Multi-Scroll and Hypercube Attractors from Josephson Junctions

Müştak E. Yalçın
Istanbul Technical University,
Faculty of Electrical and Electronic Engineering,
Electronics Engineering Department,
Maslak, TR-34469, Istanbul, Turkey
Email: mustak.yalcin@itu.edu.tr

Johan A.K. Suykens, Joos Vandewalle
Katholieke Universiteit Leuven
Department of Electrical Eng., SCD/SISTA
Kasteelpark Arenberg 10,
B-3001 Leuven, Belgium
Email: Johan.Suykens@esat.kuleuven.ac.be

***Abstract*— In this paper Josephson junctions are used in order to generate n-scroll and n-scroll hypercube attractors. We propose to use of the Josephson junction in a general Jerk circuit in such a way that there is no need for synthesizing the nonlinearity towards n-scroll and n-scroll hypercube attractors. The results are illustrated with computer simulations.**

I. Introduction

Since the discovery of the double scroll attractor in Chua's circuit [1], the double scroll attractor has been addressed by many researchers. The research on Chua's circuit led to the design of more complex attractors consisting of multiple scrolls often called multi-scroll attractors [2], [3]. The challenge in multi-scroll attractor design is to introduce a systematic modification procedure which can allow to generate more complex attractors with each modification. n-Double scroll attractors from a generalized Chua's circuit has been the first extension proposed to the double scroll [4]. Then n-scroll attractors have been introduced by Suykens *et al.* [5] allowing also for an odd and an even number of scrolls. Experimental confirmations of n-double scrolls and n-scroll attractors were given followed by the circuit realization of 2-double scroll attractor [6] and 5-scroll attractor [7]. In 2001, the family of scroll grid [8] attractors has been introduced. It enables to locate the scroll positions in any state variable direction of the system. $1D$, $2D$ and $3D$-scroll grid attractors form families of scroll grid attractors and have received considerable attention in recent years. Cafagna and Grassi [9] and Lu *et al.* [10] developed a coupled Chua's circuit and a hysteresis series approaches in order to obtain these scroll grid attractors, respectively.

A design of multi-scroll chaotic attractors is basically a problem of designing multiple equilibrium points in a scroll based chaotic attractor by modifications of the nonlinear characteristic. In [4] the modification is done by introducing additional breakpoints in the nonlinear characteristic of the Chua's circuit in order to obtain n-double scroll attractors. In [11] the same modification is done by the sine function. In [8] and [12] the authors used a collection of step functions and smooth hyperbolic tangent functions for the modifications, respectively.

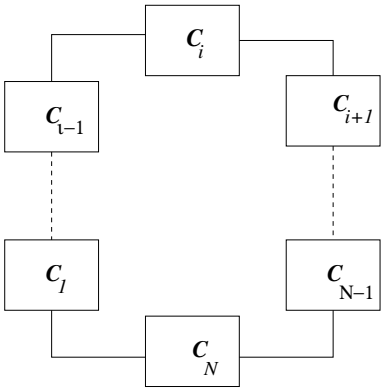


Fig. 1. One-dimensional Cellular Neural/Nonlinear Network architecture with periodic boundary condition.

In fact the design and realization of multi-scroll attractors depends on synthesizing the nonlinearity with an electrical circuit. The question arises whether there exists an electrical device that can naturally allow to design a multi-scroll chaotic attractor. In this paper we want to argue that Josephson junctions [13] are suitable candidate devices that possess such a nonlinearity.

From the application point of view, the double scroll attractor has been successfully employed towards true random bit generation [14]. Furthermore, hypercube attractors based on multi-scroll attractors have been used in networks for optimization [2].

In this paper, we introduce a new model that is based upon the nonlinearity and dynamics of the Josephson junction in order to generate n-scroll attractors. The phase difference of the junctions is one of the state variables for the resulting n-scroll attractor. We investigate n-scroll hypercube attractors in a closed chain of coupled identical n-scroll attractors (see Figure 1). This chain of coupled identical n-scroll attractors with Josephson junction describes a one-dimensional Cellular Neural/Nonlinear Network (CNN) [15].

