Introduction to Focus Issue: Synchronization in Complex Networks

Johan A. K. Suykens[1] and Grigory V. Osipov[2]
[1]*K.U. Leuven, ESAT-SCD/SISTA, Kasteelpark Arenberg 10, B-3001 Leuven (Heverlee), Belgium*
[2]*Department of Control Theory, Nizhny Novgorod University, 23, Gagarin Avenue, 603950 Nizhny Novgorod, Russia*

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Synchronization in large ensembles of coupled interacting units is a fundamental phenomenon relevant for the understanding of working mechanisms in neuronal networks, genetic networks, coupled electrical and laser networks, coupled mechanical systems, networks in social sciences, and others. It relates to mathematical and computational analysis of the existence of different states and its stability, clustering, bifurcations and chaos, robustness and sensitivity analysis, etc., at the intersection between synchronization and pattern formation in complex networks. This interdisciplinary oriented Focus Issue presents recent progress in this area with contributions on generic methods, specific model studies, and applications. © *2008 American Institute of Physics*.
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Nowadays, the main focus in the study of synchronization phenomena is moving from the analysis of a low number of interacting units to systems consisting of thousands and even millions of nodes. Such systems are ubiquitous in nature and engineering. The understanding of common properties of network synchronization, mechanisms of its appearance, and strategies of its control in dependence on dynamics of individual nodes, networks architecture, and internode coupling types is a subject of current research.

I. CONTEXT

Complex networks are systems composed of a (very) large number of nodes often characterized by a complex topology and possibly complex types of interactions.[1] The structure of a network is typically characterized in a graph theoretic way. Indicators, such as, degree distributions, average path length, and others have been used to qualitatively distinguish between different types of complex networks.[2,3] In general, one may have different network topologies, such as, according to regular networks (e.g., a ring structure, nearest neighbor coupling, globally connected), random graphs, small world networks, and scale free networks.[1,4] The type of coupling can be linear or nonlinear, constant or changing in time, etc. The dynamics of the individual nodes might in general be regular or chaotic.

The formation of collective behavior in such large networks of coupled dynamical elements is one of the new problems in the study of dynamical systems. It poses many challenges for achieving a better theoretical understanding as well as for applications in various disciplines, ranging from physics, chemistry, earth sciences via biology and neuroscience to engineering, business, and social sciences.

Due to the large number of effective degrees of freedom in spatially extended systems, a rich variety of spatio-temporal regimes is observed. Three main types of collective behavior are typically distinguished: a fully incoherent state (or highly developed spatio-temporal disorder), partially coherent states (where some of the elements in the network behave according to a common rhythm) or a fully coherent state (a regime of globally synchronized elements). The basic phenomenon of these structure formations is synchronization, i.e., a regime of coherent activity, which is universal in many dynamical systems and can be understood from the analysis of common models of oscillatory networks.

The subject of synchronization of chaotic systems has been intensively investigated since the work by Pecora and Carroll,[5] however, the main focus in the past has been on low dimensional systems and regular network topologies. Synchronization in complex networks poses new challenges towards the understanding of synchrony and its formation in large scale networks, also for more complex network topologies.

Different synchronization phenomena have been widely studied and reported in the literature. A number of related book references that address this subject are the following. The book by Pikovsky *et al.*[6] overviews many currently known synchronization phenomena (including synchronization of a periodic oscillator by external force, synchronization of chaotic systems, synchronization in the presence of noise and in oscillatory media). An introduction to the subject of synchronization (without formulas) illustrated with many biological and technological examples has been given in the book by Strogatz.[7] A popular introduction to synchronization theory illustrated by many examples can be found in Refs. 8 and 9. Synchronization by external force, synchronization of oscillators, and the influence of noise on synchronization have been studied in some parts of the monograph.[10] Kuramoto[11] studied synchronization in large populations of phase oscillators and in continuous media. Synchronization from the viewpoint of biological applications is studied in Ref. 12, and synchronization from an engineering perspective in Refs. 13–16. Blekhman[17] has addressed the synchronization in mechanical oscillators, electronic, and quantum generators, etc. Noisy influenced synchronization in periodic

