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OPTIMAL CONTROL FOR POWER GENERATING KITES

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***Abstract*— In this paper we present optimization studies for kites that produce wind energy by periodically pulling a generator on the ground while flying fast in a crosswind direction. We derive a model for a single kite and formulate an optimal control problem with periodic boundary conditions and free cycle duration. The objective function is the average power at the generator. We solve this nonlinear and unstable optimal control problem numerically with a direct multiple shooting method. Here, the main result is that we can attain about 5MW with a 500m^2-kite at $10\frac{\text{m}}{\text{s}}$ wind speed. Finally, we consider a system of two kites and show that such systems can further increase the efficiency.**

Key words: **Wind Energy, Power Kites, Optimal Control, Multiple Shooting**

I. INTRODUCTION

The idea of using kites for wind power generation, previously motivated by Loyd [16], has never been closer to a large scale realization than today: Massimo Ippolito, a contractor from Torino, and the University of Torino have already tested a prototype [4] and plan to build a large scale "Kite-Wind-Generator", the University of Delft is developing kites to produce energy with "Laddermills" [11, 17, 19] and the SkySails company [21, 23] is already using wind power to pull large cargo ships [9].

The main concept for energy production with kites is that these kites periodically pull their cables to drive a generator on the ground, as it is shown in Figure 1 for a single kite. In our paper we consider the case that this generator is fixed. Of course, every kite has to be pulled back at some point in time to achieve a periodic power generating cycle. In this paper we suggest to use a lift or drag control to reduce the kite's pulling force in contrast to other methods [4, 12, 18].

The motivations to use kites instead of conventional windmills are multifaceted: First, as the power is increasing with the third power of the wind speed, it is an important fact that kites can use winds at higher altitudes. Additionally, kites avoid the statical problems with the masts and basements of windmills and they can coat a larger wind area. However, there are also some unsolved problems: For example the start of many kites in a system is challenging and we have to bring down the kites during stormy weather.

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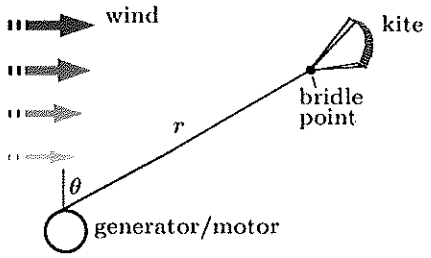


Fig. 1. Energy production with a single kite

In this paper we are interested in optimal periodic loops for a single kite to achieve a maximal average power at the generator. Our controls are the kite's roll angle, its lift coefficient and the length of the cable. To solve the corresponding optimal control problem numerically, we use the optimal control package MUSCOD-II, that is based on Bock's direct multiple shooting method and uses a full space SQP-algorithm [13, 15]. Finally, we optimize a system of two kites, which would be even more efficient, but also more difficult to control.

In section II we derive the equations of motion for a single power generating kite taking into account that the wind is increasing with the altitude. We also model the elasticity and the aerodynamic drag of the cable. In section III we formulate an optimal control problem for our kite model with periodic boundary conditions and a free cycle duration. Finally, we present and discuss the numerical results in section IV.

In section V we give a general introduction to systems with several kites, but concentrate on a special system with two kites. In section V-A we transfer the optimal control problem for the single kite to the more complicated system and present the numerical results in section V-B. Finally, we conclude the paper in section VI.

II. WIND-POWER GENERATION WITH A SINGLE KITE

We consider a kite flying in the wind that is blowing in e_x - direction, where e_x is a unit vector. Let e_z be an unit vector pointing to the sky and $e_y := e_z \times e_x$, such that $\{e_x, e_y, e_z\}$ is an orthonormal right-handed basis of the 3-dimensional Euclidean space. Furthermore, the kite is connected to a generator by a tight cable with a positive length r.

An important point in our considerations is that kites can use the powerful wind at high altitudes. Using the wind

shear model, as it is proposed in [22], the wind velocity w is depending on the altitude h over the ground:

$$w(h) = \frac{\ln\left(\frac{h}{h_r}\right)}{\ln\left(\frac{h_0}{h_r}\right)} v_0 \,, \tag{1}$$

where v_0 is the wind velocity at a suitable altitude h_0 and h_r the roughness length.

Although the dependence of the air density ρ on the altitude h will only have a small influence on our results, we take the barometric equation into account:

$$\rho(h) = \rho_0 e^{-\frac{h}{h_1}} \tag{2}$$

where ρ_0 is the air density at ground level and $h_1 \approx 8330\text{m}$ a known physical constant.

For our purposes it is efficient to use spherical coordinates $q := (r, \phi, \theta)^T$ to describe the position and the orientation of the kite. We define a local orthonormal basis $\{e_r, e_\phi, e_\theta\}$ by

$$(e_r, e_\phi, e_\theta) := R^T (e_x, e_y, e_z) \,, \tag{3}$$

where R is the following rotation matrix:

$$R := \begin{pmatrix} \sin(\theta)\cos(\phi) & \sin(\theta)\sin(\phi) & \cos(\theta) \\ -\sin(\phi) & \cos(\phi) & 0 \\ -\cos(\theta)\cos(\phi) & -\cos(\theta)\sin(\phi) & \sin(\theta) \end{pmatrix}$$

The position p of the kite is given by $p = re_r$. And its velocity is $\dot{p} = \dot{r}e_r + r\sin(\theta)e_\phi\dot{\phi} - r\dot{\theta}e_\theta$.

