ADAPTIVE COMPENSATION OF FRONT-END NON-IDEALITIES IN OFDM BASED SYSTEMS

ABSTRACT

Direct conversion architectures are very sensitive to front-end non-idealities such as in-phase/quadrature-phase (IQ) imbalances and carrier frequency offset. In this paper the joint effect of frequency selective IQ imbalance at both the transmitter and receiver under carrier frequency offset is studied for OFDM based transmission. We also consider the general case when the cyclic prefix is not sufficiently long to accommodate the channel impulse response combined with the transmitter and receiver filter, which results in inter-block-interference (IBI) between the OFDM symbols. We propose a frequency domain per-tone equalizer (PTEQ) based receiver structure obtained by transferring a time domain equalizer (TEQ) to the frequency domain. In addition to the frequency domain PTEQ design procedure, a training-based RLS type initialization scheme for direct per-tone equalization is proposed. The resulting scheme provides a very efficient post-FFT adaptive equalization and a front-end compensation performance which is shown to be close to the ideal case.

1. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) [1] is a popular, standardized modulation technique for broadband wireless systems: it is used for Wireless LAN [2], Fixed Broadband Wireless Access [3], Digital Video & Audio Broadcasting [4],[5] etc.

Hence, a lot of effort is spent in developing integrated, cost and power efficient OFDM systems. The zero-IF architecture (or direct-conversion architecture) is an attractive candidate as it can convert the radio frequency (RF) signal directly to baseband (BB) or vice-versa without any intermediate frequencies (IF). This results in an overall smaller size with lower component cost as compared to a traditional superheterodyne architecture. However the zero-IF architecture performs the in-phase/quadrature-phase (IQ) modulation and demodulation in the analog domain. This inherent two-path analog processing results in the system being extremely sensitive to mismatches between the I and Q branch, especially so when high-order modulations schemes (e.g., 64QAM, etc) are used. Due to component imperfections in practical analog electronics such imbalances are unavoidable, resulting in an overall performance degradation of the system. Along with IQ imbalance, OFDM systems are also very sensitive to carrier frequency offset (CFO). The performance degradation due to receiver IQ imbalance and CFO in OFDM systems has been investigated in [6], [7] and [8].

Rather than decreasing the IQ imbalance by increasing the design time and the component cost of the analog processing, IQ imbalance can also be tolerated and then compensated digitally. A joint compensation algorithm for frequency independent (constant over frequency) transmitter and receiver IQ imbalance has been developed in [9]. In [10], the authors analyze the performance of a joint frequency selective transmitter and receiver IQ compensation technique under a small residual CFO. However the scheme does not compensate the effect of the CFO. The compensation of CFO along with frequency independent transmitter and receiver IQ imbalance has been proposed in [11]. An adaptive frequency domain per-tone equalizer (PTEQ) for joint compensation of frequency selective transmitter and receiver IQ imbalance and for frequency selective receiver IQ imbalance along with CFO have been proposed in [12] and [13] .

This paper extends all previously developed schemes by providing a general compensation solution for joint frequency selective transmitter and receiver IQ imbalance in the presence of CFO and channel distortions. We consider the case when the cyclic prefix is not sufficiently long to accommodate the combined channel, transmitter and receiver filter impulse response, which results in Inter-Block-Interference (IBI) between adjacent OFDM symbols. In this case a PTEQ is designed to shorten the combined impulse response to fit within the cyclic prefix [14] and at the same time compensate the analog front-end imperfection. The PTEQ solution proposed is initialized by an adaptive RLS based algorithm. The PTEQ enables a true per-tone signal-to-noise ratio (SNR) optimization, because the equalization of one carrier is independent of the equalization of other carriers.

The paper is organized as follows. Section 2 describes the IQ imbalance and CFO model in OFDM systems. Section 3 explains the IQ and CFO compensation scheme. Section 4 presents the training based initialization scheme. Simulation results are shown in section 5 and finally conclusions are given in section 6.

Notation: Vectors are indicated in bold and scalar parameters in normal font. Superscripts $^*, ^T, ^H$ represent conjugate, transpose and hermitian respectively. $\mathbf{F}$ and $\mathbf{F}^{-1}$ represent the $N \times N$ discrete Fourier transform and its inverse. $\mathbf{I}_N$ is the $N \times N$ identity matrix and $0_{M\times N}$ is the $M \times N$ all zero matrix. Operators $\otimes$, $\star$ and . denote Kronecker product, convolution and component-wise vector multiplication respectively.

