A Near-Optimal Linear Crosstalk Precoder for Downstream VDSL

Raphael Cendrillon, George Ginis, Etienne Van den Bogaert, and Marc Moonen

***Abstract*—This letter presents a linear crosstalk precoder for very-high-speed digital subscriber lines (VDSL) that has a low run-time complexity. A lower bound on the data rate of the precoder is developed, and guarantees that the precoder achieves near-optimal performance in 99% of VDSL channels.**

***Index Terms*—Crosstalk cancellation, digital subscriber lines (DSL).**

I. Introduction

CROSSTALK is a major problem in very-high-speed digital subscriber line (VDSL) networks, limiting both the data rate and reach of service. In the downstream direction, the transmitting modems are colocated at the central office (CO). This allows crosstalk precoding to be applied, a technique where predistortion is added to each modem's signal prior to transmission. This predistortion is chosen such that it annihilates with crosstalk introduced in the binder, allowing each modem to operate over a crosstalk-free channel, and achieve a much higher data rate. A decision-feedback structure, based on the Tomlinson–Harashima precoder (THP), was shown to operate close to the single-user bound [1]. Unfortunately, this structure relies on a nonlinear modulo operation at the receiver side, leading to a higher run-time complexity. For example, in a standard VDSL modem operating at 4000 discrete multitone (DMT) symbols per second, with 4096 tones, the modulo operation would require an extra 16.3 million instructions per second (MIPS). This will almost double the complexity of the customer premises (CP) modem, which currently only needs to implement a frequency-domain equalizer, an operation that also requires 16.3 MIPS. Since CP modems are now a commodity, cost is an extremely sensitive issue, and any technique that helps to decrease complexity is extremely beneficial.

This letter presents an alternative linear precoder which is based on a channel diagonalizing criterion. We show that this design, which we term the diagonalizing precoder (DP), has a much lower complexity than the THP, since it does not require any additional receiver-side operations. This letter extends earlier work that considered the design of near-optimal linear crosstalk cancellers for *upstream* transmission [2]. There it was shown that the column-wise diagonal dominance (CWDD) of the upstream VDSL channel leads to near-optimal performance for zero-forcing (ZF) crosstalk cancellers. In this letter, we consider crosstalk precoding for *downstream* transmission. It is shown that, due to the row-wise diagonal dominance (RWDD) of the downstream VDSL channel, the DP achieves near-optimal performance. We develop bounds that allow the performance of the DP to be predicted without explicit knowledge of the crosstalk channels, which simplifies service provisioning considerably.

Paper approved by C.-L. Wang, the Editor for Equalization of the IEEE Communications Society. Manuscript received December 22, 2004; revised May 14, 2006 and September 3, 2006. This work was carried out under the framework of IUAP P5/22, "Dynamical Systems and Control: Computation, Identification and Modelling" and P5/11, "Mobile Multimedia Communication Systems and Networks;" under the Concerted Research Action GOA-MEFISTO-666, "Mathematical Engineering for Information and Communication Systems Technology;" under the IWT BANITS Project, "Broadband Access Networks Integrated Telecommunication Systems;" and was supported in part by Alcatel-Bell. This paper was presented in part at the IEEE International Conference on Acoustics, Speech, and Signal Processing, Montreal, QC, Canada, May 2004.