To derive the kite's equations of motion, we use the Lagrangian formalism. Let ρ_c be the density, d_c the diameter and $A_Q := \frac{\pi}{4}d_c^2$ the cross sectional area of the cable. Considering that the crosswind component is down rating and decreasing, when coming closer to the generator, we approximate the kinetic energy of the homogeneous cable by:

$$T_c := \int_0^1 \frac{1}{2}\rho_c A_Q r_0 \|s\dot{p}\|^2 \mathrm{d}s = \frac{1}{6} m_c \|\dot{p}\|^2 \,, \tag{4}$$

where r_0 is the length and m_c the mass of the cable. Let us define the effective inertial mass $m := m_k + \frac{m_c}{3}$ and the effective gravitational mass $\overline{m} := m_k + \frac{m_c}{2} - \Omega\rho$ with m_k being the mass and Ω the volume of the kite. The total kinetic energy is

$$T_{kin} := \frac{1}{2} m_k \|\dot{p}\|^2 + T_c = \frac{m}{2}\|\dot{p}\|^2 \,, \tag{5}$$

and the potential energy can be written as

$$V := \int_0^h \overline{m} g \,\mathrm{d}h' + \frac{EA_Q}{2r_0}(r - r_0)^2 \tag{6}$$

where $h = p \cdot e_z$ is the altitude of the kite, g the gravitational constant and E the elastic modulus of the cable.[1]

Now, the equations of motion are

$$\frac{\mathrm{d}}{\mathrm{d}t}\nabla_{\dot{q}}\mathcal{L} - \nabla_q \mathcal{L} = Q \tag{7}$$

with $\mathcal{L} := T_{kin} - V$ being the Lagrangian function and Q being a generalized force that is considered in more detail in the next subsection. We simplify the equations of motion:

$$\begin{aligned} &\tfrac{\mathrm{d}}{\mathrm{d}t}\nabla_{\dot{q}}\mathcal{L} = \tfrac{\mathrm{d}}{\mathrm{d}t}\nabla_{\dot{q}}\tfrac{m}{2}\left(\dot{r}^2 + r^2\sin^2(\theta)\dot{\phi}^2 + r^2\dot{\theta}^2\right) \\ &= m\begin{pmatrix} \ddot{r} + \frac{\dot{m}}{m}\dot{r} \\ r^2\sin^2(\theta)\left(\ddot{\phi} + 2\cot(\theta)\dot{\phi}\dot{\theta} + 2\frac{\dot{r}}{r}\dot{\phi} + \frac{\dot{m}}{m}\dot{\phi}\right) \\ r^2\left(\ddot{\theta} + 2\frac{\dot{r}}{r}\dot{\theta} + \frac{\dot{m}}{m}\dot{\theta}\right) \end{pmatrix} \end{aligned}$$

and

$$\nabla_q \mathcal{L} = m\begin{pmatrix} r\sin^2(\theta)\dot{\phi}^2 + r\dot{\theta}^2 \\ 0 \\ r^2\sin(\theta)\cos(\theta)\dot{\phi}^2 \end{pmatrix} + \widetilde{Q}$$

where we used the definition

$$\widetilde{Q} := -\overline{m}g\begin{pmatrix} \cos(\theta) \\ 0 \\ -r\sin(\theta) \end{pmatrix} - \frac{EA_Q}{r_0}\begin{pmatrix} r - r_0 \\ 0 \\ 0 \end{pmatrix}$$

Hence, we can summarize the equations of motion (7) in the form

$$S^2(\ddot{q} + a) = \frac{Q + \widetilde{Q}}{m} \tag{8}$$

with a pseudo acceleration a that will be quite small in comparison with $\ddot{q}$, since we use very long cables

$$a := \begin{pmatrix} -r\sin^2(\theta)\dot{\phi}^2 - r\dot{\theta}^2 + \frac{\dot{m}}{m}\dot{r} \\ 2\cot(\theta)\dot{\phi}\dot{\theta} + 2\frac{\dot{r}}{r}\dot{\phi} + \frac{\dot{m}}{m}\dot{\phi} \\ -\frac{\sin(2\theta)}{2}\dot{\phi}^2 + 2\frac{\dot{r}}{r}\dot{\theta} + \frac{\dot{m}}{m}\dot{\theta} \end{pmatrix} \tag{9}$$

and a left-hand side matrix

$$S := \begin{pmatrix} 1 & 0 & 0 \\ 0 & r\sin(\theta) & 0 \\ 0 & 0 & -r \end{pmatrix} \tag{10}$$

that is in our case simply diagonal and regular for positive r and $\theta \in (0, \pi)$. In the next subsection we will consider the force Q and simplify equation (8) again.

