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Decoupled compensation of IQ imbalance in MIMO OFDM systems

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ABSTRACT

The direct-conversion architecture is an attractive front-end design for multi-input multi-output (MIMO) orthogonal frequency division multiplexing (OFDM) systems. These systems are typically small in size and provide a good flexibility to support growing number of wireless standards. However, direct-conversion based OFDM systems are generally very sensitive to front-end component imperfections. These imperfections are unavoidable especially when cheaper components are used in the manufacturing process and can lead to radio frequency (RF) impairments such as in-phase/quadrature-phase (IQ) imbalance. These RF impairments can result in a severe performance degradation. In this paper, we propose training based efficient compensation schemes for MIMO OFDM systems impaired with transmitter and receiver frequency selective IQ imbalance. The proposed schemes can decouple the compensation of the transmitter and receiver IQ imbalance from the compensation of the channel distortion. It is shown that the proposed schemes result in an overall lower training overhead and a lower computational requirement as compared to a joint estimation/compensation of IQ imbalance and the channel distortion.

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1. Introduction

Orthogonal frequency division multiplexing (OFDM) is a widely adopted modulation scheme for high data rate communication systems [1]. An OFDM based physical layer has been adopted in a variety of wireless communication systems, such as wireless local area networks (WLANs) [2], wireless metropolitan area networks (WiMAX) [3], digital video broadcasting (DVB-T) [4], etc. OFDM can be combined with multiple antenna techniques in order to improve the capacity or the robustness of the link [5]. An OFDM based so-called multi-input multi-output (MIMO) transmission system can take benefit of its spatial diversity obtained by multiple transmit and receive antennas to elegantly cope with the interference caused by a dense multipath fading environment. However, as the MIMO OFDM architecture has to support multiple parallel radio frequency (RF) front-ends, it is extremely important to keep these front-ends simple with minimal analog electronics so as to maintain the cost, size and power consumption within an acceptable limit.

The direct-conversion architecture is an attractive front-end architecture for MIMO OFDM systems as it is typically small in size and can be easily integrated on a single chip, unlike a traditional superheterodyne architecture [6]. It also provides a very good flexibility to support growing number of wireless standards required in today's communication systems. However, direct-conversion based OFDM systems are generally very sensitive to front-end component imperfections. These imperfections are unavoidable especially when cheaper components are used in the manufacturing process and can lead to RF impairments such as in-phase/quadrature-phase (IQ) imbalance. The IQ imbalance is mainly due to a mismatch of the analog components between the I branch and the Q branch of the front-end and can result in a severe performance degradation, rendering the communication systems useless. Rather than decreasing the IQ imbalance by utilizing expensive analog components, it is easier and more

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flexible to tolerate these distortions in the analog domain and then compensate them digitally.

The effects of IQ imbalance have been studied and estimation/compensation schemes for SISO OFDM systems have been developed in [7–12]. In [7] efficient digital estimation schemes have been developed for the case of frequency independent transmitter/receiver IQ imbalance while in [8–10] these estimation schemes have been extended to also include CFO in the presence of frequency independent receiver IQ imbalance. However, for a wideband communication system it is important to also consider frequency selective IQ imbalance. Such frequency selective distortion arises mainly due to mismatched filters on the I branch and the Q branch of the front-end. In [11], efficient estimation schemes for frequency selective receiver IQ imbalance have been developed, while in [12] a joint channel and frequency selective transmitter/receiver IQ imbalance estimation scheme has been provided.

The influence and compensation of IQ imbalance in MIMO systems has been studied in [13–17]. In [13], the authors propose a compensation scheme for receiver IQ imbalance in a space–time block coded single carrier systems over a flat fading channel. In [14], the authors propose a joint compensation scheme for channel distortion and receiver IQ imbalance, while in [15,16] joint channel and transmitter/receiver IQ imbalance compensation schemes have been proposed for MIMO OFDM systems. In [17], we have considered the joint compensation of both frequency selective transmitter/receiver IQ imbalance, CFO and channel distortion in MIMO OFDM systems. The disadvantage of the joint transmitter/receiver IQ imbalance and channel estimation/compensation schemes is that the parameters of the entire compensation structure have to be re-estimated for every variation of the channel characteristics, even when the IQ imbalance parameters are constant. As the channel characteristics vary much faster than the IQ imbalance, it will be advantageous if we can develop solutions where the compensation of the IQ imbalance is decoupled from the compensation of the channel distortion. This results in decoupled schemes where in time varying scenarios only the channel compensation parameters have to be re-estimated while the IQ imbalance compensation parameters are indeed kept constant. In [18,19] we proposed decoupled schemes for single antenna systems while in this work we extend the case to also include MIMO systems. We propose novel estimation/compensation schemes that decouple the compensation of transmitter and receiver frequency selective IQ imbalance parameters from the compensation of the channel distortion. The IQ imbalance parameters can vary over different transmitter and receiver antenna branches. It is shown that the decoupled schemes for MIMO systems have an overall lower training overhead and a reduced computational requirements, compared to joint estimation/compensation schemes.

The paper is organized as follows: the input–output MIMO OFDM system model is presented in Section 2. Section 3 reviews a joint estimation/compensation scheme for MIMO OFDM systems impaired with frequency selective transmitter and receiver IQ imbalance. Based on this scheme we then propose decoupled estimation/compensation schemes in Section 4. Computer simulations are shown in Section 5 and finally conclusions are drawn in Section 6.

Notation: Vectors are indicated in bold and scalar parameters in normal font. Superscripts $\{\}^*$,$\{\}^T$,$\{\}^H$ represent conjugate, transpose and Hermitian respectively. $\mathbf{F}_N$ and $\mathbf{F}_N^{-1}$ represent the $N\times N$ discrete Fourier transform and its inverse. $\mathbf{I}_N$ is the $N\times N$ identity matrix and $\mathbf{0}_{M\times N}$ is the $M\times N$ all zero matrix. Operators $\otimes$, $\star$, . and $\div$ denote the Kronecker product, convolution, component-wise vector multiplication and component-wise vector division respectively. In order to further aid the readability of the notations, two appendices have been added at the end of the paper. Appendix A illustrates the transformation of an equation from non-matrix form to matrix form, and Appendix B then lists some of the regularly used matrix notations in this paper.

2. System model

We consider a point-to-point MIMO OFDM system. Let N_t and N_r denote the number of transmit and receive antennas. We will generally assume that $N_r \geq N_t$. Then $\mathbf{S}_{(K)}$ (for $K=1\ \ldots N_t$) is the frequency domain OFDM symbol of size $(N\times 1)$, to be transmitted over the Kth transmit antenna, where N is the number of subcarriers. The frequency domain symbol is transformed to the time domain by the inverse discrete Fourier transform (IDFT). A cyclic prefix (CP) of length ν is then added, resulting in a time domain baseband symbol $\mathbf{s}_{(K)}$ given as

$$\mathbf{s}_{(K)} = \mathbf{P}_{\mathbf{CI}}\mathbf{F}_N^{-1}\mathbf{S}_{(K)} \tag{1}$$

where $\mathbf{P}_{\mathbf{CI}}$ is the cyclic prefix insertion matrix given by

$$\mathbf{P}_{\mathrm{CI}} = \left[\begin{array}{c|c} 0_{(\nu\times N-\nu)} & \mathbf{I}_\nu \\ \hline \multicolumn{2}{c}{\mathbf{I}_N} \end{array}\right]$$

The time domain symbol $\mathbf{s}_{(K)}$ is parallel-to-serial converted and then fed to the transmitter front-end. We consider a single local oscillator (LO) supporting all the transmit (receive) antennas at the transmitter (receiver) front-end. As the LO produces only a single carrier frequency, the IQ imbalance induced by the LO is generally considered to be frequency independent (FI), i.e. it is constant over the entire OFDM symbol [11]. Due to design restrictions, the trace lengths between the LO and the individual antenna branches may not be exactly equal and this may result in a different FI IQ imbalance for each transmit antenna. We model the transmit FI IQ imbalance as an amplitude and phase mismatch of $g_{t(K)}$ and $\phi_{t(K)}$ at the Kth transmit antenna.

The other analog components in the front-end such as the digital-to-analog converters (DAC), amplifiers, low pass filters (LPFs) and mixers generally result in an overall frequency selective (FS) IQ imbalance. We represent the transmit FS IQ imbalance by two mismatched filters with frequency responses given as $\mathbf{H}_{ti(K)}=\mathbf{F}_N\mathbf{h}_{ti(K)}$ and $\mathbf{H}_{tq(K)}=\mathbf{F}_N\mathbf{h}_{tq(K)}$ at the in-phase and quadrature-phase branch of the Kth transmit antenna. Here $\mathbf{h}_{ti(K)}$ and $\mathbf{h}_{tq(K)}$ represent the impulse response of the mismatched filters. As IQ imbalance mismatch is mainly due to the components' defects, or from certain design criteria (for example: different trace lengths), the parameters associated with IQ imbalance are generally assumed to remain constant as long as the component characteristics do not change. As this characteristic change or the component

deterioration is a very slow and gradual process, thus the IQ imbalance parameters are also generally considered to remain constant for the purpose of their study and estimation. This is a general assumption throughout the literature. See the reference list and especially [6,9,11]. Thus in this paper we have also considered IQ imbalance parameters to remain constant for the purpose of our study.

Now following the derivation in [11], the equivalent baseband symbol $\mathbf{p}_{(K)}$ at the Kth transmit antenna can be given as

$$\mathbf{p}_{(K)} = \mathbf{g}_{ta(K)} \star \mathbf{s}_{(K)} + \mathbf{g}_{tb(K)} \star \mathbf{s}^*_{(K)} \tag{2}$$

where

$$\mathbf{g}_{ta(K)} = \mathbf{F}^{-1}\mathbf{G}_{ta(K)} = \mathbf{F}^{-1}\left\{\frac{[\mathbf{H}_{ti(K)} + g_{t(K)}e^{-\jmath\phi_{t(K)}}\mathbf{H}_{tq(K)}]}{2}\right\}$$
$$\mathbf{g}_{tb(K)} = \mathbf{F}^{-1}\mathbf{G}_{tb(K)} = \mathbf{F}^{-1}\left\{\frac{[\mathbf{H}_{ti(K)} - g_{t(K)}e^{\jmath\phi_{t(K)}}\mathbf{H}_{tq(K)}]}{2}\right\} \tag{3}$$

Here $\mathbf{g}_{ta(K)}$ and $\mathbf{g}_{tb(K)}$ are mostly truncated to length L_t (and then possibly padded with $N-L_t$ zero elements). They represent the combined FI and FS IQ imbalance for the Kth transmit antenna.

An expression similar to Eq. (2) can be used to model IQ imbalance at the receiver. Let $\mathbf{z}_{(J)}$ represent the down-converted baseband symbol for the Jth receive antenna. This symbol is distorted by combined FS and FI IQ imbalance modeled by filters $\mathbf{g}_{ra(J)}$ and $\mathbf{g}_{rb(J)}$ of length L_r, where $\mathbf{g}_{ra(J)}$ and $\mathbf{g}_{rb(J)}$ are defined similar to $\mathbf{g}_{ta(K)}$ and $\mathbf{g}_{tb(K)}$ in Eq. (2). The down-converted received symbol $\mathbf{z}_{(J)}$ can then be written as

$$\mathbf{z}_{(J)} = \mathbf{g}_{ra(J)} \star \sum_{K=1}^{N_t} (\mathbf{h}_{(J,K)} \star \mathbf{p}_{(K)}) + \mathbf{g}_{rb(J)} \star \sum_{K=1}^{N_t} (\mathbf{h}^*_{(J,K)} \star \mathbf{p}^*_{(K)}) + \underbrace{\mathbf{g}_{ra(J)} \star \mathbf{n}_{(J)} + \mathbf{g}_{rb(J)} \star \mathbf{n}^*_{(J)}}_{\mathbf{n}_{c(J)}} \tag{4}$$

where $\mathbf{h}_{(J,K)}$ is the baseband equivalent to the frequency selective quasi-static multipath channel of length L between the Kth transmit and Jth receive antenna. The channel is considered to be static for the duration of one entire packet consisting of a training symbol sequence and a data symbol sequence. Here $\mathbf{n}_{(J)}$ is a zero mean additive white Gaussian noise (AWGN) vector and $\mathbf{n}_{c(J)}$ is a noise vector derived from $\mathbf{n}_{(J)}$ that has been modified by IQ imbalance at the Jth receive antenna. Substituting Eq. (2) in (4) leads to

$$\mathbf{z}_{(J)} = \sum_{K=1}^{N_t} \underbrace{[(\mathbf{g}_{ra(J)} \star \mathbf{h}_{(J,K)} \star \mathbf{g}_{ta(K)} + \mathbf{g}_{rb(J)} \star \mathbf{h}^*_{(J,K)} \star \mathbf{g}^*_{tb(K)})}_{\mathbf{d}_{a(J,K)}} \star \mathbf{s}_{(K)} + \underbrace{(\mathbf{g}_{ra(J)} \star \mathbf{h}_{(J,K)} \star \mathbf{g}_{tb(K)} + \mathbf{g}_{rb(J)} \star \mathbf{h}^*_{(J,K)} \star \mathbf{g}^*_{ta(K)})}_{\mathbf{d}_{b(J,K)}} \star \mathbf{s}^*_{(K)}] + \mathbf{n}_{c(J)} \tag{5}$$

where $\mathbf{d}_{a(J,K)}$ and $\mathbf{d}_{b(J,K)}$ are the combined transmit IQ, channel and receive IQ impulse responses for the Kth transmit and Jth receive antenna. Both $\mathbf{d}_{a(J,K)}$ and $\mathbf{d}_{b(J,K)}$ are of length L_t+L+L_r-2 and are assumed to be always shorter than the CP, i.e. $(L_t+L+L_r-2 \le \nu)$.

The received symbol $\mathbf{z}_{(J)}$ is serial-to-parallel converted and the part corresponding to the CP is removed. It is then transformed to the frequency domain by the discrete Fourier transform (DFT) operation. The frequency domain received symbol $\mathbf{Z}_{(J)}$ can then be written as

$$\mathbf{Z}_{(J)} = \mathbf{F}_N \mathbf{P}_{\mathbf{CR}} \mathbf{z}_{(J)} = \sum_{k=1}^{N_t} [\mathbf{D}_{a(J,K)} \cdot \mathbf{S}_{(K)} + \mathbf{D}_{b(J,K)} \cdot \mathbf{S}^*_{m(K)}] + \mathbf{N}_{c(J)}$$
$$= \sum_{k=1}^{N_t} [(\mathbf{G}_{ra(J)} \cdot \mathbf{H}_{(J,K)} \cdot \mathbf{G}_{ta(K)} + \mathbf{G}_{rb(J)} \cdot \mathbf{H}^*_{m(J,K)} \cdot \mathbf{G}^*_{tb_m(K)}) \cdot \mathbf{S}_{(K)} + (\mathbf{G}_{ra(J)} \cdot \mathbf{H}_{(J,K)} \cdot \mathbf{G}_{tb(K)} + \mathbf{G}_{rb(J)} \cdot \mathbf{H}^*_{m(J,K)} \cdot \mathbf{G}^*_{ta_m(K)}) \cdot \mathbf{S}^*_{m(K)}] + \mathbf{N}_{c(J)} \tag{6}$$

where $\mathbf{P}_{\mathbf{CR}}$ is the CP removal matrix given as

$$\mathbf{P}_{\mathbf{CR}} = [0_{(N\times\nu)} | \mathbf{I}_N]$$

Here $\mathbf{Z}_{(J)}$, $\mathbf{D}_{a(J,K)}$, $\mathbf{D}_{b(J,K)}$, $\mathbf{H}_{(J,K)}$, $\mathbf{N}_{c(J)}$ are frequency domain representations of $\mathbf{z}_{(J)}$, $\mathbf{d}_{a(J,K)}$, $\mathbf{d}_{b(J,K)}$, $\mathbf{h}_{(J,K)}$, $\mathbf{n}_{c(J)}$, and $(\,)_m$ denotes the mirroring operation in which the vector indices are reversed, such that $\mathbf{S}_m[l]=\mathbf{S}[l_m]$ where $l_m = 2+N-l$ for $l=2\ \ldots N$ and $l_m=l$ for $l=1$.