R. Cendrillon is with Marvell Hong Kong Ltd., Mongkok, Hong Kong (e-mail: raphael@cendrillon.org).

G. Ginis is with ASSIA Inc., Redwood City, CA 94065-1198 USA (e-mail: gginis@assia-inc.com).

E. Van den Bogaert is with Alcatel Bell, Antwerp 2018, Belgium (e-mail: etienne.van_den_bogaert@alcatel.be).

M. Moonen is with the Katholieke Universiteit Leuven, Heverlee 3001, Belgium (e-mail: moonen@esat.kuleuven.be).

Digital Object Identifier 10.1109/TCOMM.2007.896121

II. System Model

Assuming that the modems are synchronized and DMT modulation is employed, we can model transmission independently on each tone $\mathbf{y}_k = \mathbf{H}_k\mathbf{x}_k + \mathbf{z}_k$. We assume perfect knowledge of the crosstalk channels. In practice, these must be identified using multiple-input multiple-output (MIMO) channel-identification techniques and communicated back to the transmitter side. Since the VDSL channel is slowly time-varying, this can be done infrequently and requires little additional overhead [3], [4]. The vector $\mathbf{x}_k \triangleq [x_k^1, \ldots, x_k^N]^T$ contains transmitted signals on tone k, where the tone index k lies in the range $1, \ldots, K$. There are N lines in the binder, and x_k^n is the signal transmitted on line n at tone k. The vectors $\mathbf{y}_k$ and $\mathbf{z}_k$ have similar structures. The vector $\mathbf{y}_k$ contains the received signals on tone k. The vector $\mathbf{z}_k$ contains the additive noise on tone k and is comprised of thermal noise, alien crosstalk, radio frequency interference (RFI), etc. The $N \times N$ matrix $\mathbf{H}_k$ is the crosstalk channel matrix on tone k. The element $h_k^{n,m} \triangleq [\mathbf{H}_k]_{n,m}$ is the channel from transmitter m to receiver n on tone k. The transmit correlation on tone k is defined $\mathbf{S}_k \triangleq \mathcal{E}\{\mathbf{x}_k\mathbf{x}_k^H\}$. We denote the transmit power spectral density (PSD) of user n on tone k as $s_k^n \triangleq \mathcal{E}\{|x_k^n|^2\}$. We assume that the transmit PSD on each line must obey a spectral mask constraint[1]

$$s_k^n \leq s_k^{\text{mask}} \quad \forall k, n. \tag{1}$$

The noise power experienced by receiver n on tone k is defined as $\sigma_{k,n} \triangleq \mathcal{E}\{|z_k^n|^2\}$. Since the transmitting modems are colocated, the crosstalk signal transmitted from a disturber into a victim must propagate through the full length of the victim's line. This is depicted in Fig. 1, where CO2 is the disturber and CP1 is the victim. The insulation between twisted pairs increases the attenuation. As a result, the crosstalk channel matrix $\mathbf{H}_k$ is RWDD, since on each row of $\mathbf{H}_k$ the diagonal element has the largest magnitude $|h_k^{n,m}| \ll |h_k^{n,n}|, \forall m \neq n$.

[1]The techniques described in this letter can also readily be combined with dynamic spectrum management.

0090-6778/$25.00 © 2007 IEEE

Authorized licensed use limited to: KU Leuven Libraries. Downloaded on July 29,2020 at 11:17:05 UTC from IEEE Xplore. Restrictions apply.

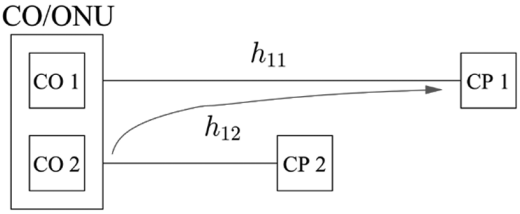


Fig. 1. RWDD $|h_{11}| \gg |h_{12}|$.

RWDD implies that the crosstalk channel $h_k^{n,m}$ from a disturber m into a victim n is always weaker than $h_k^{n,n}$, which is the direct channel of the victim.[2] The degree of RWDD can be characterized with the parameter α_k

$$|h_k^{n,m}| \leq \alpha_k |h_k^{n,n}| \quad \forall m \neq n. \tag{2}$$

Note that crosstalk precoding requires joint processing of the signals prior to transmission, so the transmitting modems must be colocated. Hence, in all channels where crosstalk precoding can be applied, the RWDD property holds. RWDD has been verified through extensive measurement campaigns of real binders. In 99% of lines α_k is bounded

$$\alpha_k \leq K_{xf} \cdot f_k \cdot \sqrt{d_{\text{coupling}}} \tag{3}$$

where $K_{xf} = -22.5$ dB and f_k is the frequency on tone k in MHz [5]. Here d_{coupling} is the coupling length between the disturber and the victim in kilometers. To find a value for α_k that is independent of the particular binder configuration, d_{coupling} can be set to 1.2 km, which is the maximum deployment length for VDSL.[3] Sections III–V show that RWDD ensures a well-conditioned crosstalk channel matrix. This results in the near-optimality of the DP.