1) Forces on the kite: Let us define the effective wind vector w_e in the kite's system by

$$w_e := we_x - \dot{p} = we_x - \dot{r}e_r - r\sin(\theta)\dot{\phi}e_\phi + r\dot{\theta}e_\theta \,.$$

In the following consideration we assume that the transversal axis e_t of the kite is always perpendicular to this effective wind vector w_e by suitable yaw control. Furthermore, we introduce a controllable roll angle Ψ defined by

$$\sin(\Psi) := e_t \cdot e_r \,, \tag{11}$$

where e_t is a unit vector pointing from the left to the right wing tip [5]. Finally, the direction e_n of the aerodynamic lift is given by

$$e_n = \frac{w_e}{\|w_e\|} \times e_t \,. \tag{12}$$

The aerodynamic force on the kite is the sum of lift and drag

$$F_{aer} := \frac{1}{2}\rho c_L A \|w_e\|^2 e_n + \frac{1}{2}\rho c_D A \|w_e\| w_e \,, \tag{13}$$

where C_L and $c_D := c_{D,0} + \epsilon C_L^2$ are the lift and drag coefficient, respectively, and A the area of the kite. Here we assume that the kite's drag coefficient can be written

[1] Note that only simple longitudinal elongation of the cable are considered.

as a sum of the parasite drag $c_{\mathrm{D},0}$ and an induced drag that is approximated by $\epsilon C_{\mathrm{L}}^2$, where ϵ is a constant. Note that this model is only valid for $C_{\mathrm{L,min}} \leq C_L \leq C_{\mathrm{L,max}}$, where the bounds $C_{\mathrm{L,min}}$ and $C_{\mathrm{L,max}}$ are specified in Table I.

If the kite is connected to a control unit at the bridle point by several lines (cf. Figure 1), not only the kite's roll angle but also the angle of attack can be influenced, where the angle of attack is simply the angle between the kite's airfoil and the effective wind vector. Consequently, we may assume that the lift coefficient C_L is directly controlled, since C_L is mainly depending on the angle of attack. Note that we can also think of alternative methods to control the lift. For example, additional elevators or stabilators at the kite might be helpful or we can even vary the shape of the airfoil.

Unfortunately, the air friction of the cable slows the kite down. We estimate the torque of this friction by:

$$\begin{aligned} M_{\mathrm{f}} &:= \int_0^1 (sre_r) \times \frac{\rho c_{\mathrm{D,C}} d_{\mathrm{c}}}{2} (s\|w_{\mathrm{e}}\|)^2 \frac{w_{\mathrm{e}}}{\|w_{\mathrm{e}}\|} \mathrm{d}s \\ &= (re_r) \times \frac{c_{\mathrm{D,C}} \rho A_{\mathrm{c}}}{8} \|w_{\mathrm{e}}\| w_{\mathrm{e}} \,, \end{aligned} \tag{14}$$

where d_{c} is the diameter, $A_{\mathrm{c}} = rd_{\mathrm{c}}$ the cross wind area and $c_{\mathrm{D,C}}$ the drag coefficient of the cable. The associated friction force is:

$$F_f := \frac{c_{\mathrm{D,C}} \rho A_{\mathrm{c}}}{8} \|w_{\mathrm{e}}\| w_{\mathrm{e}} \,. \tag{15}$$

Furthermore, we take an internal friction b of the cable into account that is increasing with the cross sectional area $A_{\mathrm{Q}} := \frac{\pi}{4} d_{\mathrm{c}}^2$ of the cable: $b = \frac{b_0 A_{\mathrm{Q}}}{r_0}$, where b_0 is a constant of the material. The associated friction force is

$$F_{\mathrm{int}} := -b(\dot{r} - \dot{r_0}) e_r \tag{16}$$

Finally, we can find the generalized force Q that is needed in equation (7) or (8) :

$$Q = \mathcal{T} \left(F_{\mathrm{aer}} + F_{\mathrm{f}} + F_{\mathrm{int}} \right) \,, \tag{17}$$

where the tensor $\mathcal{T}$ is defined by

$$\mathcal{T} := (\nabla_q p)^T = SR \,. \tag{18}$$

Let us define the total force F with respect to the local basis by

$$F := R(F_{\mathrm{aer}} + F_{\mathrm{f}} + F_{\mathrm{int}}) + S^{-1} \widetilde{Q} \,.$$

Note that the above expression for F can still be simplified for an efficient implementation. Finally, we write the equation of motion (8) in the form

$$\ddot{q} = S^{-1} \frac{F}{m} - a \,. \tag{19}$$

III. The Formulation of the Optimal Control Problem

We are interested in the tractive power P at the generator:

$$P = \left(\frac{EA_{\mathrm{Q}}}{r_0} (r - r_0) + b(\dot{r} - \dot{r}_0) \right) \dot{r}_0 . \tag{20}$$

The average of this power over one cycle is the objective function that we would like to maximize:

$$\overline{P} := \frac{1}{T} \int_0^T P dt \,. \tag{21}$$

Here, T is the time the kite needs for one loop. Let us define the differential state vector x by

$$x := \left(r_0 \,,\, r \,,\, \phi \,,\, \theta \,,\, \dot{r}_0 \,,\, \dot{r} \,,\, \dot{\phi} \,,\, \dot{\theta} \,,\, \Psi \,,\, C_{\mathrm{L}} \,,\, W \right)^T \,,$$

where W is the energy at the generator. Our control is

$$u := \left(\ddot{r}_0 \,,\, \dot{\Psi} \,,\, \dot{C}_{\mathrm{L}} \right)^T \,.$$

Here we control derivatives with respect to time of the relieved cable length, the roll angle and the lift coefficient, while $\dot{r}_0$, Ψ and C_{L} are differential states. This is a way to keep these functions continuous and we can simply require additional constraints of the form $|\ddot{r}_0| \leq \ddot{r}_{0,\max}$, $|\dot{\Psi}| \leq \dot{\Psi}_{\max}$ and $|\dot{C}_{\mathrm{L}}| \leq \dot{C}_{\mathrm{L,max}}$ to avoid too fast changes of $\dot{r}_0$, Ψ and C_{L} that might be hard to realize.