Eq. (6) shows that every received symbol $\mathbf{Z}_{(J)}$ (for $J=1\ \ldots N_r$) is a weighted sum of the N_t transmitted OFDM symbols ($\mathbf{S}_{(K)}$) plus a power leakage from symbols on the mirror subcarrier ($\mathbf{S}^*_{m(K)}$). This power leakage from the mirror subcarrier leads to inter-carrier-interference (ICI) that can severely deteriorate the performance of the system. Based on Eq. (6), we will now present joint and decoupled estimation/compensation schemes in Section 3 and 4 respectively.

3. Joint transmitter/receiver IQ imbalance and channel compensation

The frequency domain received symbol $\mathbf{Z}_{(J)}$ (Eq. (6)), and the complex conjugate of its mirror can be rewritten for $J=1\ \ldots N_r$ as

$$\underbrace{\begin{bmatrix} \begin{pmatrix} \mathbf{Z}_{(1)}[l] \\ \mathbf{Z}^*_{(1)}[l_m] \end{pmatrix} \\ \vdots \\ \begin{pmatrix} \mathbf{Z}_{(N_r)}[l] \\ \mathbf{Z}^*_{(N_r)}[l_m] \end{pmatrix} \end{bmatrix}}_{\mathbf{z}_{tot}[l]} = \underbrace{\begin{bmatrix} (\mathbf{D}_{tot}(1,1)[l]) & \ldots & (\mathbf{D}_{tot}(1,N_t)[l]) \\ \vdots & \ddots & \vdots \\ (\mathbf{D}_{tot}(N_r,1)[l]) & \ldots & (\mathbf{D}_{tot}(N_r,N_t)[l]) \end{bmatrix}}_{\mathbf{D}_{tot}[l]} \times \underbrace{\begin{bmatrix} \begin{pmatrix} \mathbf{S}_{(1)}[l] \\ \mathbf{S}^*_{(1)}[l_m] \end{pmatrix} \\ \vdots \\ \begin{pmatrix} \mathbf{S}_{(N_t)}[l] \\ \mathbf{S}^*_{(N_t)}[l_m] \end{pmatrix} \end{bmatrix}}_{\mathbf{S}_{tot}[l]} + \underbrace{\begin{bmatrix} \begin{pmatrix} \mathbf{N}_{c(1)}[l] \\ \mathbf{N}^*_{c(1)}[l_m] \end{pmatrix} \\ \vdots \\ \begin{pmatrix} \mathbf{N}_{c(N_r)}[l] \\ \mathbf{N}^*_{c(N_r)}[l_m] \end{pmatrix} \end{bmatrix}}_{\mathbf{N}_{c_{tot}}[l]} \tag{7}$$

where $\mathbf{D}_{tot}(J,K)[l] = \begin{pmatrix} \mathbf{D}_{a(J,K)}[l] & \mathbf{D}_{b(J,K)}[l] \\ \mathbf{D}^*_{b(J,K)}[l_m] & \mathbf{D}^*_{a(J,K)}[l_m] \end{pmatrix}$. Matrix $\mathbf{D}_{tot}[l]$ represents the joint transmitter/receiver IQ imbalance and channel distortion.

With $N_r \ge N_t$ and assuming $\mathbf{D}_{tot}[l]$ is known, we can compute an estimate $\widetilde{\mathbf{S}}_{tot}[l]$ of $\mathbf{S}_{tot}[l]$ based on a minimum mean square error (MMSE) criterion as

$$\widetilde{\mathbf{S}}_{tot}[l] = \mathbf{D}^H_{tot}[l](\mathbf{D}_{tot}[l]\mathbf{D}^H_{tot}[l] + \mathbf{R}_{Nc}[l])^{-1}\mathbf{Z}_{tot}[l] \tag{8}$$

where the noise covariance matrix $\mathbf{R}_{Nc}[l]=\Xi\{\mathbf{N}_{c_{tot}}[l]\mathbf{N}^H_{c_{tot}}[l]\}$ and Ξ is the expectation operator. The total number of coefficients in $\mathbf{D}_{tot}[l]$ is $4(N_t \times N_r)$, half of which are used to estimate the transmitted symbols at the desired subcarriers, i.e.

$$\mathbf{S}_{tot-}[l]=\begin{bmatrix}(\mathbf{S}_{(1)}[l])\\ \vdots \\ (\mathbf{S}_{(N_t)}[l])\end{bmatrix}.$$

Thus the total computational complexity is $2(N_t \times N_r)$ multiplications per subcarrier.

The coefficients in $\mathbf{D}_{tot}[l]$ can be estimated by considering a training based estimation scheme. We consider the availability of an M_l long sequence of the so-called long training (LT) symbols that are also constructed based on Eq. (1). Eq. (7) can then be used for all LT symbols as follows:

$$\underbrace{\begin{bmatrix}\mathbf{Z}^{(1)}_{(1)}[l] & \cdots & \mathbf{Z}^{(M_l)}_{(1)}[l]\\ \vdots & \ddots & \vdots \\ \mathbf{Z}^{(1)}_{(N_r)}[l] & \cdots & \mathbf{Z}^{(M_l)}_{(N_r)}[l]\end{bmatrix}}_{\mathbf{Z}^{Tr}_{tot-}[l]}=\underbrace{\begin{bmatrix}(\mathbf{D}_{tot-}(1,1)[l]) & \cdots & (\mathbf{D}_{tot-}(1,N_t)[l])\\ \vdots & \ddots & \vdots \\ (\mathbf{D}_{tot-}(N_r,1)[l]) & \cdots & (\mathbf{D}_{tot-}(N_r,N_t)[l])\end{bmatrix}}_{\mathbf{D}_{tot-}[l]}$$

$$\times\underbrace{\begin{bmatrix}\begin{pmatrix}\mathbf{S}^{(1)}_{(1)}[l]\\ \mathbf{S}^{*(1)}_{(1)}[l_m]\end{pmatrix} & \cdots & \begin{pmatrix}\mathbf{S}^{(M_l)}_{(1)}[l]\\ \mathbf{S}^{*(M_l)}_{(1)}[l_m]\end{pmatrix}\\ \vdots & \ddots & \vdots \\ \begin{pmatrix}\mathbf{S}^{(1)}_{(N_t)}[l]\\ \mathbf{S}^{*(1)}_{(N_t)}[l_m]\end{pmatrix} & \cdots & \begin{pmatrix}\mathbf{S}^{(M_l)}_{(N_t)}[l]\\ \mathbf{S}^{*(M_l)}_{(N_t)}[l_m]\end{pmatrix}\end{bmatrix}}_{\mathbf{S}^{Tr}_{tot}[l]}+\begin{bmatrix}\mathbf{N}^{(1)}_{c(1)}[l] & \cdots & \mathbf{N}^{(M_l)}_{c(1)}[l]\\ \vdots & \ddots & \vdots \\ \mathbf{N}^{(1)}_{c(N_r)}[l] & \cdots & \mathbf{N}^{(M_l)}_{c(N_r)}[l]\end{bmatrix} \tag{9}$$

where superscript (i) represents the training symbol number and $\mathbf{D}_{tot-}(J,K)[l]=(\mathbf{D}_{a(J,K)}[l]\ \mathbf{D}_{b(J,K)}[l])$. A total of $2(N_t \times N_r)$ elements in $\mathbf{D}_{tot-}[l]$ has to be estimated based on

$$\widetilde{\mathbf{D}}_{tot-}[l]=\mathbf{Z}^{Tr}_{tot-}[l]\mathbf{S}^{Tr^\dagger}_{tot}[l] \tag{10}$$

where † is the pseudo-inverse operation and $\widetilde{\mathbf{D}}_{tot-}[l]$ is the estimate of $\mathbf{D}_{tot-}[l]$. Eqs. (9) and (10) show that there are M_l equations in $2N_t$ unknowns (for each row of $\mathbf{D}_{tot-}[l]$). Hence to estimate $\mathbf{D}_{tot-}[l]$, we need an LT symbol sequence length $M_l \geq 2N_t$. A longer training sequence will provide improved estimates due to a better noise averaging. In the case where M_l=$2N_t$, the training symbols at subcarrier $[l]$ and $[l_m]$ are required to be linearly independent of each other and also independent of subcarrier $[l]$ and $[l_m]$ of the training symbols transmitted from other transmit antennas. This is required so that the pseudo-inversion of $\mathbf{S}^{Tr}_{tot}[l]$ in (10) always generates a unique solution.

Once $\widetilde{\mathbf{D}}_{tot-}[l]$ and hence $\widetilde{\mathbf{D}}_{tot}[l]$ is accurately known, we can obtain $\mathbf{S}_{tot}[l]$ as shown in Eq. (8). This is the principle behind the joint compensation scheme in [14–17]. It should be noted that Eq. (8) is also valid in the presence of either only transmitter IQ imbalance or only receiver IQ imbalance. In the absence of any IQ imbalance, the distortion term $\mathbf{D}_{b(J,K)}[l]$=0, and then we have to estimate only $(N_t \times N_r)$ elements in $\mathbf{D}_{tot-}[l]$.

Based on Eq. (8), we can also directly generate symbol estimates as

$$\underbrace{\begin{bmatrix}\widetilde{\mathbf{S}}_{(1)}[l]\\ \vdots \\ \widetilde{\mathbf{S}}_{(N_t)}[l]\end{bmatrix}}_{\widetilde{\mathbf{S}}_{tot-}[l]}=\underbrace{\begin{bmatrix}(\mathbf{W}_{a(1,1)}[l]\ \mathbf{W}_{b(1,1)}[l]) & \cdots & (\mathbf{W}_{a(N_r,1)}[l]\ \mathbf{W}_{b(N_r,1)}[l])\\ \vdots & \ddots & \vdots \\ (\mathbf{W}_{a(1,N_t)}[l]\ \mathbf{W}_{b(1,N_t)}[l]) & \cdots & (\mathbf{W}_{a(N_r,N_t)}[l]\ \mathbf{W}_{b(N_r,N_t)}[l])\end{bmatrix}}_{\mathbf{W}_{tot}[l]}$$

$$\times\begin{bmatrix}\begin{pmatrix}\mathbf{Z}_{(1)}[l]\\ \mathbf{Z}^*_{(1)}[l_m]\end{pmatrix}\\ \vdots \\ \begin{pmatrix}\mathbf{Z}_{(N_r)}[l]\\ \mathbf{Z}^*_{(N_r)}[l_m]\end{pmatrix}\end{bmatrix} \tag{11}$$

Here $\mathbf{W}_{a(J,K)}[l]$ and $\mathbf{W}_{b(J,K)}[l]$ are the coefficients of a frequency domain equalizer (FEQ). A total of $2(N_t \times N_r)$ coefficients per subcarrier are again needed for the estimation of $\mathbf{S}_{tot-}[l]$. The FEQ coefficients can be directly obtained from the LT symbols based on a least squares or a recursive least squares estimation scheme [20]. The FEQ scheme is illustrated in Fig. 1.

The disadvantage of the joint transmitter and receiver IQ imbalance compensation scheme is that we have to re-estimate the $\mathbf{D}_{tot}[l]$ or $\mathbf{W}_{tot}[l]$ matrix for every variation of the channel characteristics. In Section 4 we develop an alternative scheme where the transmitter and receiver IQ imbalance parameters are decoupled from the channel parameters in the $\mathbf{D}_{tot}[l]$ matrix. This results in a compensation scheme where only the channel parameters have to be re-estimated after channel variations, while the IQ parameters are kept unchanged.

4. Decoupled transmitter/receiver IQ imbalance and channel compensation

To decouple the transmitter and receiver IQ imbalance parameters from the channel parameters, we can rewrite the coefficients $\mathbf{D}_{a(J,K)}[l]$ and $\mathbf{D}_{b(J,K)}[l]$ in the matrix $\mathbf{D}_{tot}(J,K)[l]$ (Eq. (7)), as follows:

$$\mathbf{D}_{a(J,K)}[l]=\mathbf{B}_{(J,K)}[l]+\mathbf{Q}_{r(J)}[l]\mathbf{Q}^*_{t(K)}[l_m]\mathbf{B}^*_{(J,K)}[l_m]$$

$$\mathbf{D}_{b(J,K)}[l]=\mathbf{Q}_{t(K)}[l]\mathbf{B}_{(J,K)}[l]+\mathbf{Q}_{r(J)}[l]\mathbf{B}^*_{(J,K)}[l_m] \tag{12}$$

where $\mathbf{B}_{(J,K)}[l]$=$\mathbf{G}_{ra(J)}[l]\mathbf{G}_{ta(K)}[l]\mathbf{C}_{(J,K)}[l]$ is the composite channel between the Kth transmit and Jth receive antenna respectively, $\mathbf{Q}_{t(K)}[l]=\mathbf{G}_{tb(K)}[l]/\mathbf{G}_{ta(K)}[l]$ is the transmitter IQ imbalance gain parameter for the Kth transmit antenna and $\mathbf{Q}_{r(J)}[l]=\mathbf{G}_{rb(J)}[l]/\mathbf{G}^*_{ra(J)}[l_m]$ is the receiver IQ imbalance gain parameter for the Jth receive antenna.

Based on Eq. (12), the matrix $\mathbf{D}_{tot}[l]$ can be factorized as follows:

$$\mathbf{D}_{tot}[l]=\mathbf{Q}_{r_{tot}}[l]\mathbf{B}_{tot}[l]\mathbf{Q}_{t_{tot}}[l] \tag{13}$$

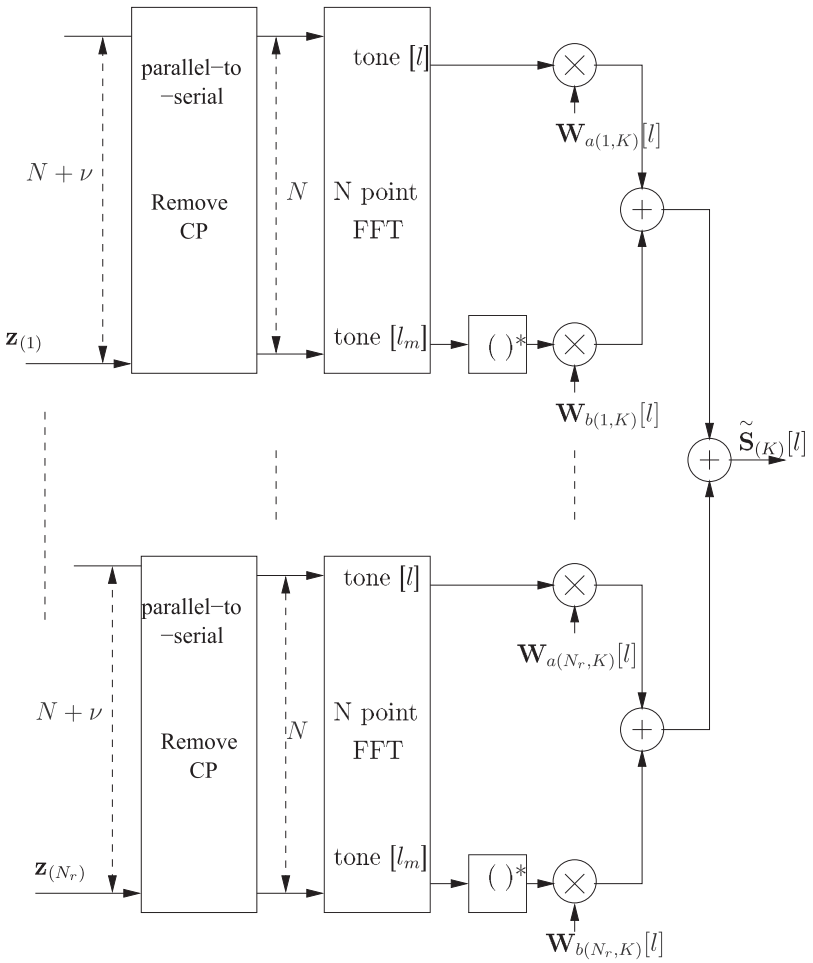


Fig. 1. Joint compensation scheme for MIMO OFDM system in the presence of transmitter and receiver IQ imbalance.

where

$$\mathbf{B}_{tot}[l]=\begin{bmatrix}(\mathbf{B}_{tot}(1,1)[l]) & \dots & (\mathbf{B}_{tot}(1,N_t)[l])\\ \vdots & \ddots & \vdots\\ (\mathbf{B}_{tot}(N_r,1)[l]) & \dots & (\mathbf{B}_{tot}(N_r,N_t)[l])\end{bmatrix}$$

$$\mathbf{Q}_{r_{tot}}[l]=\begin{bmatrix}(\mathbf{Q}_{r_{tot}}(1)[l]) & & \\ & \ddots & \\ & & (\mathbf{Q}_{r_{tot}}(N_r)[l])\end{bmatrix}$$

and

$$\mathbf{Q}_{t_{tot}}[l]=\begin{bmatrix}(\mathbf{Q}_{t_{tot}}(1)[l]) & & \\ & \ddots & \\ & & (\mathbf{Q}_{t_{tot}}(N_t)[l])\end{bmatrix}$$

where the sub-matrices are defined as $\mathbf{B}_{tot}(J,K)[l]=\begin{pmatrix}\mathbf{B}_{(J,K)}[l] & 0\\ 0 & \mathbf{B}^*_{(J,K)}[l_m]\end{pmatrix}$, $\mathbf{Q}_{r_{tot}}(J)[l]=\begin{pmatrix}1 & \mathbf{Q}_{r(J)}[l]\\ \mathbf{Q}^*_{r(J)}[l_m] & 1\end{pmatrix}$ and $\mathbf{Q}_{t_{tot}}(K)[l]=\begin{pmatrix}1 & \mathbf{Q}_{t(K)}[l]\\ \mathbf{Q}^*_{t(K)}[l_m] & 1\end{pmatrix}$. In (13) $\mathbf{Q}_{t_{tot}}[l]$ and $\mathbf{Q}_{r_{tot}}[l]$ are independent of the channel distortion. The channel distortion is contained entirely in the $\mathbf{B}_{tot}[l]$ matrix. Thus both transmitter and receiver IQ imbalances have been decoupled from the (composite) channel distortion.