III. Theoretical Capacity

We start with a bound on the capacity of the downstream VDSL channel with coordinated transmitters. This will prove useful in evaluating crosstalk precoder performance, since it provides an upper bound on the achievable data rate with any possible crosstalk precoding scheme. Denote the tone spacing as Δ_f.

Theorem 1: The achievable data rate for user n on tone k is upper bounded

$$b_k^n \leq \Delta_f \log_2 \left(1 + \Gamma^{-1} \sigma_{k,n}^{-1} s_k^{\text{mask}} |h_k^{n,n}|^2 [1 + (N-1)\alpha_k]^2\right). \tag{4}$$

Proof of Theorem 1: CO modems are colocated and use coordinated transmission, so from an information-theoretical perspective, this is a broadcast channel. Consider then the single-user bound, which is the capacity achieved when all transmitters (CO modems) are used to communicate to a single receiver (CP modem). In this case, the received signal on the CP modem is $y_k^n = \overline{\mathbf{h}}_k^n \mathbf{x}_k + z_k^n$, where $\overline{\mathbf{h}}_k^n \triangleq [\mathbf{H}_k]_{\text{row}n}$. Using the single-user bound, the achievable data rate of user n on tone k is limited

$$b_k^n \leq \Delta_f I(\mathbf{x}_k; y_k^n) = \Delta_f \log_2 \left(1 + \Gamma^{-1} \sigma_{k,n}^{-1} \overline{\mathbf{h}}_k^n \mathbf{S}_k \left(\overline{\mathbf{h}}_k^n\right)^H\right) \tag{5}$$

where $I(a;b)$ denotes the mutual information between a and b. To account for the suboptimality of practical coding schemes, we include the signal-to-noise ratio (SNR) gap to capacity Γ [6]. Define the elements of the correlation matrix $s_k^{n,m} \triangleq [\mathbf{S}_k]_{n,m}$, and the diagonal elements $s_k^n \triangleq [\mathbf{S}_k]_{n,n}$. Since $\mathbf{S}_k$ is positive semidefinite, it must be true that $s_k^{n,m} \leq \sqrt{s_k^n s_k^m}$. Hence

$$\begin{aligned}
\overline{\mathbf{h}}_k^n \mathbf{S}_k \overline{\mathbf{h}}_k^{n^H} &= \sum_i h_k^{n,i} \sum_j s_k^{i,j} \left(h_k^{n,j}\right)^* \\
&\leq \sum_i \left|h_k^{n,i}\right| \sqrt{s_k^i} \sum_j \left|h_k^{n,j}\right| \sqrt{s_k^j} \\
&= \left(\sum_i \left|h_k^{n,i}\right| \sqrt{s_k^i}\right)^2.
\end{aligned}$$

Combining this with (5) and (2) yields

$$b_k^n \leq \log_2 \left(1 + \Gamma^{-1} \sigma_{k,n}^{-1} |h_k^{n,n}|^2 \left(\sqrt{s_k^n} + \alpha_k \sum_{m \neq n} \sqrt{s_k^m}\right)^2\right).$$