Furthermore, the easiest way to express the objective function (21) is as a Mayer term, since T is unknown. Thus, we use the additional state W given by $\dot{W} := P$ to evaluate the integral in equation (21).

To achieve that the kite flies closed loops, we have to require periodic boundary conditions for all differential states but W and some additional constraints:

$$\chi(x(0), x(T)) := \begin{pmatrix} r_0(T) - r_0(0) \\ r(T) - r(0) \\ \phi(T) - \phi(0) \\ \theta(T) - \theta(0) \\ \dot{r}_0(T) - \dot{r}_0(0) \\ \dot{r}(T) - \dot{r}(0) \\ \dot{\phi}(T) - \dot{\phi}(0) \\ \dot{\theta}(T) - \dot{\theta}(0) \\ \Psi(T) - \Psi(0) \\ C_{\mathrm{L}}(T) - C_{\mathrm{L}}(0) \\ \dot{\phi}(0) \\ W(0) \end{pmatrix} = 0 \,. \tag{22}$$

The additional condition $\dot{\phi}(0) = 0$ is introduced in order to remove the indefiniteness due to the symmetry of periodic orbits with regard to phase shifts. It is necessary to keep the problem well defined. And $W(0) = 0$ is a necessary initial condition such that $W(T) = \int_0^T P \, \mathrm{d}t$ holds.

Now, the differential equation for the state vector x can be summarized as $\dot{x} = f(x, u)$ by using equation (8) :

$$f(x, u) := \begin{pmatrix} \dot{r}_0 \\ \dot{q} \\ u_1 \\ S^{-1} \frac{F}{m} - a \\ u_2 \\ u_3 \\ P \end{pmatrix} \,. \tag{23}$$

We collect all fixed parameters in Table I. The diameter d_{c} of the cable and the cycle duration T are unknown and we will optimize them, too.

TABLE I

THE KITE'S AND OTHER PARAMETERS

Name	Symbol	Value
mass of the kite	m_k	850 kg
area	A	500 m^2
volume	Ω	720 m^3
pure drag coefficient	$c_{D,0}$	0.04
induced drag constant	ϵ	0.04
	$C_{L,max}$	1.5
	$C_{L,min}$	0.3
gravitational const.	g	9.81 $\frac{m}{s^2}$
air density	ρ	1.23 $\frac{kg}{m^3}$
cable density	ρ_c	1450 $\frac{kg}{m^3}$
cable friction	$c_{D,C}$	1.0
internal friction	b_0	10^8 $\frac{kg}{s\,m^2}$
elastic modulus	E	$1.5 * 10^{11}$ Pa
reference wind	w_0	10 $\frac{m}{s}$
reference altitude	h_0	100 m
roughness length	h_r	0.1 m
barometric constant	h_1	8330m
control bounds	$\ddot{r}_{0,max}$	50 $\frac{m}{s^2}$
	$\dot{\Psi}_{max}$	0.065 $\frac{rad}{s}$
	$\dot{C}_{L,max}$	3.5 $\frac{1}{s}$
other bounds	$\dot{r}^{0,1}_{0,max}$	15 $\frac{m}{s}$
	$\dot{r}^{0,1}_{0,min}$	−40 $\frac{m}{s}$

All the important inequality constraints are collected in a vector η :

$$\eta(x,u) := \begin{pmatrix} \dot{r}_0 - \dot{r}_{0,max} \\ -\dot{r}_0 + \dot{r}_{0,min} \\ C_L - C_{L,max} \\ -C_L + C_{L,min} \\ \ddot{r}_0 - \ddot{r}_{0,max} \\ -\ddot{r}_0 - \ddot{r}_{0,max} \\ \dot{\Psi} - \dot{\Psi}_{max} \\ -\dot{\Psi} - \dot{\Psi}_{max} \\ \dot{C}_L - \dot{C}_{L,max} \\ -\dot{C}_L - \dot{C}_{L,max} \end{pmatrix} \preccurlyeq 0 \,. \tag{24}$$

Now, we define $\overline{P}(x(T),T) := \frac{W(T)}{T}$ to be our Mayer functional and summarize our optimal control problem in the following common form:

$$\begin{aligned} &\underset{x(\cdot),u(\cdot),d_c,T}{\text{maximize}} && \overline{P}(x(T),T) \\ &\text{subject to:} && \\ &\forall t \in [0,T]: && \dot{x}(t) = f(x(t),u(t),d_c) \\ &\forall t \in [0,T]: && 0 \succcurlyeq \eta(x(t),u(t),d_c) \\ & && 0 = \chi(x(0),x(T)) \end{aligned} \tag{25}$$

This optimal control problem must be solved numerically. We will treat this point in the next section.

IV. NUMERICAL OPTIMIZATION STUDY FOR THE SINGLE KITE

To study the numerical solution, we use the optimal control package MUSCOD-II [14] that is based on the direct multiple shooting method [3]. It allows us to find local solutions of the above optimal control problem (25). For more information about this package we refer to [14] and [15].