We will refer to the proposed decoupled frequency domain estimation/compensation schemes as **D-FEQ**. If we assume that estimates $\widetilde{\mathbf{B}}_{tot}[l]$,$\widetilde{\mathbf{Q}}_{t_{tot}}[l]$ and $\widetilde{\mathbf{Q}}_{r_{tot}}[l]$ of $\mathbf{B}_{tot}[l]$,$\mathbf{Q}_{t_{tot}}[l]$ and $\mathbf{Q}_{r_{tot}}[l]$ are available, then we can perform the compensation of the channel distortion and transmitter/receiver IQ imbalance either entirely at the receiver (D-FEQ$_{(R)}$) or compensate only the receiver IQ imbalance and channel distortion at the receiver, and compensate the transmitter IQ imbalance by pre-distortion of transmitted symbols (D-FEQ$_{(P)}$). The entirely receiver based and the pre-distortion based compensation schemes are explained in Sections 4.1 and 4.2, where it is assumed that $\widetilde{\mathbf{B}}_{tot}[l]$,$\widetilde{\mathbf{Q}}_{t_{tot}}[l]$ and $\widetilde{\mathbf{Q}}_{r_{tot}}[l]$ are available. Sections 4.1 and 4.2 also explain the estimation method to track the variation in the channel matrix $\widetilde{\mathbf{B}}_{tot}[l]$, while the estimation of $\widetilde{\mathbf{Q}}_{t_{tot}}[l]$ and $\widetilde{\mathbf{Q}}_{r_{tot}}[l]$ is later explained in Section 4.3.

4.1. Compensation entirely at the receiver (D-FEQ$_{(R)}$)

In this case, the estimate $\widetilde{\mathbf{D}}_{tot}[l]=(\widetilde{\mathbf{Q}}_{r_{tot}}[l]\widetilde{\mathbf{B}}_{tot}[l]\widetilde{\mathbf{Q}}_{t_{tot}}[l])$ of $\mathbf{D}_{tot}[l]$ is computed from $\widetilde{\mathbf{B}}_{tot}[l]$,$\widetilde{\mathbf{Q}}_{t_{tot}}[l]$ and $\widetilde{\mathbf{Q}}_{r_{tot}}[l]$, and hence an estimate of the transmitted symbol $\widetilde{\mathbf{S}}_{tot}[l]$ is obtained as in (8), i.e.

$$\widetilde{\mathbf{S}}_{tot}[l]=\widetilde{\mathbf{D}}^{H}{}_{tot}[l](\widetilde{\mathbf{D}}_{tot}[l]\widetilde{\mathbf{D}}^{H}{}_{tot}[l]+\mathbf{R}_{Nc}[l])^{-1}\mathbf{Z}_{tot}[l] \tag{14}$$

The total computational complexity is once again $2(N_t\times N_r)$ coefficients per subcarrier. Assuming $\widetilde{\mathbf{Q}}_{t_{tot}}[l]$ and $\widetilde{\mathbf{Q}}_{r_{tot}}[l]$ are available and fixed, based on the estimation obtained from Section 4.3 (to be derived), then the variations in the channel are tracked as

$$\underbrace{\begin{bmatrix}\widetilde{\mathbf{B}}_{r(1,1)}[l] & \dots & \widetilde{\mathbf{B}}_{r(1,N_t)}[l]\\ \vdots & \ddots & \vdots\\ \widetilde{\mathbf{B}}_{r(N_r,1)}[l] & \dots & \widetilde{\mathbf{B}}_{r(N_r,N_t)}[l]\end{bmatrix}}_{\widetilde{\mathbf{B}}_{r_{tot-}}[l]}=(\widetilde{\mathbf{Q}}_{r\nu_{tot-}}[l]\mathbf{Z}^{Tr}_{tot}[l])(\widetilde{\mathbf{Q}}_{t_{tot-}}[l]\mathbf{S}^{Tr}_{tot}[l])^{\dagger} \tag{15}$$

Here $\widetilde{\mathbf{B}}_{r(J,K)}[l]=\widetilde{\mathbf{B}}_{(J,K)}(1-\widetilde{\mathbf{Q}}_{r(J)}[l]\widetilde{\mathbf{Q}}^{*}_{r(J)}[l_m])$, the matrix $\mathbf{S}^{Tr}_{tot}[l]$ is defined in Eq. (9) and

$$\mathbf{Z}^{Tr}_{tot}[l]=\begin{bmatrix}\begin{pmatrix}\mathbf{Z}^{(1)}_{(1)}[l]\\ \mathbf{Z}^{*(1)}_{(1)}[l_m]\end{pmatrix} & \cdots & \begin{pmatrix}\mathbf{Z}^{(M_l)}_{(1)}[l]\\ \mathbf{Z}^{*(M_l)}_{(1)}[l_m]\end{pmatrix}\\ \vdots & \ddots & \vdots\\ \begin{pmatrix}\mathbf{Z}^{(1)}_{(N_r)}[l]\\ \mathbf{Z}^{*(1)}_{(N_r)}[l_m]\end{pmatrix} & \cdots & \begin{pmatrix}\mathbf{Z}^{(M_l)}_{(N_r)}[l]\\ \mathbf{Z}^{*(M_l)}_{(N_r)}[l_m]\end{pmatrix}\end{bmatrix}$$

$$\widetilde{\mathbf{Q}}_{t_{tot-}}[l]=\begin{bmatrix}(1\ \ \widetilde{\mathbf{Q}}_{t(1)}[l]) & & \\ & \ddots & \\ & & (1\ \ \widetilde{\mathbf{Q}}_{t(N_t)}[l])\end{bmatrix}$$

$$\widetilde{\mathbf{Q}}_{r\nu_{tot-}}[l]=\begin{bmatrix}(1\ \ -\widetilde{\mathbf{Q}}_{r(1)}[l]) & & \\ & \ddots & \\ & & (1\ \ -\widetilde{\mathbf{Q}}_{r(N_r)}[l])\end{bmatrix}$$

Once $\widetilde{\mathbf{B}}_{r_{tot-}}[l]$ is known, we can then obtain the coefficients of the matrix $\widetilde{\mathbf{B}}_{tot}[l]$ as $\widetilde{\mathbf{B}}_{(J,K)}[l]=\widetilde{\mathbf{B}}_{r(J,K)}[l]/(1-\widetilde{\mathbf{Q}}_{r(J)}[l]\widetilde{\mathbf{Q}}^{*}_{r(J)}[l_m])$. Eq. (15) shows that we have to estimate $(N_t\times N_r)$ coefficients per subcarrier in $\widetilde{\mathbf{B}}_{r_{tot-}}[l]$. Each row of $\widetilde{\mathbf{B}}_{r_{tot-}}[l]$ corresponds to M_l equations in N_t unknowns. Hence to obtain the estimate $\widetilde{\mathbf{B}}_{r_{tot-}}[l]$, we need an LT sequence length $M_l\geq N_t$.

4.2. Compensation with pre-distortion of transmitted symbols (D-FEQ$_{(P)}$)

The estimation of transmitted symbols in (14) is based on the joint compensation of transmitter/receiver IQ imbalance and channel distortion at the receiver. An alternative solution is to compensate for the transmitter IQ imbalance already at the transmitter. This can be obtained by distorting the transmitted symbol before the IDFT operation such that the resulting transmitted symbol is free of any transmitter IQ imbalance. A pre-distortion system requires a feedback mechanism between the receiver and the transmitter, as will be explained next.

In the pre-distortion scheme, the new OFDM symbol $\mathbf{S}_{\mathbf{n}(K)}$ is defined as $\mathbf{S}_{\mathbf{n}(K)}=\mathbf{S}_{(K)}-\widetilde{\mathbf{Q}}_{t(K)}\cdot\mathbf{S}^{*}_{m(K)}$ where $\widetilde{\mathbf{Q}}_{t(K)}$ (to be derived in Section 4.3) is the $\mathbf{Q}_{t(K)}$ estimate fed back from the receiver. In matrix form, $\mathbf{S}_{\mathbf{n}(K)}[l]$ and $\mathbf{S}^{*}_{\mathbf{n}(K)}[l_m]$ can be written as

$$\underbrace{\begin{bmatrix}\begin{pmatrix}\mathbf{S}_{\mathbf{n}(1)}[l]\\ \mathbf{S}^{*}_{\mathbf{n}(1)}[l_m]\end{pmatrix}\\ \vdots\\ \begin{pmatrix}\mathbf{S}_{\mathbf{n}(N_r)}[l]\\ \mathbf{S}^{*}_{\mathbf{n}(N_t)}[l_m]\end{pmatrix}\end{bmatrix}}_{\mathbf{S}_{\mathbf{n}tot}[l]}=\underbrace{\begin{bmatrix}(\widetilde{\mathbf{Q}}_{t\nu_{tot}(1)}[l]) & & \\ & \ddots & \\ & & (\widetilde{\mathbf{Q}}_{t\nu_{tot}(N_t)}[l])\end{bmatrix}}_{\widetilde{\mathbf{Q}}_{t\nu_{tot}}[l]}\underbrace{\begin{bmatrix}\begin{pmatrix}\mathbf{S}_{(1)}[l]\\ \mathbf{S}^{*}_{(1)}[l_m]\end{pmatrix}\\ \vdots\\ \begin{pmatrix}\mathbf{S}_{(N_r)}[l]\\ \mathbf{S}^{*}_{(N_t)}[l_m]\end{pmatrix}\end{bmatrix}}_{\mathbf{S}_{tot}[l]} \tag{16}$$

where $\widetilde{\mathbf{Q}}_{t\nu_{tot}(K)}[l]=\begin{pmatrix}1 & -\widetilde{\mathbf{Q}}_{t(K)}[l]\\ -\widetilde{\mathbf{Q}}^{*}_{t(K)}[l_m] & 1\end{pmatrix}$. Eq. (7) is now modified as

$$\begin{aligned}\mathbf{Z}_{tot}[l]&=\mathbf{D}_{tot}[l]\mathbf{S}_{\mathbf{n}tot}[l]+\mathbf{N}_{c_{tot}}[l]\\&=\mathbf{Q}_{r_{tot}}[l]\mathbf{B}_{tot}[l]\underbrace{\begin{bmatrix}(\mathbf{Q}_{td_{tot}}(1)[l]) & & \\ & \ddots & \\ & & (\mathbf{Q}_{td_{tot}}(N_t)[l])\end{bmatrix}}_{\mathbf{Q}_{td_{tot}}[l]}\mathbf{S}_{tot}[l]+\mathbf{N}_{c_{tot}}[l]\end{aligned} \tag{17}$$

where $\mathbf{Q}_{td_{tot}}(K)[l]=\begin{bmatrix}(1-\mathbf{Q}_{t(K)}[l]\widetilde{\mathbf{Q}}^{*}_{t(K)}[l_m]) & (\mathbf{Q}_t[l]-\widetilde{\mathbf{Q}}_{t(K)}[l])\\ (\mathbf{Q}^{*}_{t(K)}[l_m]-\widetilde{\mathbf{Q}}^{*}_{t(K)}[l_m]) & (1-\mathbf{Q}^{*}_t[l_m]\widetilde{\mathbf{Q}}_{t(K)}[l])\end{bmatrix}$. Under ideal conditions $\widetilde{\mathbf{Q}}_{t(K)}[l]=\mathbf{Q}_{t(K)}[l]$, and then $\mathbf{Q}_{td_{tot}}(K)[l]$ and $\widetilde{\mathbf{Q}}_{t\nu_{tot}}[l]$ are diagonalized and the remaining diagonal elements $(1-\mathbf{Q}_{t(K)}[l]\widetilde{\mathbf{Q}}^{*}_{t(K)}[l_m])$ can be merged with $\mathbf{B}_{tot}[l]$. The received symbol $\mathbf{Z}_{tot}[l]$ is then considered to be free from transmitter IQ imbalance. The estimate of the transmitted symbol $\widetilde{\mathbf{S}}_{tot}[l]$ is finally obtained as

$$\widetilde{\mathbf{S}}_{tot}[l]=\widetilde{\mathbf{B}}^{\dagger}{}_{rt_{tot}}[l]\widetilde{\mathbf{Q}}_{r\nu_{tot}}[l]\mathbf{Z}_{tot}[l] \tag{18}$$

where the matrices $\widetilde{\mathbf{Q}}_{r\nu_{tot}}[l]$ and $\widetilde{\mathbf{B}}_{rt_{tot}}[l]$ are defined as

$$\widetilde{\mathbf{Q}}_{r\nu_{tot}}[l]=\begin{bmatrix}(\widetilde{\mathbf{Q}}_{r\nu_{tot}}(1)[l]) & & \\ & \ddots & \\ & & (\widetilde{\mathbf{Q}}_{r\nu_{tot}}(N_r)[l])\end{bmatrix}$$

$$\widetilde{\mathbf{B}}_{rt_{tot}}[l]=\begin{bmatrix}(\widetilde{\mathbf{B}}_{rt_{tot}}(1,1)[l]) & \dots & (\widetilde{\mathbf{B}}_{rt_{tot}}(1,N_t)[l])\\ \vdots & \ddots & \vdots\\ (\widetilde{\mathbf{B}}_{rt_{tot}}(N_r,1)[l]) & \dots & (\widetilde{\mathbf{B}}_{rt_{tot}}(N_r,N_t)[l])\end{bmatrix}$$

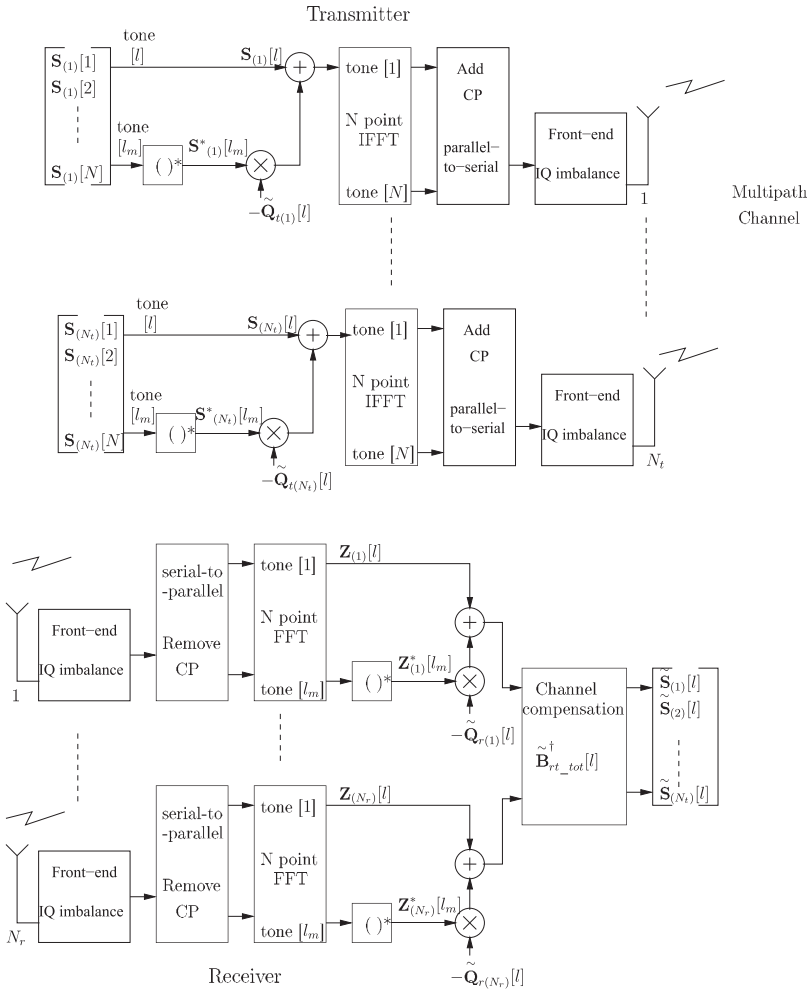


Fig. 2. D-FEQ$_{(P)}$ compensation scheme for transmitter/receiver IQ imbalance and channel distortion. The transmitter pre-distorts the transmitted symbols for the compensation of transmitter IQ imbalance, while receiver IQ imbalance and channel distortion are compensated at the receiver.