Combining this with (1) leads to (4), which completes the proof. ■

IV. Diagonalizing Precoder

This section presents the DP, which in contrast to the THP, is linear, has a lower complexity, and requires transmitter-side operations only. However, similar to the THP, the DP operates close to the theoretical channel capacity. Prior to transmission, the DP multiplies the true symbols $\widetilde{\mathbf{x}}_k \triangleq [\widetilde{x}_k^1, \ldots, \widetilde{x}_k^N]^T$ with a precoding matrix $\mathbf{P}_k$. Denote the transmitted symbols as $\mathbf{x}_k = \mathbf{P}_k \widetilde{\mathbf{x}}_k$. The DP precoding matrix is defined $\mathbf{P}_k \triangleq \beta_k^{-1} \mathbf{H}_k^{-1} \text{diag}\{h_k^{1,1}, \ldots, h_k^{N,N}\}$, where $\text{diag}\{\gamma_1, \ldots, \gamma_N\}$ denotes the diagonal matrix with elements $\gamma_1, \ldots, \gamma_N$ along the main diagonal. Here the scaling factor is defined

$$\beta_k \triangleq \max_n \left\| \left[\mathbf{H}_k^{-1} \text{diag}\left\{h_k^{1,1}, \ldots, h_k^{N,N}\right\}\right]_{\text{row}n} \right\|, \tag{6}$$

and is included to ensure that compliance with the spectral mask is maintained after precoding. That is, if the original signal $\widetilde{x}_k^n$ obeys the spectral mask, $\widetilde{s}_k^n \triangleq \mathcal{E}\{|\widetilde{x}_k^n|^2\} \leq s_k^{\text{mask}}\ \forall n$, then the signal after precoding x_k^n will obey the spectral masks as well, since

$$\begin{aligned}
s_k^n &= \mathcal{E}\left\{\left|[\mathbf{P}_k \widetilde{\mathbf{x}}_k]_{\text{row}n}\right|^2\right\} \\
&\leq \max_n \|[\mathbf{P}_k]_{\text{row}n}\|^2 \cdot \max_n s_k^n \leq s_k^{\text{mask}}.
\end{aligned}$$

During transmission, the predistortion introduced by the DP annihilates the crosstalk. The received vector is then

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{P}_k \widetilde{\mathbf{x}}_k + \mathbf{z}_k = \beta_k^{-1} \text{diag}\left\{h_k^{1,1}, \ldots, h_k^{N,N}\right\} \widetilde{\mathbf{x}}_k + \mathbf{z}_k. \tag{7}$$

[2]Contrast this with the CWDD experienced in the upstream transmission, where the crosstalk channel $h_k^{n,m}$ from a disturber into a victim is always weaker than the direct channel of the disturber $h_k^{m,m}$.

[3]Standardization groups are currently considering the deployment of VDSL2 at lengths greater than 1.2 km. However, at such distances, far-end crosstalk is no longer the dominant source of noise, and the benefits of far-end crosstalk precoding are significantly reduced.

Authorized licensed use limited to: KU Leuven Libraries. Downloaded on July 29,2020 at 11:17:05 UTC from IEEE Xplore. Restrictions apply.

Thus, application of the DP diagonalizes the channel matrix.[4] Each user experiences its direct channel, scaled by β_k and completely free from interference. RWDD in the crosstalk channel matrix implies that β_k is close to unity. As a result, each user operates close to its single-user bound, and the DP is near-optimal.[5] This observation is made rigorous through the following theorem.

Theorem 2: If $A_{\min}^{(m)} \geq \alpha_k m B_{\max}^{(m)}$, $m = 1, \ldots, N-1$, the data rate achieved by the DP can be lower bounded by

$$b_k^n \geq \Delta \log_2 \left(1 + \Gamma^{-1} \sigma_{k,n}^{-1} \tilde{s}_k^n \left|h_k^{n,n}\right|^2 f^{-1}(N, \alpha_k)\right) \qquad (8)$$

where

$$f(N, \alpha_k) \triangleq \left(\frac{A_{\max}^{(N-1)}}{A_{\min}^{(N)}}\right)^2 + (N-1)\left(\frac{B_{\max}^{(N-1)}}{A_{\min}^{(N)}}\right)^2,$$

$$\begin{bmatrix} A_{\max}^{(m)} \\ B_{\max}^{(m)} \end{bmatrix} \triangleq \left(\prod_{i=1}^{m} \begin{bmatrix} 1 & (i-1)\alpha_k \\ \alpha_k & (i-1)\alpha_k \end{bmatrix}\right) \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

and

$$A_{\min}^{(m)} \triangleq 1 - \sum_{i=1}^{m} \alpha_k (i-1) B_{\max}^{(i-1)}.$$

