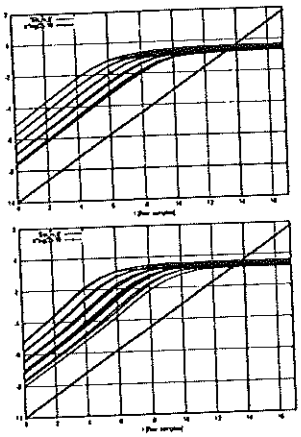


Fig. 7. Nonlinear Time Series Analysis for estimating the maximum Lyapunov exponent of (a) 1-scroll map (b) 2-scroll map

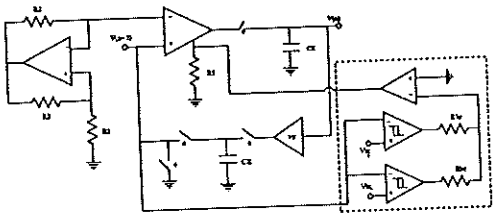


Fig. 8. The circuit realizing the 1-scroll map.

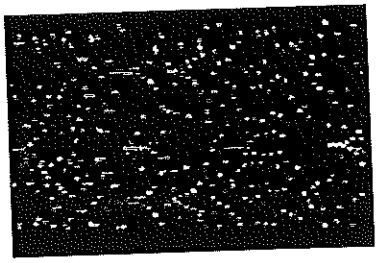


Fig. 9. Observed time waveform of the signal V_n. $(1V/div, 1V/div)$.

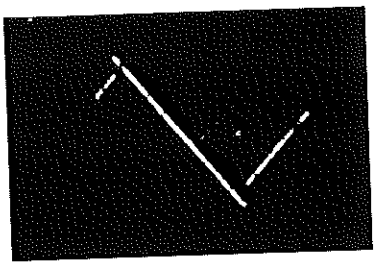


Fig. 10. (V_{n+1}, V_n) for 1-scroll map $(0.6V/div, 1V/div.)$

[2] Rovatti, R.and Mazzini, G., Tensor function analysis of quantized chaotic piecewise-affine pseudo-Markov systems. I. Second-order correlations and self similarity *IEEE Trans. Circuits and Systems-I,* vol. 49 (2), pp. 137-149, 2002.

[3] Stojanovski, T. and Kocarev, L., Chaos-based random number generators-part I: analysis *IEEE Trans. Circuits and Systems-I,* vol. 48 (3), pp. 281-288, 2001.

[4] Gotz, M., Kelber, K. and Schwarz, W., Discrete-time chaotic encryption systems. I. Statistical design approach *IEEE Trans. Circuits and Systems-I,* vol. 44 (10), pp. 963-970, 1997.

[5] Nakano, H.and Saito, T., Basic dynamics from a pulse-coupled network of autonomous integrate-and-fire chaotic circuits, *IEEE Trans. Neural Networks,* vol. 13 (1), pp. 92-100, 2002.

[6] Lasota, A. and Mackey, M.C., *Chaos, Fractals and Noise,* New York: Springer-Verlag, 1994.

[7] van Wyk, M.A., and Steeb, W-H., Chaos in one-dimensional map, *Chap. 29 in Chaos in Circuits and Systems, (Chen G., and Ueta T, eds.), World Scientific, 2002.*

[8] Wu, C.W. and Rul'kov, N.F., Studying Chaos via 1-D Maps-A Tutorial, *IEEE Trans. Circuits and Systems-I,* vol 40 (10), pp. 707-721, 1993.

[9] Chua, L.O., Komuro, M. and Matsumoto, T., The Double Scroll Family. *IEEE Trans. Circuits and Systems-I,* vol. 33 (11), pp. 1072-1118, 1986.

[10] Madan, R.N., *Chua's Circuit : A paradigm for chaos.* Singapore: World Scientific, 1993.

[11] Suykens J.A.K. and Vandewalle J., Generation of n-double scrolls (n= 1,2,3,4,...). *IEEE Trans. Circuits and Systems-I,* vol. 40 (11), pp. 861-867, 1993.

[12] Suykens J.A.K., Huang, A. and Chua, L.O., A Family of n-Scroll Attractors from a Generalized Chua's Circuit. *AEÜ (Int. J. Electronics and Communications),* vol. 51(3), pp. 131-138, 1997.

[13] Elwakil, A.S. and Kennedy, M.P., Construction of classes of circuit-independent chaotic oscillators using passive-only nonlinear devices. *IEEE Trans. Circuits and Systems-I,* vol. 48 (3), pp. 289-307, 2001.

[14] Yalçın, M. E., Özoguz, S., Suykens, J. A. K. and Vandewalle, J., $n-$Scroll Chaos Generators: a simple circuit model. *Electronics Letters,* vol. 37 (3) , pp. 147-148, 2001.

[15] Yalçın, M.E., Ozoguz, S., Suykens, J.A.K., Vandewalle, J., Families of Scroll Grid Attractors, *International Journal of Bifurcation and Chaos,* vol. 12 (1), pp. 23-41, 2002.

[16] Suykens, J.A.K., Yalçın, M.E., Vandewalle, J., A Generic Class of Chaotic and Hyperchaotic Circuits with Synchronization Methods, *In Chap. 8 of Chaos in Circuits and Systems (Chen G., and Ueta T., eds.),* World Scientific, 2002.

[17] Parker, T. S. and Chua, L. O. Practical Numerical Algorithms for Chaotic Systems, *Springer-Verlag,* 1989.

[18] Kantz, H., A robust method to estimate the maximal Lyapunov exponent of a time series, *Phys. Lett. A,* vol. 185, pp. 77, 1994.

[19] TISEAN 2.1, Nonlinear Time Series Analysis, *http://www-.mpipks-dresden.mpg.de/~tisean/TISEAN_2.1/index.html*

[20] Soliman, A.M, Applications of the current feedback operational amplifiers, *Analog Integrated Circuits & Signal Processing* vol. 11 (3), pp. 265-302, 1996.

[21] Kennedy, M.P., Robust op amp realisation of Chua's circuit, *Frequenz,* vol 46, pp. 66-80, 1992.

[22] Elwaki, A.S. and Kennedy, M.P., Improved implementation of Chua's chaotic oscillator using current feedback op amp, *Frequenz,* vol. 46, pp. 66-80, 1992.