with $\widetilde{\mathbf{Q}}_{r\nu_{tot}}(J)[l]=\begin{pmatrix} 1 & -\widetilde{\mathbf{Q}}_{r(J)}[l] \\ -\widetilde{\mathbf{Q}}^{*}_{r(J)}[l_m] & 1 \end{pmatrix}$ and $\widetilde{\mathbf{B}}_{rt_{tot}}(J,K)[l]=\begin{pmatrix} \widetilde{\mathbf{B}}_{rt(J,K)}[l] & 0 \\ 0 & \widetilde{\mathbf{B}}^{*}_{rt(J,K)}[l_m] \end{pmatrix}$. Finally the term $\widetilde{\mathbf{B}}_{rt(J,K)}[l]=\widetilde{\mathbf{B}}_{(J,K)}[l]$ $(1-\widetilde{\mathbf{Q}}_{r(J)}[l]\widetilde{\mathbf{Q}}^{*}_{r(J)}[l_m])(1-\widetilde{\mathbf{Q}}_{t(K)}[l]\widetilde{\mathbf{Q}}^{*}_{t(K)}[l_m])$.

The D-FEQ scheme with pre-distortion of transmitted symbols (D-FEQ$_{(P)}$) is shown in Fig. 2. The figure shows that the compensation of transmitter IQ imbalance takes place at every transmit antenna, and similarly the compensation of receiver IQ imbalance takes place at every receive antenna, before the inputs from all receive antenna branches are combined for the compensation of channel distortion.

The computational complexity to estimate the transmitted symbols $\widetilde{\mathbf{S}}_{tot-}[l]$ in (18) is now reduced to $N_t+N_r+(N_t\times N_r)$ coefficients per subcarrier, compared to $2(N_t\times N_r)$ coefficients in (8) and (14). It can be observed that the number of coefficients required for a lower order $(N_t\times N_r)=(2\times 2)$ MIMO system is same for the joint compensation scheme and D-FEQ schemes, but when the system order is increased $(N_t\times N_r)>(2\times 2)$, the decoupled scheme with pre-distortion provides a lower computational complexity.

Variations in the channel are now tracked as follows:

$$\underbrace{\begin{bmatrix} \widetilde{\mathbf{B}}_{rt(1,1)}[l] & \dots & \widetilde{\mathbf{B}}_{rt(1,N_t)}[l] \\ \vdots & \ddots & \vdots \\ \widetilde{\mathbf{B}}_{rt(N_r,1)}[l] & \dots & \widetilde{\mathbf{B}}_{rt(N_r,N_t)}[l] \end{bmatrix}}_{\widetilde{\mathbf{B}}_{rt_{tot-}}[l]} = (\widetilde{\mathbf{Q}}_{r\nu_{tot-}}[l]\mathbf{Z}^{Tr}_{tot}[l])(\mathbf{S}^{Tr}_{tot-}[l])^{\dagger} \tag{19}$$

where $\widetilde{\mathbf{Q}}_{r\nu_{tot-}}[l]$,$\mathbf{Z}^{Tr}_{tot}[l]$ are defined in Eq. (15) and

$$\mathbf{S}^{Tr}_{tot-}[l]=\begin{bmatrix} \mathbf{S}^{(1)}_{(1)}[l] & \dots & \mathbf{S}^{(M_l)}_{(1)}[l] \\ \vdots & \ddots & \vdots \\ \mathbf{S}^{(1)}_{(N_t)}[l] & \dots & \mathbf{S}^{(M_l)}_{(N_t)}[l] \end{bmatrix}$$

We once again need an LT sequence length $M_l\geq N_t$.

Table 1
Computational complexity in terms of the number of coefficients required per subcarrier for different RF impairment scenarios.

RF impairment scenario	D-FEQ entirely receiver based D-FEQ$_{(R)}$	D-FEQ with pre-distortion D-FEQ$_{(P)}$	Joint compensation
Transmitter/receiver IQ	$2(N_t \times N_r)$	$N_t+N_r+(N_t \times N_r)$	$2(N_t \times N_r)$
Receiver IQ	$N_r+(N_t \times N_r)$	–	$2(N_t \times N_r)$
Transmitter IQ	$2(N_t \times N_r)$	$N_t+(N_t \times N_r)$	$2(N_t \times N_r)$

Eqs. (18) and (19) are also valid in the presence of either only receiver IQ imbalance ($\mathbf{Q}_{t(K)}=0$) or only transmitter IQ imbalance ($\mathbf{Q}_{r(J)}=0$). The total computational complexity per subcarrier for the estimation of transmitted symbols $\widetilde{\mathbf{S}}_{tot-}[l]$ is reduced to $N_t+(N_t \times N_r)$ coefficients in the case of only transmitter IQ imbalance, and $N_r+(N_t \times N_r)$ coefficients for only receiver IQ imbalance. In the absence of transmitter IQ imbalance, the entire compensation takes place at the receiver with a reduced computational complexity.

Table 1 summarizes the computational complexity in terms of the number of coefficients required per subcarrier for different RF impairment scenarios. The table shows that the D-FEQ scheme with pre-distortion of transmitted symbols provides the most efficient compensation when the system is impaired with both transmitter and receiver IQ imbalances.

4.3. Estimation of transmitter and receiver IQ imbalance gain parameters

We now develop an algorithm for the estimation of the transmitter and receiver IQ imbalance gain parameters $\mathbf{Q}_{t_{(K)}}[l]$, $\mathbf{Q}_{r_{(J)}}[l]$ based on the coefficients $\mathbf{D}_{a(J,K)}[l]$, $\mathbf{D}_{b(J,K)}[l]$ and the decoupled matrix $\mathbf{D}_{tot}[l]$ shown in Eq. (12) and (13). In order to obtain the IQ imbalance gain parameter estimates, we first make an approximation, namely that the second order term $\mathbf{Q}_{r(J)}[l]\mathbf{Q}^*_{t(K)}[l_m]=0$ in $\mathbf{D}_{a(J,K)}[l]$. This approximation is based on the fact that in practice $\mathbf{G}_{ta(K)}[l] \gg \mathbf{G}_{tb(K)}[l]$ and $\mathbf{G}^*_{ra(J)}[l_m] \gg \mathbf{G}_{rb(J)}[l]$. With this approximation the channel estimate $\widetilde{\mathbf{B}}_{(J,K)}[l] \backsimeq \widetilde{\mathbf{D}}_{a(J,K)}[l]$. Eq. (12) can then be written for $\mathbf{D}_{b(J,K)}[l]$ as follows:

$$\mathbf{D}_{b(J,K)}[l]=\mathbf{Q}_{t(K)}[l]\mathbf{D}_{a(J,K)}[l]+\mathbf{Q}_{r(J)}[l]\mathbf{D}^*_{a(J,K)}[l_m] \tag{20}$$

Based on Eq. (20), we can now estimate the transmitter/receiver IQ imbalance gain parameters for the FS and FI cases as follows.

4.3.1. FI transmitter and receiver IQ imbalance

In this case the IQ imbalance gain parameter estimates for each antenna pair (J,K) are straightforwardly obtained from (20) as

$$\begin{bmatrix} \widetilde{Q}_{t(K)} \\ \widetilde{Q}_{r(J)} \end{bmatrix} = \begin{bmatrix} \widetilde{\mathbf{D}}_{a(J,K)}[2] & \widetilde{\mathbf{D}}^{*}_{a(J,K)}[N] \\ \vdots & \vdots \\ \widetilde{\mathbf{D}}_{a(J,K)}[l] & \widetilde{\mathbf{D}}^{*}_{a(J,K)}[l_m] \\ \vdots & \vdots \\ \widetilde{\mathbf{D}}_{a(J,K)}[N] & \widetilde{\mathbf{D}}^{*}_{a(J,K)}[2] \end{bmatrix}^{\dagger} \begin{bmatrix} \widetilde{\mathbf{D}}_{b(J,K)}[2] \\ \vdots \\ \widetilde{\mathbf{D}}_{b(J,K)}[l] \\ \vdots \\ \widetilde{\mathbf{D}}_{b(J,K)}[N] \end{bmatrix} \tag{21}$$

Here the multipath diversity of the channel $\mathbf{B}_{(J,K)}[l]$, and hence $\widetilde{\mathbf{D}}_{a(J,K)}[l]$, allows us to estimate $\widetilde{Q}_{t(K)}$ and $\widetilde{Q}_{r(J)}$ respectively. It should be noted that we can also obtain $\widetilde{Q}_{t(K)}$ and $\widetilde{Q}_{r(J)}$ estimates from other antenna pairs, i.e. $(\bar{\bar{J}},K)$ and $(J,\bar{\bar{K}})$ in (21). Finally $\widetilde{Q}_{t(K)}$ and $\widetilde{Q}_{r(J)}$ obtained from different antenna pairs can be averaged to obtain improved estimates.

4.3.2. FS transmitter and receiver IQ imbalance

In this case the estimate of the IQ imbalance gain parameters is to be performed for each subcarrier individually. To obtain these estimates we need at least two independent channel realizations for each antenna pair (J,K), i.e. $\mathbf{B}^{(1)}_{(J,K)}[l]$ and $\mathbf{B}^{(2)}_{(J,K)}[l]$, and hence $\widetilde{\mathbf{D}}^{(1)}_{a(J,K)}[l]$, $\widetilde{\mathbf{D}}^{(2)}_{a(J,K)}[l]$ and $\widetilde{\mathbf{D}}^{(1)}_{b(J,K)}[l]$ $\widetilde{\mathbf{D}}^{(2)}_{b(J,K)}[l]$ respectively. Here the superscript (i) in $\mathbf{X}^{(i)}_{(J,K)}[l]$ indicates a different realization of $\mathbf{X}_{(J,K)}[l]$. The estimates $\widetilde{\mathbf{Q}}_{t(K)}[l]$ and $\widetilde{\mathbf{Q}}_{r(J)}[l]$ for the (J,K) antenna pair can then be obtained as follows:

$$\begin{bmatrix} \widetilde{\mathbf{Q}}_{t(K)}[l] \\ \widetilde{\mathbf{Q}}_{r(J)}[l] \end{bmatrix} = \begin{bmatrix} \widetilde{\mathbf{D}}^{(1)}_{a(J,K)}[l] & \widetilde{\mathbf{D}}^{*(1)}_{a(J,K)}[l_m] \\ \widetilde{\mathbf{D}}^{(2)}_{a(J,K)}[l] & \widetilde{\mathbf{D}}^{*(2)}_{a(J,K)}[l_m] \end{bmatrix}^{-1} \begin{bmatrix} \widetilde{\mathbf{D}}^{(1)}_{b(J,K)}[l] \\ \widetilde{\mathbf{D}}^{(2)}_{b(J,K)}[l] \end{bmatrix} \tag{22}$$

For guaranteed invertibility we should have $\widetilde{\mathbf{D}}^{(2)}_{a(J,K)}[l] \neq \widetilde{\mathbf{D}}^{(1)}_{a(J,K)}[l]$ and/or $\widetilde{\mathbf{D}}^{*(2)}_{a(J,K)}[l_m] \neq \widetilde{\mathbf{D}}^{*(1)}_{a(J,K)}[l_m]$. If the channel does not vary for a particular subcarrier and its mirror subcarrier for any combination (J,K), then the joint compensation scheme is performed on these subcarrier pairs as in (8).

The transmitter and receiver IQ imbalance gain estimates $\widetilde{\mathbf{Q}}_{t(\bar{\bar{K}})}[l]$ and $\widetilde{\mathbf{Q}}_{r(\bar{\bar{J}})}[l]$ for a different antenna pair $(\bar{\bar{J}},\bar{\bar{K}})$, can be obtained from the estimates $\widetilde{\mathbf{Q}}_{r(J)}[l]$ and $\widetilde{\mathbf{Q}}_{t(K)}[l]$ as follows:

$$\widetilde{\mathbf{Q}}_{t(\bar{\bar{K}})}[l]=\frac{\widetilde{\mathbf{D}}_{b(J,\bar{\bar{K}})}[l]-\widetilde{\mathbf{Q}}_{r(J)}[l]\,\widetilde{\mathbf{D}}^{*}_{a(J,\bar{\bar{K}})}[l_m]}{\widetilde{\mathbf{D}}_{a(J,\bar{\bar{K}})}[l]}$$

$$\widetilde{\mathbf{Q}}_{r(\bar{\bar{J}})}[l]=\frac{\widetilde{\mathbf{D}}_{b(\bar{\bar{J}},K)}[l]-\widetilde{\mathbf{Q}}_{t(K)}[l]\widetilde{\mathbf{D}}_{a(\bar{\bar{J}},K)}[l]}{\widetilde{\mathbf{D}}^{*}_{a(\bar{\bar{J}},K)}[l_m]} \tag{23}$$

If more than one antenna pair (J,K) has at least two independent channel realizations, then $\widetilde{\mathbf{Q}}_{t(\bar{\bar{K}})}$ and $\widetilde{\mathbf{Q}}_{r(\bar{\bar{J}})}$ estimates obtained from different antenna pairs can be averaged to obtain improved estimates.

Eqs. (22) and (23) provide good estimates as long as $\mathbf{Q}_{r(J)}[l]\mathbf{Q}^*_{t(K)}[l_m] \backsimeq 0$ i.e. as long as the transmitter and receiver IQ imbalance gain parameters are relatively small. The estimates are optimal if either $\mathbf{Q}_{r(J)}[l]=0$ or $\mathbf{Q}_{t(K)}[l]=0$, as in

this case $\mathbf{B}^{(1)}_{(J,K)}[l]=\mathbf{D}^{(1)}_{a(J,K)}[l]$ and $\mathbf{B}^{(2)}_{(J,K)}[l]=\mathbf{D}^{(2)}_{a(J,K)}[l]$. We will treat this simplified case of only transmitter or only receiver IQ imbalance at the end. However, for large transmitter and receiver IQ imbalance values, the estimates obtained from (22) and (23) may not be sufficiently accurate, resulting in only a partial compensation of the transmitter and receiver IQ imbalance. The same holds true for the estimates of the FI transmitter and receiver IQ imbalance gain parameters obtained from Eq. (21). From now on we will not further consider the FI case as the description of the FS case will also apply to the FI case.