2. IQ IMBALANCE AND CFO MODEL

Let $\mathbf{S}^{(i)}$ be a frequency domain OFDM symbol of size $(N \times 1)$ where i is the time index of the symbol. We consider two successive OFDM symbols transmitted at time $i-1$ and i respectively. The ith symbol is the symbol of interest, the previous symbol is included to model IBI. These symbols are transformed to the time domain by the inverse discrete Fourier transform (IDFT). A cyclic prefix (CP) of length ν is then added to the head of each symbol. In the case of no IQ imbalance in the front end of the transmitter, the resulting time domain baseband signal $\mathbf{s}$ is given as:

$$\mathbf{s} = (\mathbf{I}_2 \otimes \mathbf{P})(\mathbf{I}_2 \otimes \mathbf{F}^{-1}) \left[\mathbf{S}^{(i-1)^T} \quad \mathbf{S}^{(i)^T}\right]^T \tag{1}$$

where $\mathbf{P}$ is the cyclic prefix insertion matrix given by:

$$\mathbf{P} = \left[\begin{array}{c|c} \mathbf{0}_{(\nu\times N-\nu)} & \mathbf{I}_\nu \\ \hline & \mathbf{I}_N \end{array}\right]$$

We categorize the IQ imbalance resulting from the front-end as frequency dependent (frequency selective) and frequency independent (constant over frequency). The imbalances caused by digital-to-analog converters (DAC), amplifiers, low pass filters (LPFs) and

mixers generally result in an overall frequency dependent IQ imbalance. We represent this imbalance at the transmitter by two mismatched filters with frequency responses given as $\mathbf{H}_{ti}$ and $\mathbf{H}_{tq}$. As the local oscillator (LO) produces only a single tone, the IQ imbalance caused by the LO can be generally categorized as frequency independent over the signal bandwidth with a transmitter amplitude and phase mismatch g_t and ϕ_t between the two branches. Following the derivation in [7], the baseband signal $\mathbf{p}$ can be specified as:

$$\mathbf{p} = \mathbf{g}_{t1} \star \mathbf{s} + \mathbf{g}_{t2} \star \mathbf{s}^* \tag{2}$$

where

$$\mathbf{g}_{t1} = \mathbf{F}^{-1}\{\mathbf{G}_{t1}\} = \mathbf{F}^{-1}\left\{\frac{[\mathbf{H}_{ti} + g_t e^{-j\phi_t}\mathbf{H}_{tq}]}{2}\right\}$$
$$\mathbf{g}_{t2} = \mathbf{F}^{-1}\{\mathbf{G}_{t2}\} = \mathbf{F}^{-1}\left\{\frac{[\mathbf{H}_{ti} - g_t e^{j\phi_t}\mathbf{H}_{tq}]}{2}\right\}$$

Here $\mathbf{g}_{t1}$ and $\mathbf{g}_{t2}$ are mostly truncated to length L_t and then padded with $N - L_t$ zero elements. They represent the combined frequency independent and dependent transmitter IQ imbalance. The convolution operations will be specified as matrix-vector products further up. Note also that $\mathbf{g}_{t2}$ vanishes if $g_t = 1$, $\phi_t = 0$ and $\mathbf{H}_{ti} = \mathbf{H}_{tq}$.

When the signal $\mathbf{p}$ is transmitted through a frequency selective time invariant channel, then the received RF signal $\tilde{\mathbf{r}}$ modulated at center frequency f_c is given as:

$$\tilde{\mathbf{r}} = \Re\{\mathbf{r}.e^{j2\pi f_c.\mathbf{t}}\}$$

where $e^{j\mathbf{x}}$ is the element-wise exponential function on the vector $\mathbf{x}$ and $\mathbf{t}$ is a time vector. $\mathbf{r}$ is the baseband representation of the received signal given as:

$$\begin{aligned}\mathbf{r} &= \mathbf{c} \star \mathbf{p} + \mathbf{n} \\ &= \mathbf{c} \star \mathbf{g}_{t1} \star \mathbf{s} + \mathbf{c} \star \mathbf{g}_{t2} \star \mathbf{s}^* + \mathbf{n} \\ &= \mathbf{c}_1 \star \mathbf{s} + \mathbf{c}_2 \star \mathbf{s}^* + \mathbf{n}\end{aligned} \tag{3}$$

where $\mathbf{c}$ is the baseband representation of the channel of length L, $\mathbf{c}_1$ and $\mathbf{c}_2$ are the combined transmitter IQ and channel impulse responses of length $L + L_t - 1$ and $\mathbf{n}$ is proper complex additive white Gaussian noise (AWGN).

Finally, an expression similar to equation (2) can be used to model IQ imbalance at the receiver. Let $\mathbf{z}$ represent the down-converted baseband complex signal after being distorted by combined frequency dependent and independent receiver IQ imbalance $\mathbf{g}_{r1}$ and $\mathbf{g}_{r2}$ of length L_r. Then $\mathbf{z}$ will be given as:

$$\mathbf{z} = \mathbf{g}_{r1} \star \mathbf{r} + \mathbf{g}_{r2} \star \mathbf{r}^* \tag{4}$$

Equation (3) can be substituted in equation (4) leading to

$$\begin{aligned}\mathbf{z} =& (\mathbf{g}_{r1} \star \mathbf{c}_1 + \mathbf{g}_{r2} \star \mathbf{c}_2^*) \star \mathbf{s} + (\mathbf{g}_{r1} \star \mathbf{c}_2 + \mathbf{g}_{r2} \star \mathbf{c}_1^*) \star \mathbf{s}^* \\ &+ \mathbf{g}_{r1} \star \mathbf{n} + \mathbf{g}_{r2} \star \mathbf{n}^* \\ =& \mathbf{d}_1 \star \mathbf{s} + \mathbf{d}_2 \star \mathbf{s}^* + \mathbf{n}_c\end{aligned} \tag{5}$$

where $\mathbf{d}_1$ and $\mathbf{d}_2$ are the combined transmitter IQ, channel and receiver IQ impulse responses of length $L_t + L + L_r - 2$ and $\mathbf{n}_c$ is now a zero mean improper complex noise vector due to the presence of receiver IQ imbalance [15].