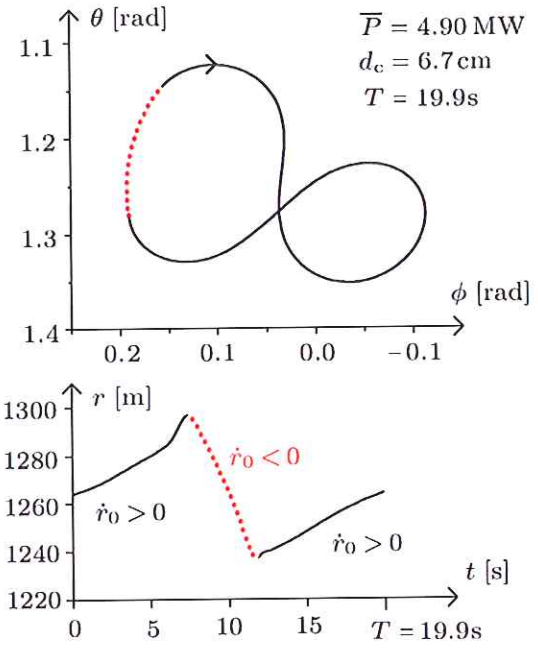


Fig. 2. Local solution ("lying eight"). The retraction phase is red colored and dotted.

A. The discussion of a local optimal solution

First of all, we like to mention that there are many locally optimal solutions of our control problem (25). In this paper, we concentrate on an optimal loop that has the form of an "eight". This loop has the advantage that the cable is not coiled up after one period, if we don't use a bearing in cable. For a more detailed discussion of how such a solution can be found, we refer to [8].

The optimal result is shown in Figure 2. The altitude of the loop is between and 275 m and 540 m and its width is 397 m. Thus, the wind shear model yields that the wind speed is between 11.5 $\frac{m}{s}$ and 12.4 $\frac{m}{s}$ at the kite's flying altitude. The cycle duration is $T = 19.9$ s and the average power at the generator is

$$\overline{P} = 4.90\text{MW} \,.$$

Note it is quite expensive to build a windmill with a power of almost 5 MW and that its wing area will typically be larger than 500m^2.

Furthermore, the average velocity of the kite during the pulling phase is 69 $\frac{m}{s}$, which is approximately 5.8 times faster than the average wind speed at the flying altitude. Noting that the kite is quite fast and the optimal eight is narrow in proportion to the dimension of our kite, we should think about a way to compensate yaw wise torques. We like to mention that this compensation should be possible with effective rudders at the back of the kite.

Figure 2 also shows that the optimal length of the cable is between 1237 m and 1297 m and its optimal diameter is $d_c = 6.7$cm. Our values for the density and the elastic modulus are based on the assumption that we use Kevlar for the cable. If we assume that the cable has a tensile strength of $\tau \approx 3$ GPa the critical diameter would be

$$d_{c,\text{critical}} := \sqrt{\frac{4F_c^{max}}{\pi\tau}} \approx 3.1\text{cm} \,,$$

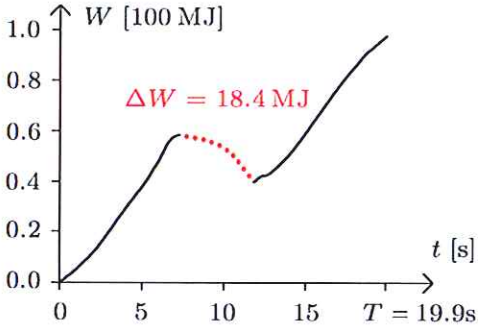


Fig. 3. The energy $W(t)$ at the generator

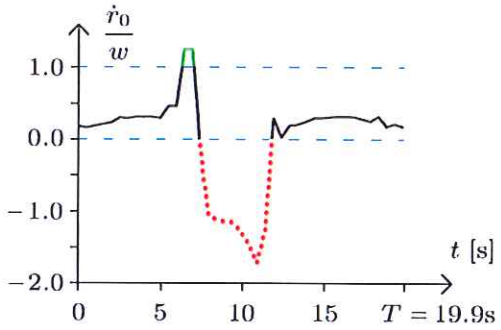


Fig. 4. The velocity $\dot{r}_0$ with respect to the wind speed

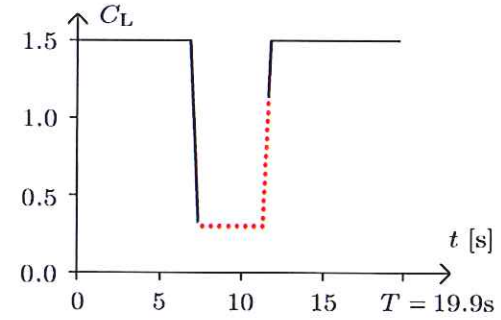


Fig. 5. The switch of the lift coefficient

as the tension in the cable is always less than $F_c^{\max} \approx 2.2\,\mathrm{MN}$ in our example. Thus, for our optimal diameter $d_c = 6.7\mathrm{cm}$ we are far from the critical value. However, the weight of the cable is between 6323 kg and 6630kg and the effective parasite drag

$$\frac{d_c r}{4A} c_{D,C} \approx 0.042$$

is larger than the parasite drag $c_{D,0} = 0.04$ of the kite[2]. Mass and drag of the cable increase with the diameter d_c while the oscillations in the cable decrease with d_c. The minimum value of $r - r_0$ is attained during the retraction phase and its maximum at the beginning of the pulling phase. In our example the elastic prolongation varied between 0.6 m and 5 m. these oscillations reduce the convertible energy noticeably in the transition between retraction and pulling phase