To further improve the estimates for large IQ imbalance values, we apply an iterative procedure as follows. The $\mathbf{D}_{tot}[l]$ matrix (ignoring the superscripts corresponding to different channel realizations) is first compensated by the raw estimates of the receiver IQ imbalance gain parameters $\widetilde{\mathbf{Q}}_{rv_{tot}}[l]$ (defined in (18)), leading to a matrix $\mathbf{D}_{1_{tot}}[l]$, given as

$$\mathbf{D}_{1_{tot}}[l]=\widetilde{\mathbf{Q}}_{rv_{tot}}[l]\mathbf{D}_{tot}[l]=\widetilde{\mathbf{Q}}_{rv_{tot}}[l]\mathbf{Q}_{r_{tot}}[l]\mathbf{B}_{tot}[l]\mathbf{Q}_{t_{tot}}[l] \tag{24}$$

where

$$\widetilde{\mathbf{Q}}_{rv_{tot}}[l]\mathbf{Q}_{r_{tot}}[l]=\begin{bmatrix}(\mathbf{Q}_{rvr_{tot}}(1)[l]) & \ddots & \\ & & (\mathbf{Q}_{rvr_{tot}}(N_r)[l])\end{bmatrix}$$

and

$$\mathbf{D}_{1_{tot}}[l]=\begin{bmatrix}(\mathbf{D}_{1tot}(1,1)[l]) & \dots & (\mathbf{D}_{1tot}(1,N_t)[l])\\ \vdots & \ddots & \vdots\\ (\mathbf{D}_{1tot}(N_r,1)[l]) & \dots & (\mathbf{D}_{1tot}(N_r,N_t t)[l])\end{bmatrix}$$

The sub-matrices are defined as: $\mathbf{Q}_{rvr_{tot}}(J)[l]=\begin{pmatrix}(1-\widetilde{\mathbf{Q}}_{r(J)}[l]\mathbf{Q}^*_{r(J)}[l_m]) & (\mathbf{Q}_{r(J)}[l]-\widetilde{\mathbf{Q}}_{r(J)}[l])\\ (\mathbf{Q}^*_{r(J)}[l_m]-\widetilde{\mathbf{Q}}^*_{r(J)}[l_m]) & (1-\widetilde{\mathbf{Q}}^*_{r(J)}[l_m]\mathbf{Q}_{r(J)}[l])\end{pmatrix}$ and $\mathbf{D}_{1_{tot}}(J,K)[l]=\begin{pmatrix}\mathbf{D}_{a1(J,K)}[l] & \mathbf{D}_{b1(J,K)}[l]\\ \mathbf{D}^*_{b1(J,K)}[l_m] & \mathbf{D}^*_{a1(J,K)}[l_m]\end{pmatrix}$ In Eq. (24), where '1' is the iteration index, the definition of $\mathbf{D}_{1_{tot}}[l]$ is in line with $\mathbf{D}_{tot}[l]$ in Eq. (7). Thus the matrix $\mathbf{D}_{1_{tot}}[l]$ can once again be decoupled, similar to (13) as follows:

$$\mathbf{D}_{1_{tot}}[l]=\mathbf{Q}_{r1_{tot}}[l]\mathbf{B}_{1_{tot}}[l]\mathbf{Q}_{t1_{tot}}[l] \tag{25}$$

where

$$\mathbf{Q}_{r1_{tot}}[l]=\begin{bmatrix}(\mathbf{Q}_{r1_{tot}}(1)[l]) & \ddots & \\ & & (\mathbf{Q}_{r1_{tot}}(N_r)[l])\end{bmatrix},$$

$$\mathbf{B}_{1_{tot}}[l]=\begin{bmatrix}(\mathbf{B}_{1_{tot}}(1,1)[l]) & \dots & (\mathbf{B}_{1_{tot}}(1,N_t)[l])\\ \vdots & \ddots & \vdots\\ (\mathbf{B}_{1_{tot}}(N_r,1)[l]) & \dots & (\mathbf{B}_{1_{tot}}(N_r,N_t)[l])\end{bmatrix},$$

$$\mathbf{Q}_{t1_{tot}}[l]=\begin{bmatrix}(\mathbf{Q}_{t1_{tot}}(1)[l]) & \ddots & \\ & & (\mathbf{Q}_{t1_{tot}}(N_t)[l])\end{bmatrix}$$

and where $\mathbf{Q}_{r1_{tot}}(J)[l]=\begin{pmatrix}1 & \mathbf{Q}_{r1(J)}[l]\\ \mathbf{Q}^*_{r1(J)}[l_m] & 1\end{pmatrix}$, $\mathbf{B}_{1_{tot}}(J,K)[l]=\begin{pmatrix}\mathbf{B}_{1(J,K)}[l] & 0\\ 0 & \mathbf{B}^*_{1(J,K)}[l_m]\end{pmatrix}$ and $\mathbf{Q}_{t1_{tot}}(K)[l]=\begin{pmatrix}1 & \mathbf{Q}_{t1(K)}[l]\\ \mathbf{Q}^*_{t1(K)}[l_m] & 1\end{pmatrix}$ with $\mathbf{Q}_{t1(K)}[l]=\mathbf{Q}_{t(K)}[l]$, $\mathbf{B}_{1(J,K)}[l]=\mathbf{B}_{(J,K)}[l](1-\widetilde{\mathbf{Q}}^*_{r(J)}[l_m]\mathbf{Q}_r[l])$ and $\mathbf{Q}_{r1(J)}[l]=(\mathbf{Q}_{r(J)}[l]-\widetilde{\mathbf{Q}}_{r(J)}[l]/(1-\widetilde{\mathbf{Q}}^*_{r(J)}[l_m]\mathbf{Q}_{r(J)}[l])$. The receiver IQ imbalance gain parameter $\mathbf{Q}_{r1(J)}[l]$ is now much smaller than $\mathbf{Q}_r[l]$ in (13).

Based on the definition of $\mathbf{D}_{1_{tot}}[l]$ in (25), the coefficients $\mathbf{D}_{a1(J,K)}[l]$ and $\mathbf{D}_{b1(J,K)}[l]$ can now be written as

$$\begin{aligned}\mathbf{D}_{a1(J,K)}[l]&=\mathbf{B}_{1(J,K)}[l]+\mathbf{Q}_{r1(J)}[l]\mathbf{Q}^*_{t1(K)}[l_m]\mathbf{B}^*_{1(J,K)}[l_m]\\ \mathbf{D}_{b1(J,K)}[l]&=\mathbf{Q}_{t1(K)}[l]\mathbf{B}_{1(J,K)}[l]+\mathbf{Q}_{r1(J)}[l]\mathbf{B}^*_{1(J,K)}[l_m]\end{aligned} \tag{26}$$

which is similar to (12).

The estimates $\widetilde{\mathbf{D}}_{a1(J,K)}[l]$ and $\widetilde{\mathbf{D}}_{b1(J,K)}[l]$ of $\mathbf{D}_{a1(J,K)}[l]$ and $\mathbf{D}_{b1(J,K)}[l]$ can be obtained by replacing $\mathbf{D}_{tot}[l]$ with the estimate $\widetilde{\mathbf{D}}_{tot}[l]$ in (24), as follows:

$$\widetilde{\mathbf{D}}_{1_{tot}}[l]=\widetilde{\mathbf{Q}}_{rv_{tot}}[l]\widetilde{\mathbf{D}}_{tot}[l]=\begin{bmatrix}(\widetilde{\mathbf{D}}_{1_{tot}}(1,1)[l]) & \dots & (\widetilde{\mathbf{D}}_{1_{tot}}(1,N_t)[l])\\ \vdots & \ddots & \vdots\\ (\widetilde{\mathbf{D}}_{1_{tot}}(N_r,1)[l]) & \dots & (\widetilde{\mathbf{D}}_{1_{tot}}(N_r,N_t)[l])\end{bmatrix} \tag{27}$$

where $\widetilde{\mathbf{D}}_{1_{tot}}(J,K)[l]=\begin{pmatrix}\widetilde{\mathbf{D}}_{a1(J,K)}[l] & \widetilde{\mathbf{D}}_{b1(J,K)}[l]\\ \widetilde{\mathbf{D}}^*_{b1(J,K)}[l_m] & \widetilde{\mathbf{D}}^*_{a1(J,K)}[l_m]\end{pmatrix}$. If two or more independent channel realizations are available only for some antenna pairs, then the estimate of the sub-matrix $\mathbf{D}_{1tot}(J,K)[l]$, given as $\widetilde{\mathbf{D}}_{1tot}(J,K)[l]$, is derived by multiplying the sub-matrix $\widetilde{\mathbf{D}}_{tot}(J,K)[l]$ with sub-matrix $\widetilde{\mathbf{Q}}_{rv_{tot}}(J)[l]$ (defined in (18)). The multiplication results in the compensation of receiver IQ imbalance only on the sub-matrix $\widetilde{\mathbf{D}}_{tot}(J,K)[l]$.

Finally $\widetilde{\mathbf{Q}}_{r1(J)}[l]$ and the improved estimate $\widetilde{\mathbf{Q}}_{t1(K)}[l]$ of $\mathbf{Q}_{t(K)}[l]$ are obtained based on an expression similar to Eq. (22), with $\widetilde{\mathbf{D}}^{(1)}_{a(J,K)}[l]$, $\widetilde{\mathbf{D}}^{(2)}_{a(J,K)}[l]$ and $\widetilde{\mathbf{D}}^{(1)}_{b(J,K)}[l]$, $\widetilde{\mathbf{D}}^{(2)}_{b(J,K)}[l]$ replaced by $\widetilde{\mathbf{D}}^{(1)}_{a1(J,K)}[l]$, $\widetilde{\mathbf{D}}^{(2)}_{a1(J,K)}[l]$ and $\widetilde{\mathbf{D}}^{(1)}_{b1(J,K)}[l]$, $\widetilde{\mathbf{D}}^{(2)}_{b1(J,K)}[l]$. Eqs. (22) and (27) may be repeated a number of times until $\widetilde{\mathbf{Q}}_{ri(J)}[l]\simeq 0$, which corresponds to $\widetilde{\mathbf{D}}_{ai(J,K)}[l]\simeq\widetilde{\mathbf{B}}_{i(J,K)}[l]$, where i is the iteration index. It should be noted that the number of iterations required may vary for different antenna pairs, depending on the amount of transmitter and receiver IQ imbalance at different antennas. Thus after performing a sufficient number of iterations, the fine estimate of *receiver* IQ imbalance gain parameter $\widetilde{\mathbf{Q}}_{rf(J)}[l]$ can be derived from $\widetilde{\mathbf{Q}}_{ri(J)}[l]$ as

$$\widetilde{\mathbf{Q}}_{rf(J)}[l]=\frac{\mathbf{Q}_{r1(J)}[l]+\widetilde{\mathbf{Q}}_{r(J)}[l]}{1+\mathbf{Q}_{r1(J)}[l]\widetilde{\mathbf{Q}}^*_{r(J)}[l_m]} \tag{28}$$

where $\mathbf{Q}_{r1(J)}[l]=(\mathbf{Q}_{r2(J)}[l]+\widetilde{\mathbf{Q}}_{r1(J)}[l])/(1+\mathbf{Q}_{r2(J)}[l]\widetilde{\mathbf{Q}}^*_{r1(J)}[l_m])$ and so on.

It should be noted that Eq. (28) has been derived by reformulating the definition of $\mathbf{Q}_{r1(J)}[l]$ in Eq. (25). For the sake of convenience, we rewrite $\mathbf{Q}_{r1(J)}[l]=(\mathbf{Q}_{r(J)}[l]-\widetilde{\mathbf{Q}}_{r(J)}[l])/(1-\widetilde{\mathbf{Q}}^*_{r(J)}[l_m]\mathbf{Q}_{r(J)}[l])$. Now by reformulating the definition of $\mathbf{Q}_{r1(J)}[l]$ in terms of $\mathbf{Q}_{r(J)}[l]$, and then replacing $\mathbf{Q}_{r(J)}[l]$ by $\widetilde{\mathbf{Q}}_{rf(J)}[l]$ we get the fine estimate of receiver IQ imbalance as shown in (28). Eq. (28) in fact results in an

iterative formula where the definition of $\mathbf{Q}_{r1(J)}[l]$ can once again be obtained by (28), but now by replacing $\mathbf{Q}_{r1(J)}[l]$ by $\mathbf{Q}_{r2(J)}[l]$ and $\widetilde{\mathbf{Q}}_{r(J)}[l]$ by $\widetilde{\mathbf{Q}}_{r1(J)}[l]$ on the right side of the equation. This gives us $\mathbf{Q}_{r1(J)}[l]=(\mathbf{Q}_{r2(J)}[l]+\widetilde{\mathbf{Q}}_{r1(J)}[l])/(1+\mathbf{Q}_{r2(J)}[l]\,\widetilde{\mathbf{Q}}^{*}_{r1(J)}[l_m])$. Similarly the definition of $\mathbf{Q}_{r2(J)}[l]$ is obtained by replacing $\mathbf{Q}_{r2(J)}[l]$ by $\mathbf{Q}_{r3(J)}[l]$ and $\widetilde{\mathbf{Q}}_{r1(J)}[l]$ by $\widetilde{\mathbf{Q}}_{r2(J)}[l]$ on the right side of Eq. (28). This gives us $\mathbf{Q}_{r2(J)}[l]=(\mathbf{Q}_{r3(J)}[l]+\widetilde{\mathbf{Q}}_{r2(J)}[l])/(1+\mathbf{Q}_{r3(J)}[l]\,\widetilde{\mathbf{Q}}^{*}_{r2(J)}[l_m])$, and so on so forth.

The number of iterations to be considered depends directly on the expected severity of IQ imbalance as anticipated by the end-user. Thus the end-user can consider any number of iterations in the algorithm. For instance, in a two step iterative process, $\mathbf{Q}_{r2(J)}[l]$ is considered to be zero and therefore $\mathbf{Q}_{r1(J)}[l]=\widetilde{\mathbf{Q}}_{r1(J)}[l]$ and then the fine estimate of receiver IQ imbalance gain parameter is given as $\widetilde{\mathbf{Q}}_{rf(J)}[l]=(\widetilde{\mathbf{Q}}_{r1(J)}[l]+\widetilde{\mathbf{Q}}_{r(J)}[l])/(1+\widetilde{\mathbf{Q}}_{r1(J)}[l]\,\widetilde{\mathbf{Q}}^{*}_{r(J)}[l_m])$. The fine estimate of *transmitter* IQ imbalance gain parameter $\widetilde{\mathbf{Q}}_{tf(K)}[l]$ is obtained from the last iteration of $\widetilde{\mathbf{Q}}_{ti(K)}[l]$. Considering large number of iterations will always give the best possible estimates but will result in additional computational costs. Later in the simulation section it is shown that even for large amount IQ imbalance values, 3–4 iterations can already provide sufficiently good estimates.

To obtain estimates of IQ imbalance gain parameters for other antenna pairs $(J,\bar{\bar{K}})$ (lacking independent channel realizations), we multiply the sub-matrix $\widetilde{\mathbf{D}}_{tot}(J,\bar{\bar{K}})[l]$ with the sub-matrix containing the fine receiver IQ imbalance estimate $\widetilde{\mathbf{Q}}_{rvf_{tot}}(J)[l]=\begin{pmatrix}1 & -\widetilde{\mathbf{Q}}_{rf(J)}[l]\\ -\widetilde{\mathbf{Q}}^{*}_{rf(J)}[l_m] & 1\end{pmatrix}$ in Eq. (27). The resulting estimates $\widetilde{\mathbf{D}}_{a1(J,\bar{\bar{K}})}[l]$ and $\widetilde{\mathbf{D}}_{b1(J,\bar{\bar{K}})}[l]$ are then considered to be free of receiver IQ imbalance. Both $\widetilde{\mathbf{D}}_{a1(J,\bar{\bar{K}})}[l]$ and $\widetilde{\mathbf{D}}_{b1(J,\bar{\bar{K}})}[l]$ are then substituted in Eq. (23), to obtain the fine estimate of *transmitter* IQ imbalance gain parameter $\widetilde{\mathbf{Q}}_{tf(\bar{\bar{K}})}[l]$.

The estimate of *receiver* IQ imbalance gain parameter $\widetilde{\mathbf{Q}}_{rf(\bar{\bar{J}})}[l]$ is obtained by first multiplying the sub-matrix $\widetilde{\mathbf{D}}_{tot}(\bar{\bar{J}},K)[l]$ with $\widetilde{\mathbf{Q}}_{rv_{tot}}(\bar{\bar{J}})[l]$ in (27). The estimates $\widetilde{\mathbf{D}}_{a1(\bar{\bar{J}},K)}[l]$ and $\widetilde{\mathbf{D}}_{b1(\bar{\bar{J}},K)}[l]$ along with the fine estimate of transmitter IQ imbalance $\widetilde{\mathbf{Q}}_{tf(K)}[l]$ are then substituted in Eq. (23) to obtain a further improved estimate of receiver IQ imbalance gain parameter, now given as $\widetilde{\mathbf{Q}}_{r1(\bar{\bar{J}})}[l]$. Eq. (27) and (23) are repeated a number of times until $\widetilde{\mathbf{Q}}_{ri(\bar{\bar{J}})}[l]=0$. At this point the fine estimate of $\widetilde{\mathbf{Q}}_{rf(\bar{\bar{J}})}[l]$ can be obtained with (28).