This paper is organized as follows. In Section I we discuss n-scroll attractors obtained from a Jerk circuit using a sine function. In Section II a new model is introduced by making

use of the Josephson junction nonlinear characteristic and the phase dynamics in the Jerk circuit. In Section III n-scroll hypercube attractors are generated from a one-dimensional array consisting of such cells.

II. n-SCROLL ATTRACTORS FROM A JERK CIRCUIT USING THE SINE FUNCTION

In [11] a sine function was replacing the nonlinear characteristic of Chua's circuit. Using the sine function different numbers of scrolls can be designed. A similar approach can be applied to Jerk circuit

$$\begin{cases} \dot{x} &= y \\ \dot{y} &= z \\ \dot{z} &= -ay - az + ag(x) \end{cases} \tag{1}$$

where

$$g(x) = \sin(2\pi bx) \tag{2}$$

and $b \in R$ [16]. An n-scroll attractor can obtained for $a = 0.3$, $b = 0.25$ [16]. The Jerk circuit consists of a linear circuit and a nonlinear circuit (See Figure 2). The design and realization of the linear circuit are simple. Therefore its design and realization mainly depends on synthesizing the nonlinearity by an electrical circuit. Here the nonlinearity is chosen to obtain a sine function and it can be realized by a commercial trigonometric function chips [11].

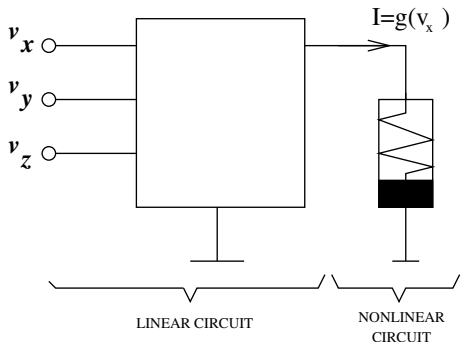


Fig. 2. The Jerk circuit consists of a linear circuit and a nonlinear circuit. The v_x, v_y, v_z voltages correspond to the x, y, z state variables in the model (1), respectively.

III. JOSEPHSON JUNCTIONS AND n-SCROLL ATTRACTORS

Josephson junctions are highly nonlinear superconducting electronic devices. It is also well-known that it are superconducting devices that can generate high frequency oscillations [17]. In [18] chaotic dynamics from Josephson junction have been reported. The aim here is basically to use the nonlinearity of a Josephson junction for (2). The current in a Josephson junction is described by

$$I = I_c \sin\phi \tag{3}$$

where

$$\dot{\phi} = kV. \tag{4}$$

Here ϕ denotes the phase difference and V is the voltage across the junction. In a superconducting Josephson junction k is defined by the fundamental constants $k = \frac{2e}{h}$ (h is Planck's constant divided by 2π and e is the charge on the electron).

Instead of designing a sine function for the n-scroll attractor, the nonlinearity of the Josephson junction (3) was applied to the model (1) in [16]. The current in the Josephson junction was chosen as $g(x)$ and the voltage across the junction set to the y state variable.

Here we introduce a new model which is described by

$$\begin{cases} \dot{\phi} &= ky \\ \dot{y} &= z \\ \dot{z} &= -ay - az + \frac{a}{2Ic} I. \end{cases} \tag{5}$$

In this case the Josephson junction is integrated within the system and the phase difference (ϕ) is taken as one of the state variables for obtaining n-scroll attractors (see Figure 3). The design and realization problem of this n-scroll attractor simply depends on the linear part of the circuit. One should note that by using the phase difference of the Josephson junction as a state variable in the model, the third-order linear circuit (Figure 2) becomes a second-order linear circuit (Figure 3).