Substituting equation (1) in equation (5) we obtain:

$$\begin{aligned}\mathbf{z} =& [\mathbf{O}_1 | \mathbf{T}_{d1}](\mathbf{I}_2 \otimes \mathbf{P})(\mathbf{I}_2 \otimes \mathbf{F}^{-1}) \begin{bmatrix} \mathbf{S}^{(i-1)} \\ \mathbf{S}^{(i)} \end{bmatrix} \\ &+ [\mathbf{O}_1 | \mathbf{T}_{d2}](\mathbf{I}_2 \otimes \mathbf{P})(\mathbf{I}_2 \otimes \mathbf{F}^{-1}) \begin{bmatrix} \mathbf{S}_m^{*(i-1)} \\ \mathbf{S}_m^{*(i)} \end{bmatrix} + \mathbf{n}_c\end{aligned} \tag{6}$$

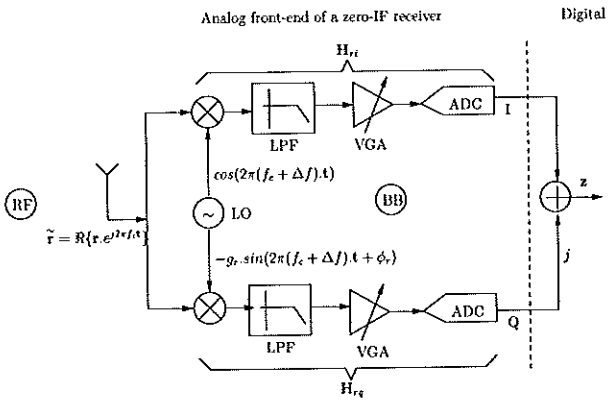


Figure 1: Model of a zero-IF receiver with IQ imbalance and CFO.

where $\mathbf{z}$ is of dimension $(N \times 1)$, $\mathbf{O}_1 = 0_{(N \times N + 2\nu - L_T - L - L_r + 3)}$. $\mathbf{T}_{dk}$ (for $k = 1,2$) is an $(N \times N + L_t + L + L_r - 3)$ Toeplitz matrix with first column $[\mathbf{d}_{k(L_t+L+L_r-3)}, 0_{(1\times N-1)}]^T$ and first row $[\mathbf{d}_{k(L_t+L+L_r-3)}, \cdots, \mathbf{d}_{k(0)}, 0_{(1\times N-1)}]$. Here $()_m$ denotes the mirroring operation in which the vector indices are reversed, such that $\mathbf{S}_m[l] = \mathbf{S}[l_m]$ where $l_m = 2 + N - l$ for $l = 2 \ldots N$ and $l_m = l$ for $l = 1$.

In the frequency domain, if the cyclic prefix is long enough $(\nu \geq L_t + L + L_r - 2)$, then equation (6) can be given as:

$$\begin{aligned}\mathbf{Z} =& \mathbf{D}_1.\mathbf{S}^{(i)} + \mathbf{D}_2.\mathbf{S}_m^{*(i)} + \tilde{\mathbf{n}}_c \\ =& (\mathbf{G}_{r1}.\mathbf{G}_{t1}.\mathbf{C} + \mathbf{G}_{r2}.\mathbf{G}_{t2m}^*.\mathbf{C}_m^*).\mathbf{S}^{(i)} \\ &+ (\mathbf{G}_{r1}.\mathbf{G}_{t2}.\mathbf{C} + \mathbf{G}_{r2}.\mathbf{G}_{t1m}^*.\mathbf{C}_m^*).\mathbf{S}_m^{*(i)} + \mathbf{G}_{r1}.\tilde{\mathbf{n}} + \mathbf{G}_{r2}.\tilde{\mathbf{n}}_m^*\end{aligned} \tag{7}$$

where $\mathbf{Z}, \mathbf{D}_1, \mathbf{D}_2, \mathbf{G}_{r1}, \mathbf{G}_{r2}, \mathbf{C}, \tilde{\mathbf{n}}_c$ and $\tilde{\mathbf{n}}$ are frequency domain representations of $\mathbf{z}, \mathbf{d}_1, \mathbf{d}_2, \mathbf{g}_{r1}, \mathbf{g}_{r2}, \mathbf{c}, \mathbf{n}_c$ and $\mathbf{n}$. Equation (7) shows that due to the transmitter and receiver IQ imbalance, power leaks from the signal on the mirror carrier $(\mathbf{S}_m^*)$ to the carrier under consideration $(\mathbf{S})$ and thus causes Inter-Carrier-Interference (ICI). In the case when the cyclic prefix is not long enough $(\nu < L_t + L + L_r - 2)$, then in addition to ICI there is also interference from the adjacent OFDM symbol $\mathbf{S}^{(i-1)}$, leading to IBI.