Finally the fine estimates of all the transmitter and receiver IQ imbalance gain parameters can be substituted in Eqs. (14) and (18) to obtain the estimates of transmitted symbols $\widetilde{\mathbf{S}}_{tot}[l]$. A complete algorithm description for the

Table 2
D-FEQ algorithm for the estimation of transmitter and receiver IQ imbalance gain parameters.

1. For *FI transmitter/receiver IQ imbalance*, the raw estimates $\widetilde{Q}_{r(J)}$, $\widetilde{Q}_{t(K)}$ are computed with (21). Also obtain $\widetilde{Q}_{t(K)}$ and $\widetilde{Q}_{r(J)}$ estimates from other antenna pairs, i.e. $(\bar{\bar{J}},K)$ and $(J,\bar{\bar{K}})$ based on (21). Average all the estimates to obtain improved estimates of $\widetilde{Q}_{r(J)}$ and $\widetilde{Q}_{t(K)}$.

   For *FS transmitter/receiver IQ imbalance*, the raw estimates $\widetilde{Q}_{r(J)}$ and $\widetilde{Q}_{t(K)}$ are computed from at least two independent realizations of $\widetilde{\mathbf{D}}^{(1)}_{a(J,K)}[l]$, $\widetilde{\mathbf{D}}^{(2)}_{a(J,K)}[l]$ and $\widetilde{\mathbf{D}}^{(1)}_{b(J,K)}[l]$, $\widetilde{\mathbf{D}}^{(2)}_{b(J,K)}[l]$ based on Eq. (22).
2. In the case of FS IQ imbalance, compute the raw estimates for other antenna pairs (lacking independent channel realizations) $\widetilde{\mathbf{Q}}_{t(\bar{\bar{K}})}[l]$ and $\widetilde{\mathbf{Q}}_{r(\bar{\bar{J}})}[l]$ from $\widetilde{\mathbf{Q}}_{r(J)}[l]$ and $\widetilde{\mathbf{Q}}_{t(K)}[l]$ based on (23).
3. Compensate $\widetilde{\mathbf{D}}_{tot}[l]$ with the raw estimates of receiver IQ imbalance parameters $\widetilde{\mathbf{Q}}_{rv_{tot}}[l]$ in (27), and obtain the matrix $\widetilde{\mathbf{D}}_{i_{tot}}[l]$ with coefficients $\widetilde{\mathbf{D}}_{ai(J,K)}[l]$ and $\widetilde{\mathbf{D}}_{bi(J,K)}[l]$, where i is the iteration number. If two or more independent channel realizations are available only for some (J,K) antenna pairs, then derive the sub-matrix estimate $\widetilde{\mathbf{D}}_{1tot}(J,K)[l]$ by multiplying the sub-matrix $\widetilde{\mathbf{Q}}_{rv_{tot}}(J)[l]$ with the sub-matrix $\widetilde{\mathbf{D}}_{tot}(J,K)[l]$.
4. Obtain $\widetilde{\mathbf{Q}}_{ri(J)}[l]$ and $\widetilde{\mathbf{Q}}_{ti(K)}[l]$ by substituting the coefficients $\widetilde{\mathbf{D}}_{ai(J,K)}[l]$ and $\widetilde{\mathbf{D}}_{bi(J,K)}[l]$ in step 1.
5. Repeat steps 1, 3 and 4, until $\widetilde{\mathbf{Q}}_{ri(J)}[l]=0$.
6. Obtain the fine estimate $\widetilde{\mathbf{Q}}_{rf(J)}[l]$ with (28). The fine estimate $\widetilde{\mathbf{Q}}_{tf(K)}[l]$ is obtained from the last iteration $\widetilde{\mathbf{Q}}_{ti(K)}[l]$.
7. Obtain the fine estimate $\widetilde{\mathbf{Q}}_{tf(\bar{\bar{K}})}[l]$ for other transmit antenna branches by first multiplying $\widetilde{\mathbf{D}}_{tot}(J,\bar{\bar{K}})[l]$ with $\widetilde{\mathbf{Q}}_{rvf_{tot}}(J)[l]$, and then substituting the estimates $\widetilde{\mathbf{D}}_{a1(J,\bar{\bar{K}})}[l]$ and $\widetilde{\mathbf{D}}_{b1(J,\bar{\bar{K}})}[l]$ in (23).
8. Obtain the fine estimate of $\widetilde{\mathbf{Q}}_{rf(\bar{\bar{J}})}[l]$ for other receive antenna branches by first multiplying the sub-matrix $\widetilde{\mathbf{D}}_{tot}(\bar{\bar{J}},K)[l]$ with $\widetilde{\mathbf{Q}}_{rv_{tot}}(\bar{\bar{J}})[l]$. Substitute the estimates $\widetilde{\mathbf{D}}_{a1(\bar{\bar{J}},K)}[l]$, $\widetilde{\mathbf{D}}_{b1(\bar{\bar{J}},K)}[l]$ and $\widetilde{\mathbf{Q}}_{tf(K)}[l]$ in (23) to obtain an improved estimate of $\widetilde{\mathbf{Q}}_{r1(\bar{\bar{J}})}[l]$. Repeat (27) and (23) until $\widetilde{\mathbf{Q}}_{ri(\bar{\bar{J}})}[l]=0$. Obtain the fine estimate of $\widetilde{\mathbf{Q}}_{rf(\bar{\bar{J}})}[l]$ with (28).

estimation of transmitter and receiver IQ imbalance gain parameters is provided in Table 2.

In the presence of only transmitter ($\mathbf{Q}_{r(J)}[l]=0$) or only receiver IQ imbalance ($\mathbf{Q}_{t(K)}[l]=0$), the IQ imbalance gain parameter estimates are obtained directly from the coefficients $\widetilde{\mathbf{D}}_{a(J,K)}[l]$ and $\widetilde{\mathbf{D}}_{b(J,K)}[l]$, i.e. without the need for an iterative scheme. The channel estimate can then be written as $\mathbf{B}_{(J,K)}[l]=\mathbf{D}_{a(J,K)}[l]$ in Eq. (12). Thus for the case of only transmitter IQ imbalance, the estimate $\widetilde{\mathbf{Q}}_{t(K)}[l]$ is obtained as

$$\widetilde{\mathbf{Q}}_{t(K)}[l]=\frac{1}{N_r}\sum_{J=1}^{N_r}\frac{\widetilde{\mathbf{D}}_{b(J,K)}[l]}{\widetilde{\mathbf{D}}_{a(J,K)}[l]} \tag{29}$$

and for the case of only receiver IQ imbalance, the estimate $\widetilde{\mathbf{Q}}_{r(J)}[l]$ is obtained as

$$\widetilde{\mathbf{Q}}_{r(J)}[l]=\frac{1}{N_t}\sum_{k=1}^{N_t}\frac{\widetilde{\mathbf{D}}_{b(J,K)}[l]}{\widetilde{\mathbf{D}}^{*}_{a(J,K)}[l_m]} \tag{30}$$

5. Simulations

We have simulated an uncoded 16QAM 2×2 MIMO OFDM transmission to evaluate the proposed D-FEQ estimation/compensation schemes in the presence of transmitter and receiver IQ imbalance. A transmission packet consists of a sequence of LT symbols followed by a data sequence. The OFDM symbol length N=64, the cyclic prefix length $v=16$. We consider a quasi-static multipath channel that is L=4 taps long. The taps of the multipath channel are chosen independently with complex Gaussian distribution and their realization may be different from one packet to another. The performance comparison is made with the joint estimation/compensation scheme (11), with an ideal system with no front-end distortion and with a system with no compensation algorithm included. During the estimation phase of transmitter and receiver IQ imbalance gain parameters, we use M_l=10 linearly independent LT symbols, while M_l=4 linearly independent LT symbols are used to track the variation of the channel characteristics, as this is also the minimal requirement for the joint estimation/compensation scheme.

Fig. 3(a) and (b) shows the performance curves bit-error-rate (BER) vs signal-to-noise ratio (SNR) obtained for a MIMO OFDM system impaired by either only transmitter IQ imbalance or only receiver IQ imbalances. The BER performance results depicted are obtained by taking the average of the BER curves over 10^4 independent channel realizations. In Fig. 3(a), the transmitter IQ imbalance parameters are $\mathbf{h}_{ti(K)}$=[0.01, 0.5 0.06], $\mathbf{h}_{tq(K)}$=[0.06 0.5, 0.01], $g_{t(K)}$=10% and $\phi_{t(K)}=10^{\circ}$. Similarly in Fig. 3(b), the receiver IQ imbalance parameters are $\mathbf{h}_{ri(J)}$=[0.01, 0.5 0.06], $\mathbf{h}_{rq(J)}$=[0.06 0.5, 0.01], $g_{r(J)}$=10% and $\phi_{r(J)}=10^{\circ}$ respectively. We have applied the same IQ imbalance values across all antenna branches so as to maintain a simple simulation scenario. The IQ imbalance parameters are estimated directly from the $\widetilde{\mathbf{D}}_{tot-}[l]$ coefficients as shown in (29) and (30). Fig. 3(a) shows that the D-FEQ scheme with pre-distortion of transmitted symbols provides the most efficient compensation performance for a system impaired with transmitter IQ imbalance. The results obtained with the pre-distortion scheme D-FEQ$_{(P)}$ are very close to the ideal case with no IQ imbalance. This is because the pre-distortion scheme compensates for transmitter IQ imbalance even before the noise is added in the system. Thus for accurate estimation of transmitter IQ imbalance gain parameter, the compensation results in no noise enhancement and the performance is very close to the ideal case. The difference between the pre-distortion scheme and the joint compensation scheme is almost 12 dB at BER 10^{-2}. The performance is much better with D-FEQ based schemes because it essentially requires very few training symbols to track the channel variations once the IQ imbalance gain parameters are known. Thus the main difference in performance is due to the efficient utilization of training symbols. In the simulation, only M_l=4 LT symbols have been used by D-FEQ schemes to estimate the channel variation once the IQ imbalance gain parameters have been estimated by using M_l=10 LT symbols. The minimum requirement for estimating the channel variation in 2×2 MIMO system by D-FEQ scheme is only M_l=2 LT symbols. Thus there are twice as many training symbols to track the channel variations and hence noise averaging is superior, resulting in improved performance. Whereas the joint compensation scheme requires all M_l=4 LT symbols to estimate the compensation coefficients. As M_l=4 is also the minimal requirement, the noise averaging is poor. Improved performance results can be obtained with joint compensation scheme but at the cost of large training overhead.

Fig. 4(a) and (b) considers the presence of both transmitter and receiver IQ imbalance. The IQ imbalance parameters are $\mathbf{h}_{ti(K)}=\mathbf{h}_{ri(J)}$=[0.01, 0.5 0.06], $\mathbf{h}_{tq(K)}=\mathbf{h}_{rq(J)}$=[0.06 0.5, 0.01], $g_{t(K)}=g_{r(J)}$=10% and $\phi_{t(K)}=30^{\circ}$, $\phi_{r(J)}=25^{\circ}$ respectively. It should be noted that these imbalance levels may be much higher than those observed in a practical receiver. However we consider such an extreme case to evaluate the robustness/effectiveness of the proposed compensation schemes. For the estimation of the IQ imbalance gain parameters, we consider two independent channel realizations only for the antenna pair (J,K)=(1,1), while the remaining antenna pairs are considered to have a static channel characteristic. Thus the estimates of the IQ imbalance gain parameters for the remaining pairs are obtained based on the estimates obtained for the (J,K)=(1,1) antenna pair. We follow the algorithm explained in Table 2 for the estimation of the IQ imbalance gain parameters.

Fig. 4(a) illustrates the number of iterations required to perform adequate estimates of the IQ imbalance gain parameters $\mathbf{Q}_{t(1)}[l]$ and $\mathbf{Q}_{r(1)}[l]$ for the (1,1) antenna pair. The simulation results are obtained at SNR=40 dB. The figure shows the convergence of the transmitter and receiver IQ imbalance gain parameter estimates to their ideal values. The curves measure the IQ imbalance gain parameter estimates as the mean of the absolute values for all N subcarriers (i.e $(1/N)\sum_{l=1}^{N}\{|\widetilde{\mathbf{Q}}_{t(1)}[l]|\}$ and $(1/N)\sum_{l=1}^{N}\{|\widetilde{\mathbf{Q}}_{r(1)}[l]|\}$). It can be observed that 3–4 iterations can already provide sufficiently good estimates. The fine estimates of the IQ imbalance gain parameters for the remaining antenna pairs are also obtained within 3–4 iterations, based on the $\widetilde{\mathbf{Q}}_{t(1)}[l]$ and $\widetilde{\mathbf{Q}}_{r(1)}[l]$ estimates.

Fig. 4(b) once again shows the BER vs SNR performance for a system impaired with transmitter and receiver IQ imbalance. It can be seen that the proposed pre-distortion

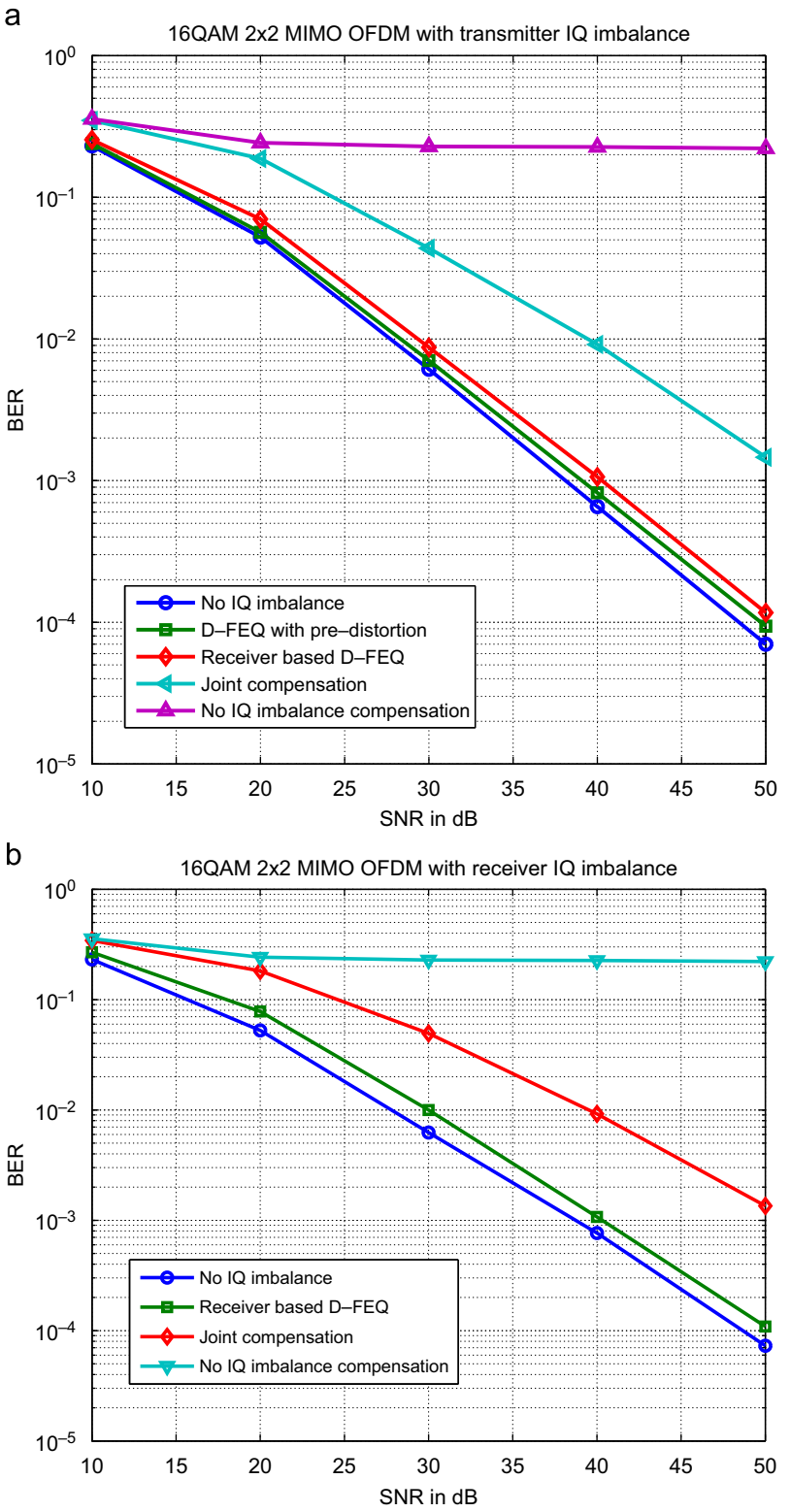


Fig. 3. BER vs SNR for an uncoded 16QAM 2×2 MIMO OFDM system. (a) D-FEQ scheme for transmitter IQ imbalance compensation, with $g_{t(K)}$=10%, $\phi_{t(K)}=10^{\circ}$, $\mathbf{h}_{ti(K)}$=[0.01, 0.5 0.06] and $\mathbf{h}_{tq(K)}$=[0.06 0.5, 0.01]. (b) D-FEQ scheme for receiver IQ imbalance compensation, with $g_{r(J)}$=10%, $\phi_{r(J)}=10^{\circ}$, $\mathbf{h}_{ri(J)}=$[0.01,0.5 0.06] and $\mathbf{h}_{rq(J)}$=[0.06 0.5, 0.01].