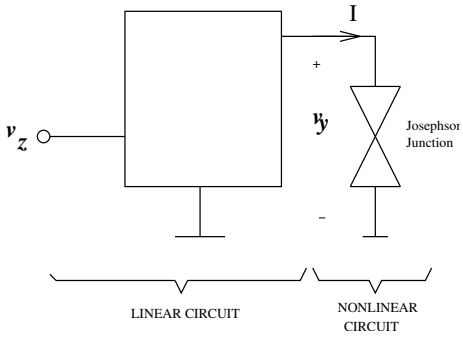


Fig. 3. n-scroll attractor circuit using a Josephson junction. The design of this n-scroll attractor circuit depends only on the linear part of the circuit.

Figure 4 shows the phase portrait of an n-scroll attractor obtained from this model for $k = 1$ and $a = 0.1$. Furthermore in Figure 5 n-scroll attractors from the same system (5) are shown for $a = 0.1$ and $k = 2$. The Josephson junction is described without an upper and lower bound [17]. Therefore the number of scrolls can not be counted for the model (5).

The Figure 6 shows the bifurcation diagram of the system (5) for the value of a.

IV. JOSEPHSON JUNCTIONS AND n-SCROLL HYPERCUBE ATTRACTORS

CNN (Cellular Neural/Nonlinear Networks) is not only a paradigm for image processing systems composed of a large number of analog processing elements that are locally interconnected, but is also employed also for generating chaotic and hyperchaotic behaviours, static and dynamic patterns, autowaves and spatial-temporal chaos [15], [2], [19]. Here we consider a one-dimensional CNN consisting of identical n-scroll attractors from the system 5 with diffusive coupling

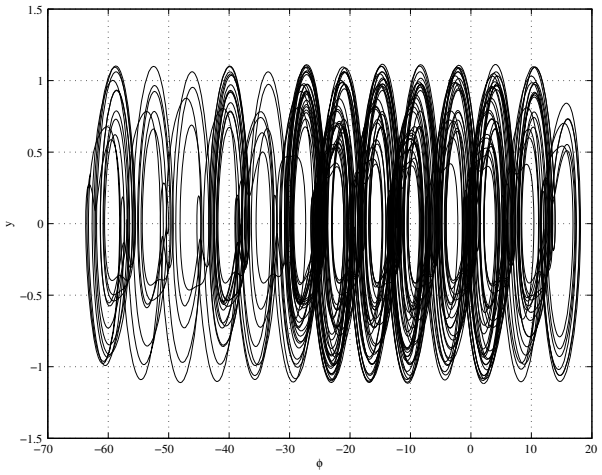


Fig. 4. n-scroll attractor from the model (5) with $k = 1$, $a = 0.1$.

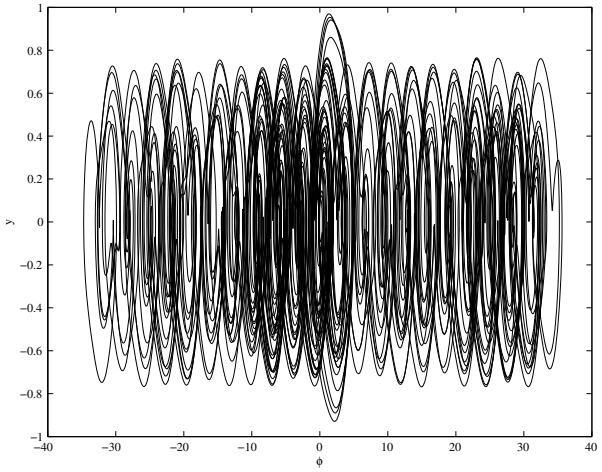


Fig. 5. n-scroll attractor from the model (5) with $k = 2$, $a = 0.1$.

between the cells

$$\begin{cases} \dot{\phi}_i &= ky_i \\ \dot{y}_i &= z_i + \lambda(y_{i-1} - 2y_i + y_{i+1}) \\ \dot{z}_i &= -ay_i - az_i + \frac{a}{2Ic} I_i, \;\; i = 1, 2, 3..., L \end{cases} \tag{6}$$

where i denotes the cell number and L is the number of the cells (see Figure 7). We impose the periodic boundary condition $y_0 = y_L$ and $y_{L+1} = y_1$ (see Figure 1). By taking n-scroll attractors as a cell one can obtain so-called n-scroll square ($L = 2$), n-scroll cube ($L = 3$) and n-scroll hypercube ($L > 3$) attractors in the common state subspace of the cells [20], [2].