Proof of Theorem 2: Equation (7) implies that after application of the DP, the signal at receiver n is $y_k^n = \beta_k^{-1} h_k^{n,n} \tilde{x}_k^n + z_k^n$. Hence, the received signal power for user n on tone k is $\beta_k^{-2} \tilde{s}_k^n |h_k^{n,n}|^2$, the received interference power is zero, and the received noise power is $\sigma_{k,n}$. So the data rate achieved by the DP is

$$b_k^n(\tilde{s}_k^n) = \log_2 \left(1 + \Gamma^{-1} \sigma_{k,n}^{-1} \beta_k^{-2} \tilde{s}_k^n \left|h_k^{n,n}\right|^2\right). \qquad (9)$$

Define $\mathbb{A}^{(N)}$ as the set of $N \times N$ diagonally dominant matrices, such that for any $\mathbf{A}^{(N)} \in \mathbb{A}^{(N)}$ it holds that

$$\begin{aligned} |a_{n,n}| &= 1 \\ |a_{n,m}| &\leq \alpha_k \quad \forall n \neq m \end{aligned}$$

where $a_{n,m} \triangleq [\mathbf{A}^{(N)}]_{n,m}$. Define the matrix $\overline{\mathbf{G}}_k \triangleq [\overline{g}_k^{n,m}]$, where $\overline{g}_k^{n,m} \triangleq h_k^{n,m}/h_k^{n,n}$. Equation (2) implies that $\overline{\mathbf{G}}_k \in \mathbb{A}^{(N)}$. Hence, [2, Th. 3] can be applied to bound the elements of $\overline{\mathbf{G}}_k^{-1}$ as follows:

$$\left|\left[\overline{\mathbf{G}}_k^{-1}\right]_{n,m}\right| \leq \begin{cases} \dfrac{A_{\max}^{(N-1)}}{A_{\min}^{(N)}}, & n = m \\ \dfrac{B_{\max}^{(N-1)}}{A_{\min}^{(N)}}, & n \neq m. \end{cases}$$

Now $\mathbf{H}_k = \operatorname{diag}\{h_k^{1,1}, \ldots, h_k^{N,N}\}\overline{\mathbf{G}}_k$, hence, $\mathbf{H}_k^{-1}\operatorname{diag}\{h_k^{1,1}, \ldots, h_k^{N,N}\} = \overline{\mathbf{G}}_k^{-1}$. Combining this with (6) implies that $\beta_k = \max_n \|[\overline{\mathbf{G}}_k^{-1}]_{\text{row}n}\|$, hence $\beta_k^2 \leq f(N, \alpha_k)$. Combining this with (9) leads to (8), which concludes the proof. ■

[4]Note that using a ZF criterion $\mathbf{P}_k = \beta_k^{-1}\mathbf{H}_k^{-1}$, as in [2], would lead to a highly suboptimal design, since the channel is not CWDD. In fact, it can be shown that with a ZF design, the scaling factor causes all modems to see the channel of the worst line within the binder. So use of the diagonalizing criterion is crucial for good performance.

[5]In this letter, we only consider crosstalk precoding with a spectral mask;, however, it can also be shown that the DP is near-optimal when dynamic spectrum management is applied. Furthermore, the DP decouples transmission on each line, allowing the transmit spectra to be optimized with a much lower complexity.