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systems was developed by Stratonovich.[18] Books that are entirely related to synchronization problems in networks of coupled oscillators are Afraimovich *et al.*[19] (focusing on networks of identical phase locked loops), Manrubia *et al.*,[20] and recently Osipov *et al.*[21]

II. THIS FOCUS ISSUE

The contributions to this Focus Issue on *Synchronization in Complex Networks* are grouped into three parts:

- Generic methods
- Specific models
- Applications

The first part on *Generic methods* addresses problems such as: What is the relation between the topology of the network and synchronization into clusters? How could one design the topology of a network as to achieve desired clusters? Under which conditions is synchronization still achieved in the case of time delays or communication constraints? How can one enhance the synchronizability of complex networks and rewire the network? Does adding nodes increase synchronizability? How can one efficiently control time-varying couplings?

The second part contains in-depth studies of *Specific models*. It includes synchronization in networks of, e.g., Kuramoto models, Hodgkin–Huxley neuron models, integrate-and-fire oscillators, and coupled map lattices.

The third part on *Applications* contains contributions on synchronization of genetic oscillators, synchronization of cells undergoing metabolic oscillations, synchronization in ciliary arrays, collective phenomena in cardiac cell cultures, phase synchrony from multielectrode arrays visual cortex measurements, and synchrony in electrocorticographic signals.

A brief summary of the contributions is given here for each of the three parts.

A. Generic methods

Chen and Duan[22] address the fundamental problem of complex network synchronizability from a graph-theoretic view. They study the relationship between network synchronizability and structural network parameters, such as, average distance and degree distribution. The effect of adding new edges to the network with respect to the network synchronizability is investigated.

Wu[23] looks at pinning control of a set of coupled dynamical systems forced by an external signal. Suitable locations of pinning are investigated and how it relates to the network topology.

In Ref. 24, Rad *et al.* study efficient rewirings, i.e., eliminating an edge and creating a new edge elsewhere, in order to achieve enhanced synchronizability. Different target functions for optimization are explored including the eigenratio of the Laplacian.

Hagberg and Schult[25] study synchronization of diffusively coupled identical oscillators and examine the effect of adding and removing edges on the network synchronization. Through the master stability function approach a separation of the network topology from the consideration of the dynamics of the oscillators is obtained.

V. Belykh *et al.*[26] study conditions for cluster partitioning into ensembles for the case of identical chaotic systems. A distinction is made between conditional and unconditional clusters. The design problem of organizing clusters into a given configuration is discussed.

In Ref. 27, Pogromsky investigates partial synchronization in networks of linearly coupled oscillators for the case of symmetry in the couplings.

Oguchi *et al.*[28] consider synchronization of a set of identical nonlinear systems unidirectionally or bidirectionally coupled with time delay. Sufficient conditions for synchronization are obtained in terms of linear matrix inequalities based on a Lyapunov–Krasovskii function approach.

Fradkov *et al.*[29] study synchronization of nonlinear systems under information constraints with a limited information capacity of the coupling channel. An upper bound on the limit synchronization error is derived as a function of the transmission error. This is analyzed with respect to different network topologies.

In Ref. 30, De Lellis *et al.* discuss adaptive strategies for the synchronization of complex networks with time-varying coupling. These include vertex-based and edge-based approaches for which global synchronization is analyzed.

Y. Wu *et al.*[31] study complete synchronization in small world networks of identical oscillators. Synchronizability of the network is analyzed with respect to a time-varying coupling.

B. Specific models

Antonsen *et al.*[32] address large systems of coupled oscillators subjected to a periodic external drive for equal strength, all-to-all sine coupling of phase oscillators. Stationary states and their stability are determined. With sufficient drive the system produces regular rhythms.

Ott and Antonsen[33] show that in the infinite size limit certain systems of globally coupled phase oscillators display low dimensional dynamics. An exact closed form solution for the nonlinear time evolution of the Kuramoto problem is obtained. Also for extensions to this model low dimensional behavior is demonstrated.