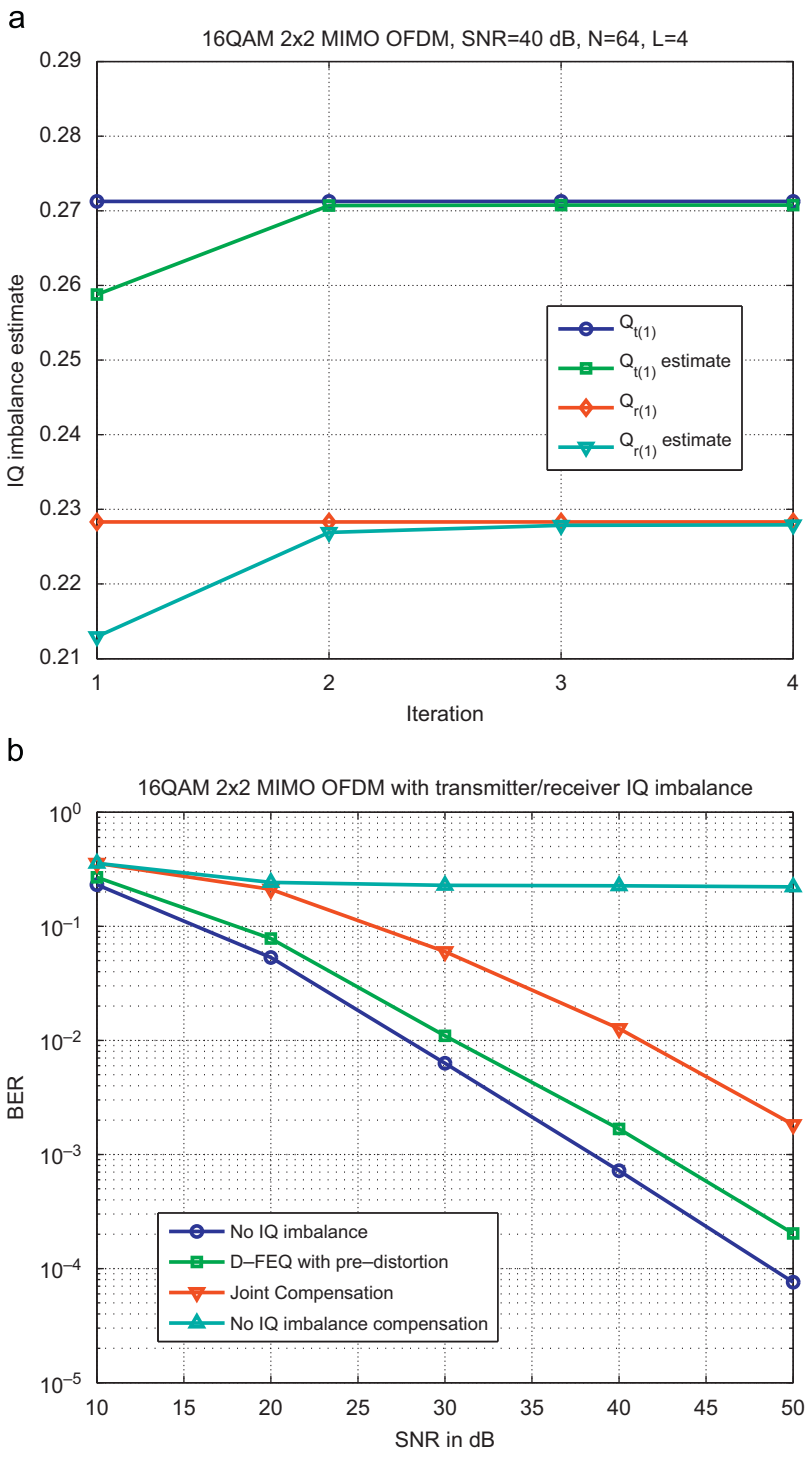


Fig. 4. Performance results for an uncoded 16QAM 2×2 MIMO OFDM system with transmitter and receiver IQ imbalance, with $g_{t(K)}=g_{r(J)}=10\%$, $\phi_{t(K)}=30^{\circ}, \phi_{rJ}=25^{\circ}$, $\mathbf{h}_{ti(K)}=\mathbf{h}_{ri(J)}=[0.01, 0.5\ 0.06]$ and $\mathbf{h}_{tq(K)}=\mathbf{h}_{rq(J)}=[0.06\ 0.5, 0.01]$. The transmitter and receiver IQ imbalance gain parameter estimates are obtained from two independent channel realizations for the antenna pair $(J,K)=(1,1)$. The channel characteristics of the remaining antenna pairs are considered static during the estimation phase of the IQ imbalance gain parameters.

based D-FEQ$_{(P)}$ compensation scheme is still very robust. The SNR difference between the proposed scheme and the joint compensation scheme is almost 12 dB at BER 10^{-2}. Thus the proposed compensation scheme provides a very efficient compensation even with a very low training overhead.

6. Conclusion

In this paper, we have proposed training based estimation/compensation schemes for a MIMO OFDM system impaired with transmitter and receiver IQ imbalance. The proposed schemes can decouple the compensation of transmitter and receiver IQ imbalance from the compensation of the channel distortion. Once the IQ imbalance parameters are known, then only the channel compensation parameters have to be re-estimated while the IQ imbalance compensation parameters are kept constant. The proposed schemes result in an overall lower training overhead and a lower computational requirement as compared to a joint estimation/compensation of the transmitter/receiver IQ imbalance and the channel distortion. Simulation results show that the proposed schemes provide a very efficient compensation with performance close to the ideal case without any IQ imbalance.

Appendix A

This appendix illustrates the transformation of an equation from the non-matrix form to the matrix form as represented in this paper. For the purpose of illustration we have considered Eq. (6) as it is also the most frequently referred equation in the paper. We hope that the reader should be able to transform any other equation from non-matrix to matrix form or vice versa based on this illustration.

We first re-write Eq. (6) for convenience:

$$\mathbf{Z}_{(J)} = \sum_{k=1}^{N_t} [\mathbf{D}_{a(J,K)} \cdot \mathbf{S}_{(K)} + \mathbf{D}_{b(J,K)} \cdot \mathbf{S}^*_{m(K)}] + \mathbf{N}_{c(J)} \tag{31}$$

Now in Eq. (31), the vectors $\mathbf{Z}_{(J)}, \mathbf{D}_{a(J,K)}, \mathbf{S}_{(K)}, \mathbf{D}_{b(J,K)}, \mathbf{S}^*_{m(K)}, \mathbf{N}_{c(J)}$ are of size $(N \times 1)$ and thus can be re-written for subcarrier $[l] = 1 \ldots N$ as

$$\mathbf{Z}_{(J)}[l] = \sum_{k=1}^{N_t} [\mathbf{D}_{a(J,K)}[l] \cdot \mathbf{S}_{(K)}[l] + \mathbf{D}_{b(J,K)}[l] \cdot \mathbf{S}^*_{(K)}[l_m]] + \mathbf{N}_{c(J)}[l] \tag{32}$$

where

$$\mathbf{Z}_{(J)}[l] = \begin{pmatrix} \mathbf{Z}_{(J)}[1] \\ \vdots \\ \mathbf{Z}_{(J)}[N] \end{pmatrix}, \quad \mathbf{D}_{a(J,K)}[l] = \begin{pmatrix} \mathbf{D}_{a(J,K)}[1] \\ \vdots \\ \mathbf{D}_{a(J,K)}[N] \end{pmatrix}, \quad \mathbf{D}_{b(J,K)}[l] = \begin{pmatrix} \mathbf{D}_{b(J,K)}[1] \\ \vdots \\ \mathbf{D}_{b(J,K)}[N] \end{pmatrix}$$

$$\mathbf{S}_{(K)}[l] = \begin{pmatrix} \mathbf{S}_{(K)}[1] \\ \vdots \\ \mathbf{S}_{c(K)}[N] \end{pmatrix}, \quad \mathbf{S}^*_{(K)}[l_m] = \begin{pmatrix} \mathbf{S}^*_{(K)}[N] \\ \vdots \\ \mathbf{S}^*_{(K)}[1] \end{pmatrix}, \quad \mathbf{N}_{c(J)}[l] = \begin{pmatrix} \mathbf{N}_{c(J)}[1] \\ \vdots \\ \mathbf{N}_{c(J)}[N] \end{pmatrix}$$

Eq. (32) can be written in matrix form for $K = 1 \ldots N_t$ as

$$\mathbf{Z}_{(J)}[l] = [(\mathbf{D}_{a(J,1)}[l] \ \mathbf{D}_{b(J,1)}[l]) \ \ldots \ (\mathbf{D}_{a(J,N_t)}[l] \ \mathbf{D}_{b(J,N_t)}[l])] \times \begin{bmatrix} \begin{pmatrix} \mathbf{S}_{(1)}[l] \\ \mathbf{S}^*_{(1)}[l_m] \end{pmatrix} \\ \vdots \\ \begin{pmatrix} \mathbf{S}_{(N_t)}[l] \\ \mathbf{S}^*_{(N_t)}[l_m] \end{pmatrix} \end{bmatrix} + \mathbf{N}_{c(J)}[l] \tag{33}$$

Similarly the complex conjugate of Eq. (31) can be re-written for subcarrier $[l] = 1 \ldots N$ as

$$\mathbf{Z}^*_{(J)}[l_m] = \sum_{k=1}^{N_t} [\mathbf{D}^*_{a(J,K)}[l_m] \cdot \mathbf{S}^*_{(K)}[l_m] + \mathbf{D}^*_{b(J,K)}[l_m] \cdot \mathbf{S}_{(K)}[l]] + \mathbf{N}^*_{c(J)}[l_m] \tag{34}$$

where

$$\mathbf{Z}^*_{(J)}[l_m] = \begin{pmatrix} \mathbf{Z}^*_{(J)}[N] \\ \vdots \\ \mathbf{Z}_{(J)}[1] \end{pmatrix}, \quad \mathbf{D}^*_{a(J,K)}[l_m] = \begin{pmatrix} \mathbf{D}^*_{a(J,K)}[N] \\ \vdots \\ \mathbf{D}^*_{a(J,K)}[1] \end{pmatrix}$$

$$\mathbf{D}^*_{b(J,K)}[l_m] = \begin{pmatrix} \mathbf{D}^*_{b(J,K)}[N] \\ \vdots \\ \mathbf{D}^*_{b(J,K)}[1] \end{pmatrix}, \quad \mathbf{N}^*_{c(J)}[l_m] = \begin{pmatrix} \mathbf{N}^*_{c(J)}[N] \\ \vdots \\ \mathbf{N}^*_{c(J)}[1] \end{pmatrix}$$

The matrix form of Eq. (34) can then be written as

$$\mathbf{Z}^*_{(J)}[l_m] = [(\mathbf{D}^*_{b(J,1)}[l_m] \ \mathbf{D}^*_{a(J,1)}[l_m]) \ \ldots \ (\mathbf{D}^*_{b(J,N_t)}[l_m] \ \mathbf{D}^*_{a(J,N_t)}[l_m])] \times \begin{bmatrix} \begin{pmatrix} \mathbf{S}_{(1)}[l] \\ \mathbf{S}^*_{(1)}[l_m] \end{pmatrix} \\ \vdots \\ \begin{pmatrix} \mathbf{S}_{(N_t)}[l] \\ \mathbf{S}^*_{(N_t)}[l_m] \end{pmatrix} \end{bmatrix} + \mathbf{N}^*_{c(J)}[l_m] \tag{35}$$

Eqs. (33) and (35) can now be merged and then written for $J = 1 \ldots N_r$ as

$$\underbrace{\begin{bmatrix} \begin{pmatrix} \mathbf{Z}_{(1)}[l] \\ \mathbf{Z}^*_{(1)}[l_m] \end{pmatrix} \\ \vdots \\ \begin{pmatrix} \mathbf{Z}_{(N_r)}[l] \\ \mathbf{Z}^*_{(N_r)}[l_m] \end{pmatrix} \end{bmatrix}}_{\mathbf{z}_{tot}[l]} = \underbrace{\begin{bmatrix} (\mathbf{D}_{tot}(1,1)[l]) & \ldots & (\mathbf{D}_{tot}(1,N_t)[l]) \\ \vdots & \ddots & \vdots \\ (\mathbf{D}_{tot}(N_r,1)[l]) & \ldots & (\mathbf{D}_{tot}(N_r,N_t)[l]) \end{bmatrix}}_{\mathbf{D}_{tot}[l]} \times \underbrace{\begin{bmatrix} \begin{pmatrix} \mathbf{S}_{(1)}[l] \\ \mathbf{S}^*_{(1)}[l_m] \end{pmatrix} \\ \vdots \\ \begin{pmatrix} \mathbf{S}_{(N_t)}[l] \\ \mathbf{S}^*_{(N_t)}[l_m] \end{pmatrix} \end{bmatrix}}_{\mathbf{s}_{tot}[l]} + \underbrace{\begin{bmatrix} \begin{pmatrix} \mathbf{N}_{c(1)}[l] \\ \mathbf{N}^*_{c(1)}[l_m] \end{pmatrix} \\ \vdots \\ \begin{pmatrix} \mathbf{N}_{c(N_r)}[l] \\ \mathbf{N}^*_{c(N_r)}[l_m] \end{pmatrix} \end{bmatrix}}_{\mathbf{N}_{c_{tot}}[l]} \tag{36}$$

where $\mathbf{D}_{tot}(J,K)[l] = \begin{pmatrix} \mathbf{D}_{a(J,K)}[l] & \mathbf{D}_{b(J,K)}[l] \\ \mathbf{D}^*_{b(J,K)}[l_m] & \mathbf{D}^*_{a(J,K)}[l_m] \end{pmatrix}$.

Eq. (36) is the frequency domain matrix form representation of the received symbol and its complex conjugate as shown in Eq. (7).

Appendix B

This appendix lists some of the regularly used matrix notations in the paper. As a general rule the estimates of a matrix notation $\mathbf{X}$ has been represent as $\widetilde{\mathbf{X}}$ throughout the paper.