A carrier frequency offset occurs in the system when there is a deviation in the generation of the standard carrier frequency between the transmitter and receiver local oscillators. Assume a CFO Δf is present in the OFDM system together with transmitter and receiver IQ imbalance, then the resulting baseband signal can be written as [11]:

$$\begin{aligned}\mathbf{z} =& \mathbf{g}_{r1} \star (\mathbf{r}.e^{j2\pi\Delta f.\mathbf{t}}) + \mathbf{g}_{r2} \star (\mathbf{r}^*.e^{-j2\pi\Delta f.\mathbf{t}}) \\ =& (\tilde{\mathbf{g}}_{r1} \star \mathbf{r}).e^{j2\pi\Delta f.\mathbf{t}} + (\tilde{\mathbf{g}}_{r2} \star \mathbf{r}^*).e^{-j2\pi\Delta f.\mathbf{t}}\end{aligned} \tag{8}$$

where $\tilde{\mathbf{g}}_{rk}$ (for $k = 1,2$) $= \mathbf{g}_{rk}.(e^{j2\pi\Delta f.(0\ldots(L_r-1))})^{-1}$. Figure 1 illustrates the front-end of a zero-IF receiver with IQ imbalance and CFO. The joint effect of both transmitter and receiver IQ imbalance along with CFO results in a severe performance degradation, as will be shown in section 5, and so a digital compensation scheme is needed.

3. IQ IMBALANCE AND CFO COMPENSATION

We assume that the CFO can be estimated accurately in the OFDM system. In practice, several CFO estimation schemes indeed exist that are found to be sufficiently robust against the IQ imbalance, see e.g. [8] and [16]. Thus, given a good estimate of Δf, we can perform an element wise multiplication of the estimated negative frequency offset $e^{-j2\pi\Delta f.\mathbf{t}}$ with the received distorted symbol as well

as with its conjugate, leading to:

$$\mathbf{z}_1 \triangleq \mathbf{z}.e^{-j2\pi\Delta f.t} = \underbrace{(\tilde{\mathbf{g}}_{r1} \star \mathbf{r})}_{\mathbf{p}} + \underbrace{(\tilde{\mathbf{g}}_{r2} \star \mathbf{r}^*).e^{-2j2\pi\Delta f.t}}_{\mathbf{q}} \quad (9)$$

and

$$\mathbf{z}_2 \triangleq \mathbf{z}^*.e^{-j2\pi\Delta f.t} = \underbrace{(\tilde{\mathbf{g}}^*_{r2} \star \mathbf{r})}_{\mathbf{p}} + \underbrace{(\tilde{\mathbf{g}}^*_{r1} \star \mathbf{r}^*).e^{-2j2\pi\Delta f.t}}_{\mathbf{q}} \quad (10)$$

Both the signals $\mathbf{z}_1$ and $\mathbf{z}_2$ consist of two parts named '**p**' the desired part and '**q**' the undesired part. This is because in the frequency domain, the former gives rise to the desired signal, while the latter yields a mirror image and causes ICI (because of the complex conjugate), subject to leakage caused by the exponential term $(e^{-2j2\pi\Delta f.t})$.

In order to eliminate the undesired part **q** from equations (9) and (10), we first propose a compensation scheme based on two time domain equalizers (TEQs) $\mathbf{w}_1$ and $\mathbf{w}_2$ each with L' taps. If the cyclic prefix is sufficiently long ($\nu \geq L_t + L + L_r - 2$) then typically $L' = L_r$ taps is sufficient. For the case of an insufficiently long cyclic prefix ($\nu < L_t + L + L_r - 2$), L' is chosen longer in order to shorten the combined channel, transmitter and receiver filter impulses to fit within the cyclic prefix.

The TEQ $\mathbf{w}_1$ is applied to the signal $\mathbf{z}_1$ and TEQ $\mathbf{w}_2$ to the signal $\mathbf{z}_2$. Now $\mathbf{z}$ in equation (6) is of size $(N + L' - 1 \times 1)$, where $\mathbf{O}_1 = 0_{(N+L'-1\times N+2\nu-L_t-L-L_r-L'+4)}, \mathbf{T}_{dk}$ (for $k = 1,2$) is of size $(N + L' - 1 \times N + L_t + L + L_r + L' - 4)$ with first column $[\mathbf{d}_{k(L_t+L+L_r-3)}, 0_{(1\times N+L'-1)}]^T$ and first row $[\mathbf{d}_{k(L_t+L+L_r-3)}, \cdots, \mathbf{d}_{k(0)}, 0_{(1\times N+L'-2)}]$. This leads to:

$$\begin{aligned}
\mathbf{z}_t =& \mathbf{w}_1 \star \mathbf{z}_1 + \mathbf{w}_2 \star \mathbf{z}_2 \\
=& \underbrace{((\mathbf{w}_1 \star \tilde{\mathbf{g}}_{r1} \star \mathbf{g}_{t1} + \mathbf{w}_2 \star \tilde{\mathbf{g}}^*_{r2} \star \mathbf{g}_{t1}) \star \mathbf{c})}_{\mathbf{f}_1} \star \mathbf{s} \\
&+ \underbrace{((\mathbf{w}_1 \star \tilde{\mathbf{g}}_{r1} \star \mathbf{g}_{t2} + \mathbf{w}_2 \star \tilde{\mathbf{g}}^*_{r2} \star \mathbf{g}_{t2}) \star \mathbf{c})}_{\mathbf{f}_2} \star \mathbf{s}^* \\
&+ \underbrace{\mathbf{w}_1 \star ((\tilde{\mathbf{g}}_{r2} \star \mathbf{r}^*).e^{-2j2\pi\Delta f.t}) + \mathbf{w}_2 \star ((\tilde{\mathbf{g}}^*_{r1} \star \mathbf{r}^*).e^{-2j2\pi\Delta f.t})}_{\mathbf{f}_3} \\
&+ (\mathbf{w}_1 \star \tilde{\mathbf{g}}_{r1} + \mathbf{w}_2 \star \tilde{\mathbf{g}}^*_{r2}) \star \mathbf{n}
\end{aligned} \quad (11)$$

where $\mathbf{z}_t$ is of size $(N \times 1)$ and the design target for $\mathbf{w}_1$ and $\mathbf{w}_2$ is that $\mathbf{f}_3$ vanishes, while $\mathbf{f}_1$ and $\mathbf{f}_2$ are (approximately) shorter than the cyclic prefix, such that the resulting vector $\mathbf{z}_t$ is free of CFO and IBI. $\mathbf{z}_t$ then contains only frequency selective transmitter and receiver IQ imbalance along with channel and noise distortions. In conjunction with the TEQ scheme, a DFT is applied to the filtered sequence $\mathbf{z}_t$. Finally a two tap frequency domain equalizer (FEQ) is applied at every sub-carrier and its conjugate mirror to compensate the IQ imbalance and the channel distortions. This scheme is shown in **Figure 2(a)**. We define $\tilde{\mathbf{S}}^{(i)}[l]$ as the estimate for lth sub-carrier of the ith OFDM symbol. This estimate is then obtained as:

$$\tilde{\mathbf{S}}^{(i)}[l] = \mathbf{v}_1[l].\mathbf{F}[l](\mathbf{W}_1^T\mathbf{z}_1 + \mathbf{W}_2^T\mathbf{z}_2) + \mathbf{v}_2[l].(\mathbf{F}[l_m](\mathbf{W}_1^T\mathbf{z}_1 + \mathbf{W}_2^T\mathbf{z}_2))^* \quad (12)$$

where $\mathbf{v}_1[l]$ and $\mathbf{v}_2[l]$ are the two taps of the FEQ for the lth sub-carrier, and $\mathbf{F}[l]$ is the lth row of the DFT matrix $\mathbf{F}$. $\mathbf{W}_k$ (for $k = 1,2$) is an $(N + L' - 1 \times N)$ Toeplitz matrix with first column $[\mathbf{w}_{k,L'-1}, \cdots, \mathbf{w}_{k,0}, 0_{(1\times N-1)}]$ and first row $[\mathbf{w}_{k,L'-1}, 0_{(1\times N-1)}]$.

Following the derivation in [14], the two TEQs thus obtained can be transferred to the frequency domain resulting in two per-tone equalizers (PTEQs) each employing one DFT and $L' - 1$ difference terms. With this transformation the difficult channel shortening and CFO compensation problem (design of $\mathbf{w}_1$ and $\mathbf{w}_2$) is avoided and replaced by a simple per-tone optimization problem. Equation (12) is then modified as follows:

$$\tilde{\mathbf{S}}^{(i)}[l] = \mathbf{v}_1^T[l]\mathbf{F}_i[l]\mathbf{z}_1 + \mathbf{v}_2^T[l](\mathbf{F}_i[l_m]\mathbf{z}_1)^* + \mathbf{v}_3^T[l]\mathbf{F}_i[l]\mathbf{z}_2 + \mathbf{v}_4^T[l](\mathbf{F}_i[l_m]\mathbf{z}_2)^* \quad (13)$$

where $\mathbf{v}_k[l]$ (for $k = 1 \ldots 4$) are PTEQs of size $(L' \times 1)$. $\mathbf{F}_i[l]$ is defined as:

$$\mathbf{F}_i[l] = \left[\begin{array}{c|c|c} \mathbf{I}_{L'-1} & \mathbf{0}_{L'-1\times N-L'+1} & -\mathbf{I}_{L'-1} \\ \hline \mathbf{0}_{1\times L'-1} & \mathbf{F}[l] & \end{array}\right]$$