In Ref. 34, So *et al.* study a network of two large interacting heterogeneous populations of limit-cycle oscillators under a switching connectivity. For sufficiently high switching frequency the system behaves as if the connectivity were static.

Ott *et al.*[35] show the existence of echo phenomena in large systems of coupled oscillators below the transition to phase coherence. This is exemplified by the Kuramoto model.

D'Huys *et al.*[36] address the effect of coupling delays on the synchronization properties of network motifs. The influence of coupling delay and symmetry on synchronization patterns in uni- and bidirectionally coupled rings and open chains of Kuramoto oscillators are analyzed.

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In Ref. 37, Wang *et al.* show analytically for a generalized Kuramoto model, how applying gradient coupling can enhance the synchronization in nonidentical scale-free networks.

Ahnert and Pikovsky[38] characterize different types of traveling waves in a chain of dispersively coupled phase oscillators. Stability of the waves is studied.

Nowotny *et al.*[39] discuss neural synchrony including synchronization with activity dependent coupling, synchronization of bursts generated by a group of nonsymmetrically coupled inhibitory neurons, partial synchronization of small composite structures, and synchronization on a mesoscopic scale.

In Ref. 40, Shilnikov *et al.* study polyrhythmic synchronization in bursting networking motifs. A universal burst-duration based mechanism of the emergence of synchronous bursting patterns is described for larger networks, such as, pattern generators that control locomotion activities.

Komarov *et al.*[41] study collective phenomena in small and large arrays of nonidentical cells coupled by models of electrical and chemical synapses and a Hodgkin–Huxley-type model. Various regimes of phase synchronization are observed.

Mauroy and Sepulchre[42] investigate clustering behavior in a model of integrate-and-fire oscillators with excitory pulse coupling. Global convergence to phase-locked cluster behavior is proven for identical oscillators. The robustness of the clustering behavior is addressed for nonidentical oscillators.

In Ref. 43, Anishchenko *et al.* address synchronization of quasiperiodic oscillations by an external harmonic signal. Synchronization of a resonant limit cycle on a two-dimensional torus is studied.

Zhusubaliyev and Mosekilde[44] investigate the formation and destruction of multilayered tori in coupled map systems. Torus destruction involves nonlocal bifurcations and represents a transition to chaos scenario. The tori formation is related to noninvertibility of the maps.

Cencini *et al.*[45] study chaotic synchronizations of spatially extended systems of nonequilibrium phase transitions. Short- and long-range interactions are addressed in two replicas of coupled map lattices with a coupling decaying as a power law.

C. Applications

Zhou *et al.*[46] address synchronization of genetic oscillators. It is shown how a collective rhythm is achieved through synchronization-induced intercellular communication and how an ensemble is synchronized by a common noisy signaling molecule.

Gonze *et al.*[47] study in-phase and out-phase synchronization in a model based on global coupling of cells undergoing metabolic oscillations.

In Ref. 48, Niedermayer *et al.* investigate synchronization, phase locking, and metachronal wave formation in ciliary chains. Beating cilia are described as phase oscillators moving on circular trajectories with variable radius, where the radial degree of freedom enables hydrodynamically induced synchronization.

Kryukov *et al.*[49] study collective phenomena in heterogeneous cardiac cell cultures consisting of a mixture of passive, oscillatory, and excitable cells. For oscillatory and excitable media, transitions from incoherent behavior to partially coherent behavior and finally to global synchronization are discussed.

Manyakov and Van Hulle[50] investigate phase synchrony with respect to local field potentials recorded with a multi-electrode array implanted in the monkey visual cortex. An information-theoretic measure is used to characterize the phase synchrony based on normalized mutual information between pairs of local field potential phases.

In Ref. 51, Kozma and Freeman address intermittent spatio-temporal desynchronization and sequenced synchrony in electrocorticographic signals from the brain surface. Filtered signals confirm the presence of a shared mean frequency in a frame of synchronized oscillation.

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