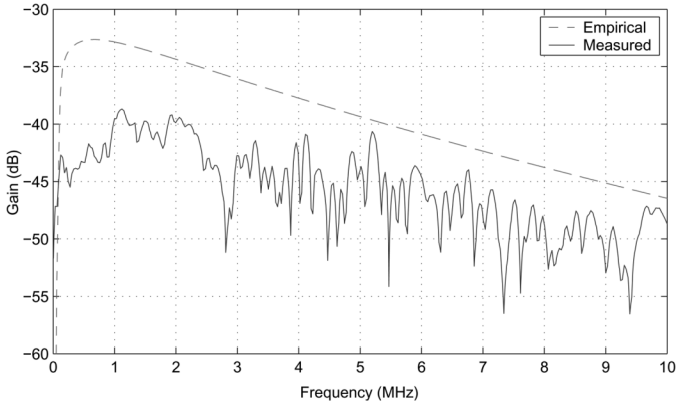


Fig. 2. Crosstalk channel transfer functions (1 km cable, 0.5 mm pairs).

Note that with the THP, the achievable data rate is difficult to predict, since it depends on the magnitude of the crosstalk channels. Crosstalk channels are not well understood, and actual channels can deviate significantly from the few empirical models that exist, see, for example, Fig. 2, making service provisioning difficult. Using the bound (8) allows us to overcome this problem. The bound show that actual crosstalk channel gains are not important so long as RWDD is observed. RWDD is a well-understood phenomenon and models of the degree of RWDD, α_k, are available based on extensive measurement campaigns [5]. For example, the value for α_k from (3) is based on worst 1% case models, hence, for 99% of lines, α_k will be smaller and a data rate above the bound (8) is achieved. The bound is, therefore, a useful tool not just for theoretical analysis, but for the provisioning of services, as well.

V. Performance

This section evaluates the performance of the DP in a binder of eight VDSL lines. The line lengths range from 150 m to 1200 m in 150 m increments. For all simulations, the line diameter is 0.5 mm (24-AWG). Direct and crosstalk channel transfer functions are generated using semiempirical models [5]. The target symbol-error probability is 10^{-7}, the coding gain set to 3 dB, and the noise margin is 6 dB. As per the VDSL standards, the tone-spacing Δ_f is set to 4.3125 kHz. The modems use 4096 tones, the 998 frequency-division duplexing (FDD) bandplan, and a spectral mask s_k^{mask} set to -60 dBm/Hz. Background noise is generated using ETSI noise model A [5].

Fig. 3 shows the downstream data rate achieved on each of the lines with the different crosstalk-precoding schemes. As can be seen, the DP achieves substantial gains, typically 30 Mb/s or more, over conventional systems with no crosstalk precoding. The DP achieves near-optimal performance, operating close to the single-user bound. This is a direct result of the RWDD of $\mathbf{H}_k$, which ensures that the scaling parameter β_k is always close to unity.

Fig. 4 shows the data rate of the DP as a percentage of the channel capacity. As can be seen, performance does not drop below 99% of the channel capacity. The lower bound on the performance of the DP (8) is also included for comparison. The

Authorized licensed use limited to: KU Leuven Libraries. Downloaded on July 29,2020 at 11:17:05 UTC from IEEE Xplore. Restrictions apply.

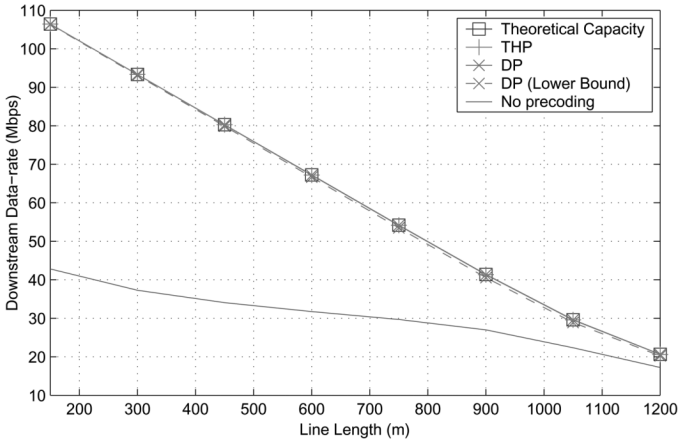


Fig. 3. Data rate with different precoders.