Figure 8 and 9 show n-scroll cube attractor from the model 6 for the coupling parameter $\lambda = 0.001$ and $\lambda = 0.01$, respectively. As it is reported in [20], in order to obtain n-scroll cube attractors weak coupling is needed. When the coupling weight is increased, one starts observing synchronization [21] between the cells such as for $\lambda = 0.5$.

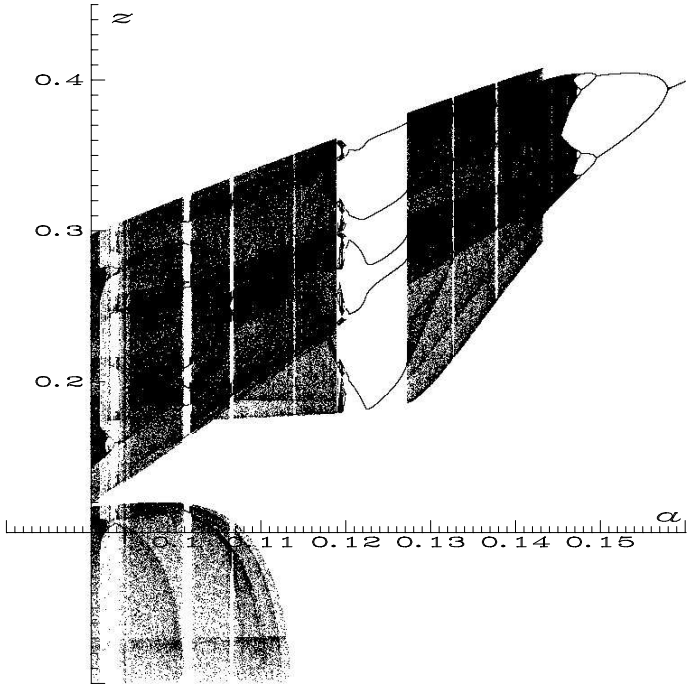


Fig. 6. Bifurcation diagram of the system (5).

V. Conclusion

In this paper a model for generating n-scroll attractors has been studied which is based on a model with a sine function nonlinearity. This model allows to use a Josephson junction nonlinearity. Using the phase difference of the junction as a state variable in the model n-scroll attractors have been obtained and the bifurcation diagram of the model is given. By taking a one-dimensional CNN consisting of n-scroll attractors from the model with a Josephson junction and weak diffusive coupling between the cells, n-scroll hypercube attractors are obtained. The results have been illustrated with computer simulations.

Acknowledgment

This research work was partially carried out at the ESAT laboratory and the Interdisciplinary Center of Neural Networks ICNN of the Katholieke Universiteit Leuven, in the framework of the Belgian Programme on Interuniversity Poles of Attraction, initiated by the Belgian State, Prime Minister's Office for Science, Technology and Culture (IUAP P4-02, IUAP P4-24, IUAP-V), the Concerted Action Project Ambiorics of the Flemish Community and the FWO projects G.0226.06, G.0211.05, G.0499.04, G.0407.02. JS is an associate professor with K.U. Leuven.

References

[1] L. O. Chua, "Chua's circuit 10 years later," *Int. J. Circuit Theory and Applications*, vol. 22, pp. 279–305, 1994.

[2] M. E. Yalçın, J. A. K. Suykens, and J. Vandewalle, *Cellular neural networks, multi-scroll chaos and synchronization*, ser. Nonlinear Science A. Singapore: World Scientific, 2005, vol. 50.