Another question is if the cable touches the ground, since we have only considered the cable to be a straight line in our model. Let us define the angle θ_0 of the cable at the generator by $\theta_0 := \arccos(e_z \cdot e_r^0)$ with e_r^0 being a unit vector pointing along the cable at the ground station, which is mainly depending on the ratio between the tension $\|F_c\|$ and the gravitational force $m_c g$ of the cable. In a rough quasi statical approximation [8] we have

$$\theta_0 \approx \operatorname{arccot}\left(\cot(\theta) - \frac{m_c g}{\|F_c\| \sin(\theta)}\right) \tag{26}$$

During the pulling phase the ratio $\frac{m_c g}{\|F_c\|} \approx 0.03$ is quite small and the assumption that the cable is a straight line is virtually satisfied. Thus, the only critical moment is the retraction phase. For the loop in Figure 2 we found

$$\max_{t\in[0,T]} \theta_0 \approx 1.50 < \frac{\pi}{2}\,.$$

This approximation indicates that the cable is not touching the ground.

We consider the result for the energy at the generator a little more in detail. In Figure 3 the function $W(t)$ is plotted. We have to invest the energy $\Delta W = 18.4\,\mathrm{MJ}$ during the retraction phase to achieve that r is periodic. Thus, in this phase the generator has to act as a motor, if we have only one kite at this generator. It might be a good idea to use several kites that are connected to the same generator acting with phase shifts similar to the cylinders in a motor, or to use a "Kite-Wind-Generator" [4].

Let us consider the velocity $\dot{r}_0$ with respect to the wind speed, which is shown in Figure 4. In the black colored part of the pulling phase in Figure 4 the average of $\frac{\dot{r}_0}{w}$ is 0.248, which is close to the rough theoretical approximation $\nu_p^* \approx 0.240$ in [8]. But, during a short period before the retraction phase, $\dot{r}_0$ has a peak. It is interesting that we even have $\dot{r}_0 > w$ for this period, which is illustrated as the green part of the plot. Of course, the kite is slowed down during this short period still producing energy before it is pulled back. Note that the tension in the cable was always positive, such that it is always pulling and not pushing, although we have $\dot{r}_0 > w$ in the green colored part.

In Figure 5 the lift coefficient C_L is plotted. During the pulling phase the upper bound $C_L \le C_L^{\max} = 1.5$ is active until it switches to the lower bound $C_L^{\min} = 0.3$ during the retraction phase.

To understand from the physical point of view why it is optimal to switch C_L to the lower bound during the retraction phase, we have to keep in mind that the kite's velocity is mainly depending on the effective glide ratio[3] (cf. [8, 9, 16])

$$G := \frac{C_L}{c_{D,0} + \epsilon C_L^2 + \frac{d_c r}{4A} c_{D,C}}\,,$$

that is monotonically increasing with C_L on the interval $[C_L^{\min}, C_L^{\max}]$. Of course, another method to reduce G and consequently to slow down the kite is to increase the drag - for example with simple flaps at the wings of the kite.

[2] On the other hand, our value for $c_{D,C}$ was 1.0 during the simulation, but it might be possible to design cables with much lower drag.

[3] Sometimes the effective glide ratio is also referred as the "efficiency factor".

Another question is if it is possible to produce energy by only controlling $\dot{r}_0$ and $\dot{\Psi}$ such that a control of the lift or drag can be avoided. This might be easier to realize in a real-world-implementation. One answer, how to do this with a single kite can be found in [4], but in this paper we will only discuss this question for a two kite system, that will be introduced in section V.

Finally, we like to mention that we cannot find the global optimum of the optimal control problem with our methods. Nevertheless, the approximation in [8] yields a theoretical upper bound of 5.17 MW for the maximal power, if we use $w \approx 11.9\frac{m}{s}$ for the average wind speed at the flying altitude of the kite. As our results are quite close to this approximation, they cannot be far from the global maximum of the average tractive power.

V. Wind power generating systems with more than one kite

In the last section we have seen how a single kite can be used efficiently to produce energy at a generator on the ground. Now, we like to discuss how we can use several kites to even increase the power per kite. We have already mentioned that we can use several generators and kites acting with phase shifts, such that finally a constant amount of power can be produced, although each kite has a pull and a retraction phase. An additional idea is to use several kites that are connected to one generator. As an example it could be more efficient to construct stacked kites as a cheap way to increase the wing area. But in this paper we like to discuss a special constellation of two kites:

The main cable coming from the generator is split into two cables each with a kite at its end. This is on the one hand a method to reduce the drag of the cables, as the main cable is almost static for antisymmetric trajectories of the two kites and only the two smaller cables have high velocities. On the other hand, we can reduce the energy that has to be invested during the retraction phase, as the kites can simply fly to a position where they can almost compensate their forces mutually.