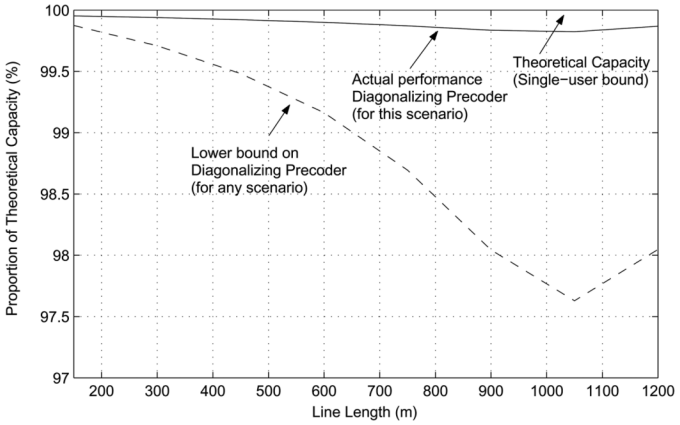


Fig. 4. Proportion of theoretical capacity achieved by DP.

bound is quite tight, and guarantees that the DP will achieve at least 97% of the channel capacity not only in the scenario considered here, but in all other binder configurations, as well. Like the DP, the THP also achieves near-optimal performance; however, the THP requires twice as many operations at the receiver side. Furthermore, the performance of the THP cannot be guaranteed, since no bound is available that does not require explicit knowledge of the crosstalk channel gains.

VI. Conclusions

This letter investigated crosstalk precoder design for downstream VDSL. Existing precoders suffer from high run-time complexity. A novel linear precoder based on the channel diagonalizing criterion was proposed. This precoder has a low complexity and does not require additional receiver-side operations, which is important, since it helps keep CP modem complexity as low as possible. A lower bound on the data rate of the DP was derived that depends only on the direct channel gain and background noise. As a result, the performance of the DP can be accurately predicted, simplifying service provisioning considerably. The bound also shows that the DP operates close to the theoretical channel capacity regardless of binder configuration. The DP is, therefore, a low-complexity design with guaranteed near-optimal performance.

References

[1] G. Ginis and J. Cioffi, "Vectored transmission for digital subscriber line systems," *IEEE J. Sel. Areas Commun.*, vol. 20, no. 3, pp. 1085–1104, Jun. 2002.

[2] R. Cendrillon, G. Ginis, E. Van den Bogaert, and M. Moonen, "A near-optimal linear crosstalk canceler for upstream VDSL," *IEEE Trans. Signal Process.*, vol. 54, no. 8, pp. 3136–3146, Aug. 2006.

[3] C. Zeng, C. Aldana, A. Salvekar, and J. Cioffi, "Crosstalk identification in xDSL systems," *IEEE J. Sel. Areas Commun.*, vol. 19, no. 4, pp. 1488–1496, Aug. 2001.

[4] S. Galli, C. Valenti, and K. Kerpez, "A frequency-domain approach to crosstalk identification in xDSL systems," *IEEE J. Sel. Areas Commun.*, vol. 19, no. 4, pp. 1497–1506, Aug. 2001.

[5] *Very High Speed Digital Subscriber Line (VDSL); Functional Requirements*, ETSI TS 101 270-1, Rev. V.1.3.1, 2003.

[6] G. Forney and M. Eyuboglu, "Combined equalization and coding using precoding," *IEEE Commun. Mag.*, vol. 29, no. 12, pp. 25–34, Dec. 1991.

Authorized licensed use limited to: KU Leuven Libraries. Downloaded on July 29,2020 at 11:17:05 UTC from IEEE Xplore. Restrictions apply.