[3] J. Lu and G. Chen., "Multi-scroll chaos generation: Theories, methods and applications," *Int. J. Bifurcation and Chaos*, vol. 16, no. 4, 2006, in press.

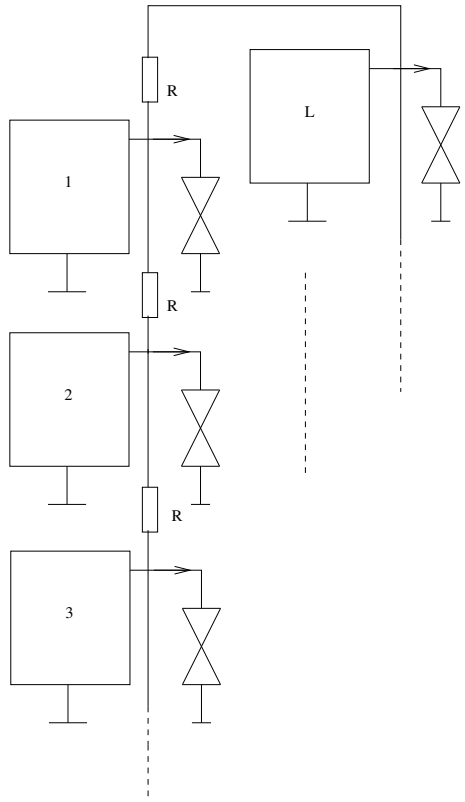


Fig. 7. One-dimensional CNN consisting of identical n-scroll attractor circuit using Josephson junction.

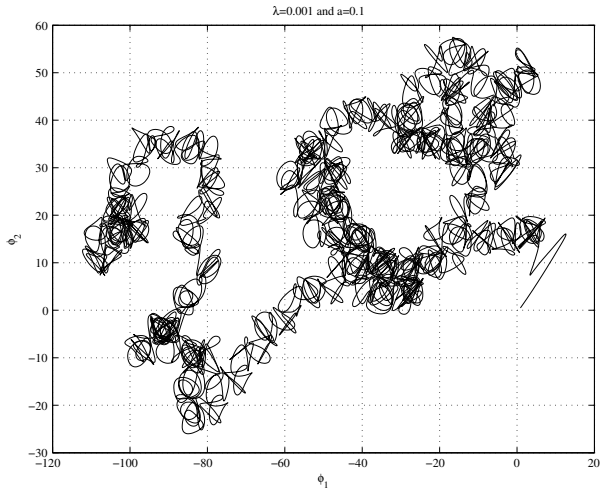


Fig. 8. n-scroll hypercube attractor for $\lambda = 0.001$.

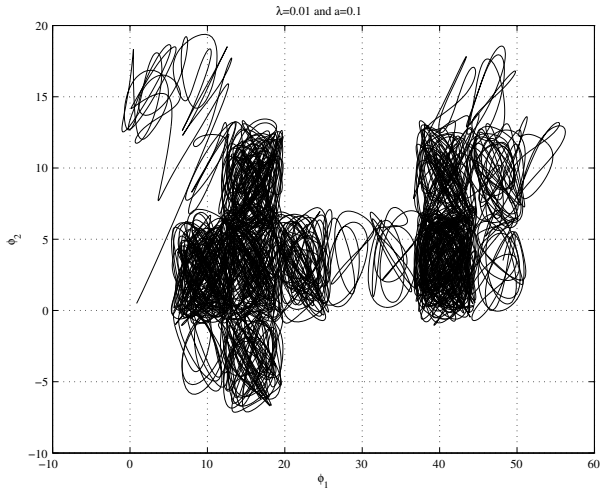


Fig. 9. n-scroll hypercube attractor for $\lambda = 0.01$.

[4] J. A. K. Suykens and J. Vandewalle, "Generation of n-double scrolls (n= 1,2,3,4,...)," *IEEE Trans. Circuits and Systems-I*, vol. 40, pp. 861–867, 1993.