A. The setup for the optimal control problem

In this section we discuss the main differences of the optimal control problem for the two kite system in comparison to the problem for a single kite. First, we summarize the positions p_1, p_2, p_3 and velocities $\dot{p}_1, \dot{p}_2, \dot{p}_3$ of the juncture point (where the three cable pieces meet) and the kites in one differential state vector. Therefore, we used Cartesian coordinates: The wind is blowing along the x-axis and the z-axis is pointing to the sky. Thus, the difference to the single kite is that we model the motion of three instead of one point. Furthermore, we have five controls now: the derivatives of the roll angles and lift coefficients for both kites and the second derivative of the length of the main cable.

Periodic boundary conditions are required for the positions and velocities of all modeled points. Additionally, we need some inequality constraints. To keep a certain security distance between the kites we require that each kite has a minimum distance of 150 m to the cable of the other one. The lengths and diameters of the cables are optimized, too.

All the other assumptions, parameters and bounds for the setup of the optimal control problem are analogous to the single kite. In particular, both kites have the same size as the single kite in the previous section and the objective function is the average power at the generator. In the following section we discuss the numerical results.

B. Numerical optimization study of the system with two kites

Similar to the single kite we can try to find optimal loops for the two-kite system. Again, we are interested in solutions that neither coil the main cable at the generator nor the two smaller cables[4]. Furthermore, we introduced a constraint $T \leq T_{\max}$ and observed that this bound was always active at the optimal solution for $T_{\max} \leq 40$s. Nevertheless, we did not remove this bound, since the precision of our numerical solutions would decrease for too long periods. Of course, there are methods to avoid these difficulties e.g. by dividing the trajectory into several parts or using indirect methods, but another observation was that the upper bound for a period time greater than 40s does not influence the objective much. Therefore, we decided to discuss the optimal loop for $T_{\max} = 40$s. A local result for the projection of the kite trajectories on the y, z-plane is shown in Figure 6. The trajectory of one kite is black and the trajectory of the other one blue colored and dashed, but during the retraction phase both trajectories are red colored and dotted.

The result for the average power of the loop in Figure 6 is

$$\overline{P} = 14.86\,\text{MW}\,.$$

Thus, each kite is producing approximately 7.43 MW, while the single kite with the same effective area in section IV produces only less than 5 MW. Both kites fly at altitudes between 300 m and 700 m, where the average wind speed at the kites is $12.3\frac{\text{m}}{\text{s}}$.

During the pulling phase both kites are flying almost antisymmetric such that the velocity of the juncture point is always very low. This is one of the main reasons why the two kite system is more efficient than a single kite, since the loss of energy due to the aerodynamic friction of the main cable to the generator is quite low. Only the shorter cable pieces between the kites and the juncture point slow down the kites. This can also be seen more directly in Figure 7, where the trajectory of the juncture point on the $x-z$-plane is shown.

Neglecting the oscillations of the juncture point, it has an almost constant altitude during the pulling phase, but it is falling down during the retraction phase of the kites due to the lower tension in the cables. Nevertheless, the cable is not touching the ground, since the angle θ_0 between the cable close to the generator and the z-axis takes the

[4]We used additional constraints that require the winding numbers of the cables to be 0. [8]

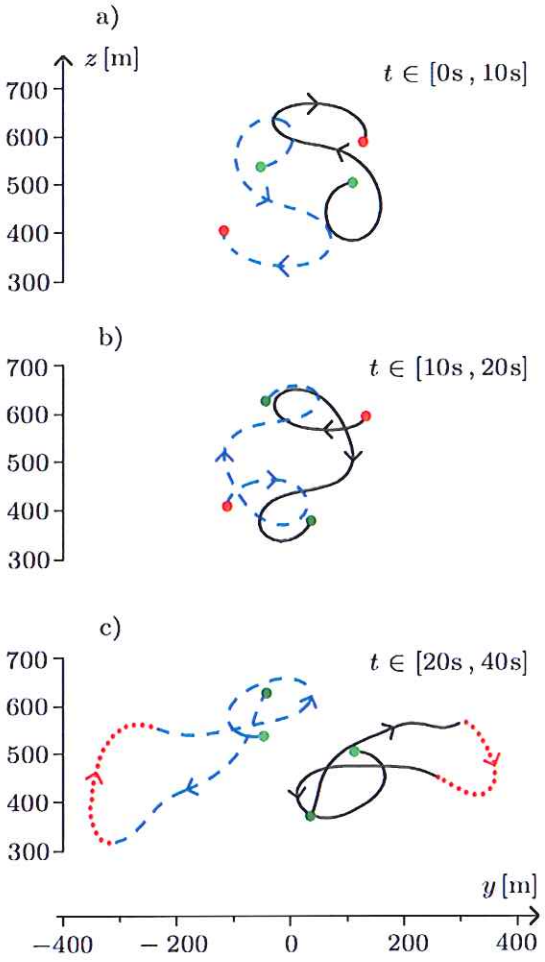


Fig. 6. Local optimal kite trajectories on the $y-z$-plane

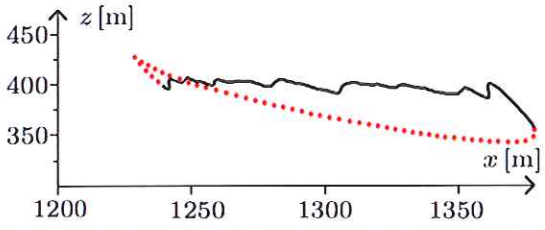


Fig. 7. The trajectory of the juncture point in the x-z-plane

maximum value

$$\max_{t\in[0,T]} \theta_0 \approx 1.55 < \frac{\pi}{2} \,. \tag{27}$$

in a quasi static approximation [8].