where the first block row in $\mathbf{F}_i[l]$ is seen to extract the difference terms, while the last row corresponds to the single DFT. The PTEQ block scheme is shown in **Figure 2(b)**. The PTEQ coefficients for the lth sub-carrier can be obtained based on the following MSE minimization:

$$min_{\begin{pmatrix}\mathbf{v}_1[l],\mathbf{v}_2[l]\\ \mathbf{v}_3[l],\mathbf{v}_4[l]\end{pmatrix}} \mathbb{E}\left\{\left|\mathbf{S}^{(i)}[l] - \begin{bmatrix}\mathbf{v}_1[l]\\ \mathbf{v}_2[l]\\ \mathbf{v}_3[l]\\ \mathbf{v}_4[l]\end{bmatrix}^T \begin{bmatrix}\mathbf{F}_i[l]\mathbf{z}_1\\ (\mathbf{F}_i[l_m]\mathbf{z}_1)^*\\ \mathbf{F}_i[l]\mathbf{z}_2\\ (\mathbf{F}_i[l_m]\mathbf{z}_2)^*\end{bmatrix}\right|^2\right\} \quad (14)$$

where $\mathbb{E}\{.\}$ is the expectation operator.

4. TRAINING BASED INITIALIZATION

In this section, we specify a training based RLS algorithm for direct initialization of the PTEQ coefficients based on (14). The RLS algorithm provides optimal convergence and achieves initialization with an acceptably small number of training symbols. Let d be the number of OFDM symbols dedicated for training. The RLS algorithm to compute the coefficient vector for sub-carrier l is shown in **Algorithm 1**. Let $\mathbf{v}_k^{(i)}[l]$ (for $k = 1 \ldots 4$) represent the equalization vectors at time instant i. The regularization factor δ is a small positive constant and $\mathbf{D}^{(i)}$ is the training symbol transmitted at time instant i.

Algorithm 1 RLS direct equalization

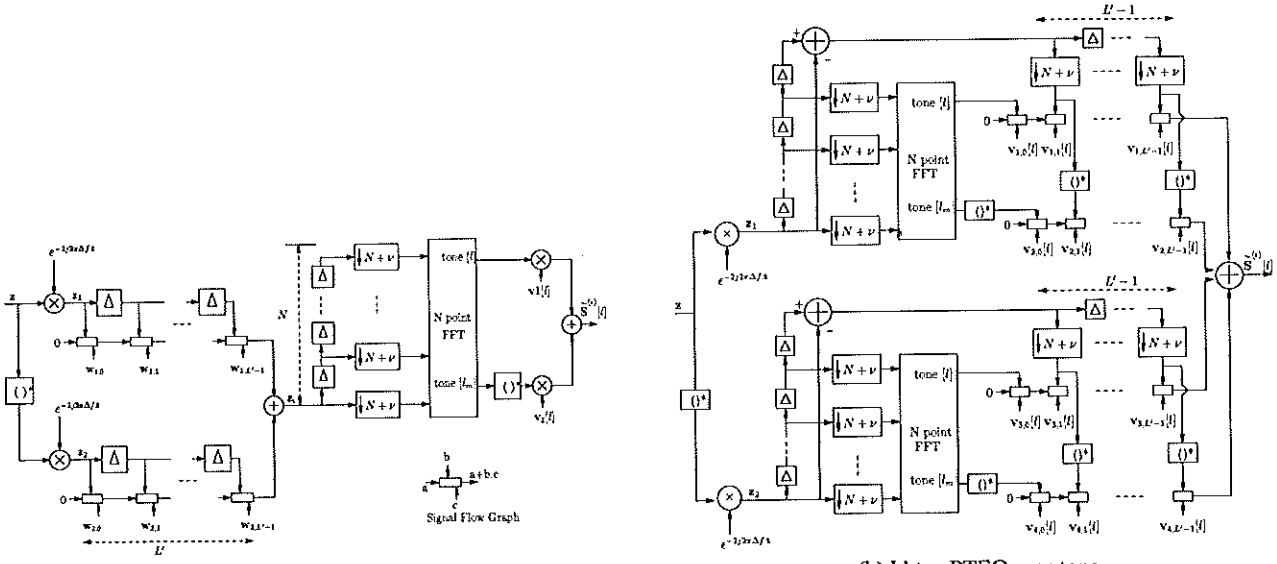
Initialize the algorithm by setting: $\mathbf{v}_k^{(i=0)} = 0_{L'\times N}$
For all the carriers in OFDM symbol $l = 1 \ldots N$ compute