[5] J. A. K. Suykens, A. Huang, and L. O. Chua, "A family of n-scroll attractors from a generalized Chua's circuit," *Archiv für Elektronik und Ubertragungstechnik*, vol. 51, no. 3, pp. 131–138, 1997.

[6] P. Arena, S. Baglio, L. Fortuna, and G. Manganaro, "Generation of n-double scrolls via cellular neural networks," *Int. J. Circuit Theory and Applications*, vol. 24, pp. 241–252, 1996.

[7] M. E. Yalçın, J. A. K. Suykens, and J. Vandewalle, "Experimental confirmation of 3- and 5-scroll attractors from a generalized Chua's circuit," *IEEE Trans. Circuits and Systems-I*, vol. 47, no. 3, pp. 425–429, 2000.

[8] M. E. Yalçın, S. Ozoğuz, J. A. K. Suykens, and J. Vandewalle, "Families of scroll grid attractors," *Int. J. Bifurcation and Chaos*, vol. 12, no. 1, pp. 23–41, 2002.

[9] D. Cafagna and G. Grassi, "Hyperchaotic coupled chua circuits: An approach for generating new n x m-scroll attractors," *Int. J. Bifurcation and Chaos*, vol. 13, no. 9, 2003.

[10] J. H. Lu, F. L. Han, X. Yu, and G. Chen, "Generating 3-D multi-scroll chaotic attractors: A hysteresis series switching method," *Automatica*, vol. 40, pp. 1677–1687, 2004.

[11] K. S. Tang, G. Q. Zhong, G. Chen, and K. F. Man, "Generation of n-scroll attractors via sine function," *IEEE Trans. Circuits and Systems-I*, vol. 48, no. 11, pp. 1369–1372, 2001.

[12] S. Ozoğuz, A. S. Elwakil, and K. N. Salama, "n-scroll chaos generator using nonlinear transconductor," *Elecronics Letters*, vol. 38, no. 14, pp. 685–686, 2002.

[13] K. K. Likharev, *Dynamics of Josepson junction and circuit*. New York: Gordon and Breach, 1986.

[14] M. E. Yalçın, J. A. K. Suykens, and J. Vandewalle, "True random bit generation from a double scroll attractor," *IEEE Trans. Circuits and Systems-I*, vol. 51, no. 7, pp. 1395–1404, 2004.

[15] L. O. Chua, *CNN: a Paradigm for Complexity*. Singapore: World Scientific, 1998.

[16] M. E. Yalçın and J. A. K. Suykens, "Generation of n-scroll attractors by josephson junctions," in *Proceedings of the 2005 Int. Symposium on Nonlinear Theory and its Applications (NOLTA 2005)*, Bruges, Belgium, Oct. 18-21 2005, pp. 501–504.

[17] S. H. Strogatz, *Nonlinear Dynamics and Chaos*. Addison Wesley, 1996.

[18] S. K. Dana, D. C. Sengupta, and K. D. Edoh, "Chaotic dynamics in Josephson junction," *IEEE Trans. Circuits and Systems-I*, vol. 48, no. 8, pp. 990–996, 2001.

[19] M. E. Yalçın and J. A. K. Suykens, "Spatiotemporal pattern formation on the ace16k cnn chip," *Int. J. Bifurcation and Chaos*, vol. 16, no. 5, 2006.

[20] J. A. K. Suykens and L. O. Chua, "$n-$double scroll hypercubes in $1D-$CNNs," *Int. J. Bifurcation and Chaos*, vol. 7, no. 8, pp. 1873–1885, 1997.

[21] J. A. K. Suykens, M. E. Yalçın, and J. Vandewalle, "Chaotic systems synchronization," in *Chaos Control : Theory and Applications*, ser. Lecture Notes in Control and Information Sciences, G. Chen and X. Yu, Eds. Springer-Verlag, 2003, vol. 292, pp. 117–136.