Figure 8 shows the length r of the main cable between the generator and the juncture point. It is slightly longer than the cable of the single kite in section IV, but in the same order. Its diameter is $d_c = 8\text{cm}$ - thus the upper bound for d_c is active. We did not remove this bound since we did not model the sagging of the cable directly and due to its high weight the cable would touch the ground otherwise. For the diameters of the other cables $d_c^1 = d_c^2 = 4.0\,\text{cm}$, the lower bound was active. Again, the bound shouldn't be removed, since the cable would

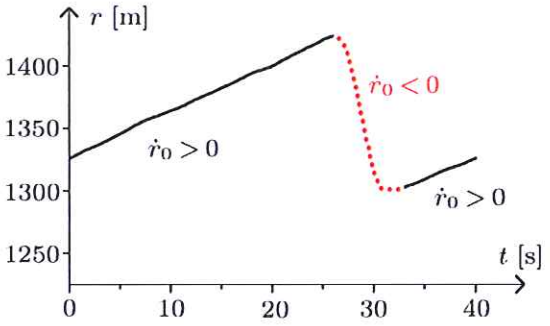


Fig. 8. The length of the main cable between the generator and the juncture point

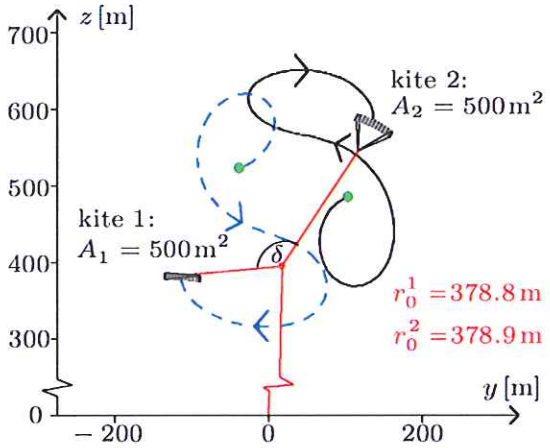


Fig. 9. A visualization of the kites and their trajectories for $t \in [0\text{s}, 10\text{s}]$

break otherwise. The lengths of the cables between the juncture point and kite 1 or 2 are $r_0^1 = 378.8\,\text{m}$ and $r_0^2 = 378.9\,\text{m}$ respectively. It is interesting to note that they have almost equal lengths. A visualization of the kites and their trajectories on the $y-z$-plane is shown in Figure 9.

Finally, we like to discuss the angle δ between the cables that are connecting the kites with the juncture point. As it can be seen in Figure 10 this angle is very large during the retraction phase. The physical interpretation of this effect is that the kite nearly compensate their forces if this angle δ is close to 180°. In fact, this is the prime reason why the two kite system is more efficient than the single kite. Since the ratio of the produced energy to invested energy is 8.6 for the two-kite system, while it is only 5.3 for the single kite.

This observation also shows that the switch of the lift

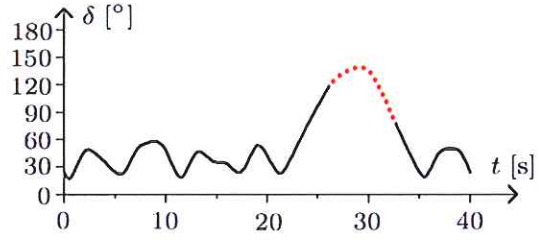


Fig. 10. The angle δ between the cables of the two kites

coefficient for both kites is not as important as for the single kite. If we do not control C_L, such that we have $C_{\mathrm{L}}^1 = C_{\mathrm{L}}^2 = 1.5$ for all $t \in [0, T]$, the result for the maximal average tractive power at the generator is still

$$\overline{P} \approx 13.42\mathrm{MW}\,,$$

which is only slightly below the attainable power with lift control.

VI. Conclusions and Outlook

We have presented optimal periodic loops for kites, which are connected to a generator at the ground, such that a maximal amount of power can be produced. To solve the corresponding optimal control problems we used a direct multiple shooting algorithm. The main result was that we can produce almost $5\,\mathrm{MW}$ with a $500\mathrm{m}^2$-kite by flying a loop with the form of a lying eight. During the retraction phase the kite has been slowed down by a switch of the lift coefficient. It is flying upwards during this phase and we have to invest a low amount of energy. In the pulling phase the lift coefficient is maximal and we can produce energy.

Furthermore, we presented a way to reduce the aerodynamic drag of the cables by using a system of two kites. Another advantage was that this system can increase the efficiency during the retraction phase, as the two kites can compensate their forces by flying in opposite positions. The result of our optimization study was that two kites each with an area of $500\mathrm{m}^2$ can produce an average power of $14.86\,\mathrm{MW}$. Additionally, we have seen that the two kite system can also be used without a lift or drag control of the kites still producing $13.42\,\mathrm{MW}$.

Since all our presented loops for the kite systems are unstable, for a real-world implementation a feedback controller must be designed. This might be done by Nonlinear Model Predictive Control (NMPC) [1], as it is investigated in [6, 7] for kite control. We are actually working on NMPC-studies in combination with a Moving Horizon Estimator (MHE) [2, 20] for power generating kites [10]. Finally, we have to validate and improve our model, as soon as we have data from real world experiments.

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