$$\mathbf{B}^{(i=0)} = \delta^{-1}\mathbf{I}_{4L'\times 4L'}$$

For each iteration $i = 1 \ldots d$ compute

$$\begin{aligned}
\mathbf{u}^{(i)}[l] &= \begin{bmatrix}\mathbf{F}_i[l]\mathbf{z}_1^{(i)} & (\mathbf{F}_i[l_m]\mathbf{z}_1^{(i)})^* & \mathbf{F}_i[l]\mathbf{z}_2^{(i)} & (\mathbf{F}_i[l_m]\mathbf{z}_2^{(i)})^*\end{bmatrix}^T \\
\xi^{(i)} &= \mathbf{D}^{(i)}[l] \\
&\quad - \begin{bmatrix}\mathbf{v}_1^{T(i-1)}[l] & \mathbf{v}_2^{T(i-1)}[l] & \mathbf{v}_3^{T(i-1)}[l] & \mathbf{v}_4^{T(i-1)}[l]\end{bmatrix}\mathbf{u}^{(i)}[l] \\
\mathbf{B}_1^{(i)} &= \frac{\mathbf{B}^{(i)}\mathbf{u}^{(i)}[l]}{1 + \mathbf{u}^{H(i)}[l]\mathbf{B}^{(i)}\mathbf{u}^{(i)}[l]} \\
\mathbf{v}_1^{*(i)}[l] &= \mathbf{v}_1^{*(i-1)}[l] + (\mathbf{B}^{(i)}_{1,(1\ldots L')}\xi^{*(i)})^T \\
\mathbf{v}_2^{*(i)}[l] &= \mathbf{v}_2^{*(i-1)}[l] + (\mathbf{B}^{(i)}_{1,(L'+1\ldots 2L')}\xi^{*(i)})^T \\
\mathbf{v}_3^{*(i)}[l] &= \mathbf{v}_3^{*(i-1)}[l] + (\mathbf{B}^{(i)}_{1,(2L'+1\ldots 3L')}\xi^{*(i)})^T \\
\mathbf{v}_4^{*(i)}[l] &= \mathbf{v}_4^{*(i-1)}[l] + (\mathbf{B}^{(i)}_{1,(3L'+1\ldots 4L')}\xi^{*(i)})^T \\
\mathbf{B}^{*(i)} &= \mathbf{B}^{(i-1)} - \mathbf{B}_1^{(i-1)}\mathbf{u}^{H(i)}[l]\mathbf{B}^{(i-1)}
\end{aligned}$$

At the end of the training the weights $\mathbf{v}_k[l]$ are substituted in formula (13) to obtain the transmitted symbol estimates $\tilde{\mathbf{S}}^{(i)}[l]$.



(a) L' tap TEQs with 2 tap FEQ per-tone

(b) L' tap PTEQs per-tone

Figure 2: Compensation scheme for OFDM with frequency selective IQ imbalance and CFO

5. SIMULATION RESULTS

A typical OFDM system (similar to IEEE 802.11a) is simulated to evaluate the performance of the compensation scheme for frequency dependent and independent transmitter and receiver IQ imbalance under CFO. The performance comparison is made with an ideal system with no front-end distortion and with a system with no compensation algorithm included.

The parameters used in the simulation are as follows: OFDM symbol length $N = 64$, cyclic prefix length $\nu = 8$, filter impulse responses $\mathbf{h}_{ti} = \mathbf{h}_{ri} = [0.1,\ 0.9]$ and $\mathbf{h}_{tq} = \mathbf{h}_{rq} = [0.9,\ 0.1]$, frequency independent amplitude imbalance $g_t = g_r = 5\%$ and phase imbalance $\phi_t = \phi_r = 5°$. It should be noted that the imbalance level in this case may be higher than the level observed in a practical receiver. However we consider such an extreme case to evaluate the robustness/effectiveness of the proposed compensation scheme. We consider a CFO $\Delta f = 100kHz$. The CFO is usually considered as the ratio ζ of the actual carrier frequency offset Δf to the sub-carrier spacing $1/T.N$, i.e., $\zeta = T.N.\Delta f$, where T is the sampling period. In our case, the sub-carrier spacing is $312.5kHz$, thus ζ is 0.32.

There are 3 different channel profiles: 1) an additive white Gaussian noise (AWGN) channel with a single tap unity gain, 2) a multipath channel with $L = 4$ taps. In both the cases 1 and 2, $(L_t + L + L_r - 2 < \nu)$ and a PTEQ with $L' = L_r$ taps is sufficient for compensation. In our case $L_r = 2$. 3) A multipath channel with $L = 10$ taps. In this case $(L_t + L + L_r - 2 > \nu)$, and a PTEQ with $L' = 10$ and $L' = 15$ taps is used for compensation. The taps of the multipath channel are chosen independently with complex Gaussian distribution.

Figure 3 (a),(b) and (c) show the performance curves (BER vs SNR) for an uncoded 64QAM OFDM system. Every channel realization is independent of the previous one and the BER results depicted are obtained by averaging the BER curves over 10^4 independent channels. In the presence of frequency selective transmitter and receiver IQ imbalance and CFO with no compensation scheme in place, the OFDM system is completely unusable. Even for the case when there is only frequency independent IQ imbalance, the BER is very high. For the case $(\nu > L_t + L + L_r - 2)$, close to ideal performance is obtained with the compensation scheme in place. For the case $(\nu < L_t + L + L_r - 2)$, good performance is obtained when $L' = 15$. This is also illustrated in **Figure 3 (d)** which shows the convergence of PTEQ weights for 100 BPSK training symbols in a noiseless scenario. For $L' = 15$, all the PTEQ weights for one randomly chosen OFDM tone (tone=10), converge after 60 training symbols (solid lines with circles). For $L' = 2$, the PTEQ weights do not properly converge (solid lines), hence for this case a 2 tap PTEQ is not sufficient. Thus a PTEQ with a sufficient number of taps is essential to shorten the combined channel, transmitter and receiver filter impulse response and also to compensate for the channel and front-end distortions. The compensation performance depends on how accurately the adaptive equalizer coefficients can converge to the ideal values.

6. CONCLUSION

In this paper the joint effect of transmitter and receiver frequency selective IQ imbalance, CFO and multipath channel distortions has been studied and an algorithm has been developed to compensate for such distortions in the digital domain. The PTEQ solution proposed is also applicable in those cases where the cyclic prefix is not sufficiently long to accommodate the channel, transmitter and receiver filter impulse responses. The algorithm provides a very efficient, post-FFT adaptive equalization and compensation which provides close to ideal performance.

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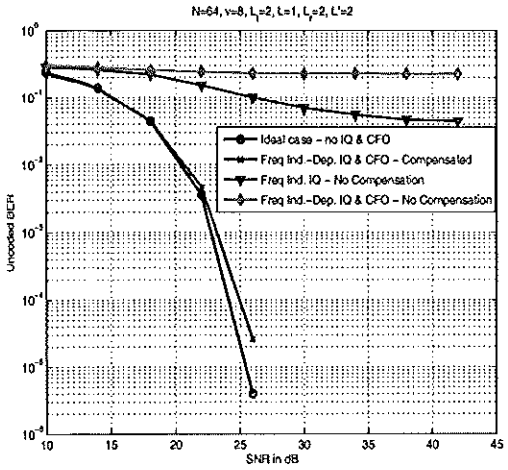
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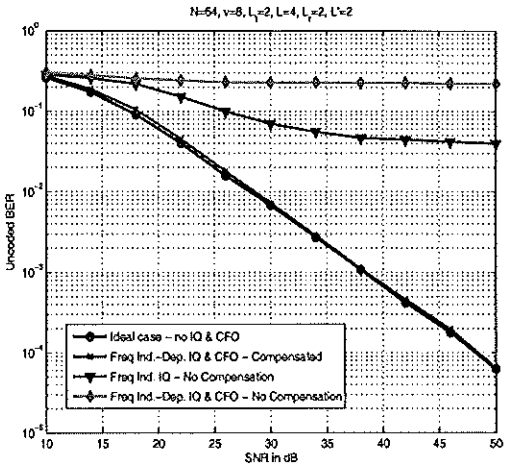
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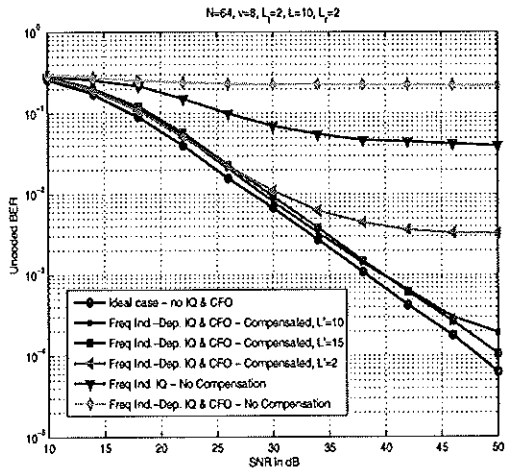
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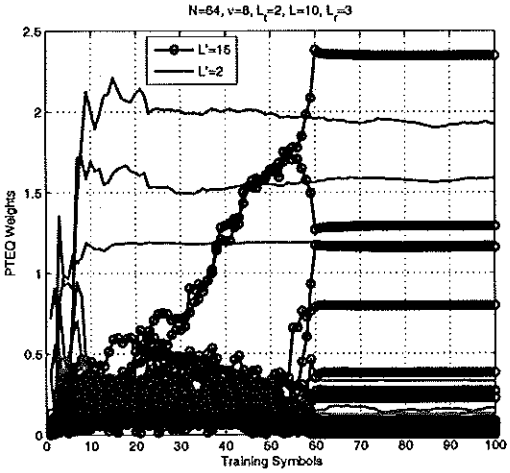
(a) BER vs SNR for AWGN flat channel (non-fading)



(b) BER vs SNR for 4-tap Rayleigh fading channel (fading)



(c) BER vs SNR for 10-tap Rayleigh fading channel (fading)



(d) PTEQ weights convergence for 10-tap Rayleigh fading channel (fading and noiseless scenario)

Figure 3: Simulation results for OFDM with 64QAM constellation.

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