S family of matrix notations:

$$\mathbf{S}_{tot}[l]=\begin{bmatrix}\begin{pmatrix}\mathbf{S}_{(1)}[l]\\ \mathbf{S}^{*}_{(1)}[l_m]\end{pmatrix}\\ \vdots\\ \begin{pmatrix}\mathbf{S}_{(N_t)}[l]\\ \mathbf{S}^{*}_{(N_t)}[l_m]\end{pmatrix}\end{bmatrix},\quad \mathbf{S}_{\mathbf{n}tot}[l]=\begin{bmatrix}\begin{pmatrix}\mathbf{S}_{\mathbf{n}(1)}[l]\\ \mathbf{S}^{*}_{\mathbf{n}(1)}[l_m]\end{pmatrix}\\ \vdots\\ \begin{pmatrix}\mathbf{S}_{\mathbf{n}(N_r)}[l]\\ \mathbf{S}^{*}_{\mathbf{n}(N_t)}[l_m]\end{pmatrix}\end{bmatrix}$$

$$\mathbf{S}^{Tr}_{tot-}[l]=\begin{bmatrix}\mathbf{S}^{(1)}_{(1)}[l] & \dots & \mathbf{S}^{(M_l)}_{(1)}[l]\\ \vdots & \ddots & \vdots\\ \mathbf{S}^{(1)}_{(N_t)}[l] & \dots & \mathbf{S}^{(M_l)}_{(N_t)}[l]\end{bmatrix}$$

Z family of matrix notations:

$$\mathbf{z}_{tot}[l]=\begin{bmatrix}\begin{pmatrix}\mathbf{Z}_{(1)}[l]\\ \mathbf{Z}^{*}_{(1)}[l_m]\end{pmatrix}\\ \vdots\\ \begin{pmatrix}\mathbf{Z}_{(N_r)}[l]\\ \mathbf{Z}^{*}_{(N_r)}[l_m]\end{pmatrix}\end{bmatrix},\quad \mathbf{Z}^{Tr}_{tot-}[l]=\begin{bmatrix}\mathbf{Z}^{(1)}_{(1)}[l] & \dots & \mathbf{Z}^{(M_l)}_{(1)}[l]\\ \vdots & \ddots & \vdots\\ \mathbf{Z}^{(1)}_{(N_r)}[l] & \dots & \mathbf{Z}^{(M_l)}_{(N_r)}[l]\end{bmatrix}$$

$$\mathbf{Z}^{Tr}_{tot}[l]=\begin{bmatrix}\begin{pmatrix}\mathbf{Z}^{(1)}_{(1)}[l]\\ \mathbf{Z}^{*(1)}_{(1)}[l_m]\end{pmatrix} & \cdots & \begin{pmatrix}\mathbf{Z}^{(M_l)}_{(1)}[l]\\ \mathbf{Z}^{*(M_l)}_{(1)}[l_m]\end{pmatrix}\\ \vdots & \ddots & \vdots\\ \begin{pmatrix}\mathbf{Z}^{(1)}_{(N_r)}[l]\\ \mathbf{Z}^{*(1)}_{(N_r)}[l_m]\end{pmatrix} & \cdots & \begin{pmatrix}\mathbf{Z}^{(M_l)}_{(N_r)}[l]\\ \mathbf{Z}^{*(M_l)}_{(N_r)}[l_m]\end{pmatrix}\end{bmatrix}$$

D family of matrix notations:

$$\mathbf{D}_{a(J,K)}[l]=\mathbf{B}_{(J,K)}[l]+\mathbf{Q}_{r(J)}[l]\mathbf{Q}^{*}_{t(K)}[l_m]\mathbf{B}^{*}_{(J,K)}[l_m]$$
$$\mathbf{D}_{b(J,K)}[l]=\mathbf{Q}_{t(K)}[l]\mathbf{B}_{(J,K)}[l]+\mathbf{Q}_{r(J)}[l]\mathbf{B}^{*}_{(J,K)}[l_m]$$
$$\mathbf{D}_{a1(J,K)}[l]=\mathbf{B}_{1(J,K)}[l]+\mathbf{Q}_{r1(J)}[l]\mathbf{Q}^{*}_{t1(K)}[l_m]\mathbf{B}^{*}_{1(J,K)}[l_m]$$
$$\mathbf{D}_{b1(J,K)}[l]=\mathbf{Q}_{t1(K)}[l]\mathbf{B}_{1(J,K)}[l]+\mathbf{Q}_{r1(J)}[l]\mathbf{B}^{*}_{1(J,K)}[l_m]$$

$$\mathbf{D}_{tot}[l]=\begin{bmatrix}(\mathbf{D}_{tot}(1,1)[l]) & \dots & (\mathbf{D}_{tot}(1,N_t)[l])\\ \vdots & \ddots & \vdots\\ (\mathbf{D}_{tot}(N_r,1)[l]) & \dots & (\mathbf{D}_{tot}(N_r,N_t)[l])\end{bmatrix}$$

$$\mathbf{D}_{tot}(J,K)[l]=\begin{pmatrix}\mathbf{D}_{a(J,K)}[l] & \mathbf{D}_{b(J,K)}[l]\\ \mathbf{D}^{*}_{b(J,K)}[l_m] & \mathbf{D}^{*}_{a(J,K)}[l_m]\end{pmatrix}$$

$$\mathbf{D}_{tot-}[l]=\begin{bmatrix}(\mathbf{D}_{tot-}(1,1)[l]) & \dots & (\mathbf{D}_{tot-}(1,N_t)[l])\\ \vdots & \ddots & \vdots\\ (\mathbf{D}_{tot-}(N_r,1)[l]) & \dots & (\mathbf{D}_{tot-}(N_r,N_t)[l])\end{bmatrix}$$

$$\mathbf{D}_{tot-}(J,K)[l]=(\mathbf{D}_{a(J,K)}[l]\ \ \mathbf{D}_{b(J,K)}[l])$$

$$\mathbf{D}_{1_{tot}}[l]=\begin{bmatrix}(\mathbf{D}_{1tot}(1,1)[l]) & \dots & (\mathbf{D}_{1tot}(1,N_t)[l])\\ \vdots & \ddots & \vdots\\ (\mathbf{D}_{1tot}(N_r,1)[l]) & \dots & (\mathbf{D}_{1tot}(N_r,N_tt)[l])\end{bmatrix}$$

$$\mathbf{D}_{1_{tot}}(J,K)[l]=\begin{pmatrix}\mathbf{D}_{a1(J,K)}[l] & \mathbf{D}_{b1(J,K)}[l]\\ \mathbf{D}^{*}_{b1(J,K)}[l_m] & \mathbf{D}^{*}_{a1(J,K)}[l_m]\end{pmatrix}$$

B family of matrix notations:

$$\mathbf{B}_{(J,K)}[l]=\mathbf{G}_{ra(J)}[l]\mathbf{G}_{ta(K)}[l]\mathbf{C}_{(J,K)}[l]$$

$$\mathbf{B}_{1(J,K)}[l]=\mathbf{B}_{(J,K)}[l](1-\overset{\sim}{\mathbf{Q}}^{*}_{r(J)}[l_m]\mathbf{Q}_{r}[l])$$

$$\mathbf{B}_{rt(J,K)}[l]=\mathbf{B}_{(J,K)}[l](1-\mathbf{Q}_{r(J)}[l]\overset{\sim}{\mathbf{Q}}^{*}_{r(J)}[l_m])(1-\mathbf{Q}_{t(K)}[l]\mathbf{Q}^{*}_{t(K)}[l_m])$$

$$\mathbf{B}_{r(J,K)}[l]=\mathbf{B}_{(J,K)}(1-\mathbf{Q}_{r(J)}[l]\mathbf{Q}^{*}_{r(J)}[l_m]);$$

$$\mathbf{B}_{tot}[l]=\begin{bmatrix}(\mathbf{B}_{tot}(1,1)[l]) & \dots & (\mathbf{B}_{tot}(1,N_t)[l])\\ \vdots & \ddots & \vdots\\ (\mathbf{B}_{tot}(N_r,1)[l]) & \dots & (\mathbf{B}_{tot}(N_r,N_t)[l])\end{bmatrix}$$

$$\mathbf{B}_{tot}(J,K)[l]=\begin{pmatrix}\mathbf{B}_{(J,K)}[l] & 0\\ 0 & \mathbf{B}^{*}_{(J,K)}[l_m]\end{pmatrix}$$

$$\mathbf{B}_{r_{tot-}}[l]=\begin{bmatrix}\mathbf{B}_{r(1,1)}[l] & \dots & \mathbf{B}_{r(1,N_t)}[l]\\ \vdots & \ddots & \vdots\\ \mathbf{B}_{r(N_r,1)}[l] & \dots & \mathbf{B}_{r(N_r,N_t)}[l]\end{bmatrix}$$

$$\mathbf{B}_{rt_{tot}}[l]=\begin{bmatrix}(\mathbf{B}_{rt_{tot}}(1,1)[l]) & \dots & (\mathbf{B}_{rt_{tot}}(1,N_t)[l])\\ \vdots & \ddots & \vdots\\ (\mathbf{B}_{rt_{tot}}(N_r,1)[l]) & \dots & (\mathbf{B}_{rt_{tot}}(N_r,N_t)[l])\end{bmatrix}$$

$$\mathbf{B}_{rt_{tot}}(J,K)[l]=\begin{pmatrix}\mathbf{B}_{rt(J,K)}[l] & 0\\ 0 & \mathbf{B}^{*}_{rt(J,K)}[l_m]\end{pmatrix}$$

$$\mathbf{B}_{rt_{tot-}}[l]=\begin{bmatrix}\mathbf{B}_{rt(1,1)}[l] & \dots & \mathbf{B}_{rt(1,N_t)}[l]\\ \vdots & \ddots & \vdots\\ \mathbf{B}_{rt(N_r,1)}[l] & \dots & \mathbf{B}_{rt(N_r,N_t)}[l]\end{bmatrix}$$

$$\mathbf{B}_{1_{tot}}[l]=\begin{bmatrix}(\mathbf{B}_{1_{tot}}(1,1)[l]) & \dots & (\mathbf{B}_{1_{tot}}(1,N_t)[l])\\ \vdots & \ddots & \vdots\\ (\mathbf{B}_{1_{tot}}(N_r,1)[l]) & \dots & (\mathbf{B}_{1_{tot}}(N_r,N_t)[l])\end{bmatrix}$$

$$\mathbf{B}_{1_{tot}}(J,K)[l]=\begin{pmatrix}\mathbf{B}_{1(J,K)}[l] & 0\\ 0 & \mathbf{B}^{*}_{1(J,K)}[l_m]\end{pmatrix}$$

$\mathbf{Q}_t$ family of matrix notations:

$$\mathbf{Q}_{t(K)}[l]=\frac{\mathbf{G}_{tb(K)}[l]}{\mathbf{G}_{ta(K)}[l]},\quad \mathbf{Q}_{t1(K)}[l]=\mathbf{Q}_{t(K)}[l]$$

$$\mathbf{Q}_{t_{tot}}[l]=\begin{bmatrix}(\mathbf{Q}_{t_{tot}}(1)[l]) & & \\ & \ddots & \\ & & (\mathbf{Q}_{t_{tot}}(N_t)[l])\end{bmatrix}$$

$$\mathbf{Q}_{t_{tot}}(K)[l]=\begin{pmatrix}1 & \mathbf{Q}_{t(K)}[l]\\ \mathbf{Q}^{*}_{t(K)}[l_m] & 1\end{pmatrix}$$

$$\mathbf{Q}_{t_{tot-}}[l]=\begin{bmatrix}(1\ \ \mathbf{Q}_{t(1)}[l]) & & \\ & \ddots & \\ & & (1\ \ \mathbf{Q}_{t(N_t)}[l])\end{bmatrix};$$

$$\mathbf{Q}_{tv_{tot}}[l]=\begin{bmatrix}(\mathbf{Q}_{tv_{tot}(1)}[l]) & & \\ & \ddots & \\ & & (\mathbf{Q}_{tv_{tot}(N_t)}[l])\end{bmatrix}$$

$$\mathbf{Q}_{tv_{tot}(K)}[l]=\begin{pmatrix}1 & -\mathbf{Q}_{t(K)}[l]\\ -\mathbf{Q}^{*}_{t(K)}[l_m] & 1\end{pmatrix}$$

$$\mathbf{Q}_{td_{tot}}[l]=\begin{bmatrix}(\mathbf{Q}_{td_{tot}}(1)[l]) & & \\ & \ddots & \\ & & (\mathbf{Q}_{td_{tot}}(N_t)[l])\end{bmatrix}$$

$$\mathbf{Q}_{td_{tot}}(K)[l]=\begin{bmatrix}(1-\mathbf{Q}_{t(K)}[l]\,\tilde{\mathbf{Q}}^{*}_{t(K)}[l_m]) & (\mathbf{Q}_t[l]-\tilde{\mathbf{Q}}_{t(K)}[l])\\ (\mathbf{Q}^{*}_{t(K)}[l_m]-\tilde{\mathbf{Q}}^{*}_{t(K)}[l_m]) & (1-\mathbf{Q}^{*}_{t}[l_m]\tilde{\mathbf{Q}}_{t(K)}[l])\end{bmatrix}$$

$$\mathbf{Q}_{t1_{tot}}[l]=\begin{bmatrix}(\mathbf{Q}_{t1_{tot}}(1)[l]) & & \\ & \ddots & \\ & & (\mathbf{Q}_{t1_{tot}}(N_t)[l])\end{bmatrix}$$

$$\mathbf{Q}_{t1_{tot}}(K)[l]=\begin{pmatrix}1 & \mathbf{Q}_{t1(K)}[l]\\ \mathbf{Q}^{*}_{t1(K)}[l_m] & 1\end{pmatrix}$$

$\mathbf{Q}_r$ family of matrix notations:

$$\mathbf{Q}_{r(J)}[l]=\frac{\mathbf{G}_{rb(J)}[l]}{\mathbf{G}^{*}_{za(J)}[l_m]},\quad \mathbf{Q}_{r1(J)}[l]=\frac{\mathbf{Q}_{r(J)}[l]-\tilde{\mathbf{Q}}_{r(J)}[l]}{1-\tilde{\mathbf{Q}}^{*}_{r(J)}[l_m]\mathbf{Q}_{r(J)}[l]}$$

$$\tilde{\mathbf{Q}}_{rf(J)}[l]=\frac{\mathbf{Q}_{r1(J)}[l]+\tilde{\mathbf{Q}}_{r(J)}[l]}{1+\mathbf{Q}_{r1(J)}[l]\,\tilde{\mathbf{Q}}^{*}_{r(J)}[l_m]}$$

$$\mathbf{Q}_{r_{tot}}[l]=\begin{bmatrix}(\mathbf{Q}_{r_{tot}}(1)[l]) & & \\ & \ddots & \\ & & (\mathbf{Q}_{r_{tot}}(N_r)[l])\end{bmatrix}$$

$$\mathbf{Q}_{r_{tot}}(J)[l]=\begin{pmatrix}1 & \mathbf{Q}_{r(J)}[l]\\ \mathbf{Q}^{*}_{r(J)}[l_m] & 1\end{pmatrix}$$

$$\mathbf{Q}_{rv_{tot-}}[l]=\begin{bmatrix}(1\ -\mathbf{Q}_{r(1)}[l]) & & \\ & \ddots & \\ & & (1\ -\mathbf{Q}_{r(N_r)}[l])\end{bmatrix}$$

$$\mathbf{Q}_{rv_{tot}}[l]=\begin{bmatrix}(\mathbf{Q}_{rv_{tot}}(1)[l]) & & \\ & \ddots & \\ & & (\mathbf{Q}_{rv_{tot}}(N_r)[l])\end{bmatrix}$$

$$\mathbf{Q}_{rv_{tot}}(J)[l]=\begin{pmatrix}1 & -\mathbf{Q}_{r(J)}[l]\\ -\mathbf{Q}^{*}_{r(J)}[l_m] & 1\end{pmatrix}$$

$$\mathbf{Q}_{r1_{tot}}[l]=\begin{bmatrix}(\mathbf{Q}_{r1_{tot}}(1)[l]) & & \\ & \ddots & \\ & & (\mathbf{Q}_{r1_{tot}}(N_r)[l])\end{bmatrix}$$

$$\mathbf{Q}_{r1_{tot}}(J)[l]=\begin{pmatrix}1 & \mathbf{Q}_{r1(J)}[l]\\ \mathbf{Q}^{*}_{r1(J)}[l_m] & 1\end{pmatrix}$$

$$\mathbf{Q}_{rvf_{tot}}(J)[l]=\begin{pmatrix}1 & -\mathbf{Q}_{rf(J)}[l]\\ -\mathbf{Q}^{*}_{rf(J)}[l_m] & 1\end{pmatrix}$$

$$\tilde{\mathbf{Q}}_{rv_{tot}}[l]\mathbf{Q}_{r_{tot}}[l]=\begin{bmatrix}(\mathbf{Q}_{rvr_{tot}}(1)[l]) & & \\ & \ddots & \\ & & (\mathbf{Q}_{rvr_{tot}}(N_r)[l])\end{bmatrix}$$

$$\mathbf{Q}_{rvr_{tot}}(J)[l]=\begin{pmatrix}(1-\tilde{\mathbf{Q}}_{r(J)}[l]\mathbf{Q}^{*}_{r(J)}[l_m]) & (\mathbf{Q}_{r(J)}[l]-\tilde{\mathbf{Q}}_{r(J)}[l])\\ (\mathbf{Q}^{*}_{r(J)}[l_m]-\tilde{\mathbf{Q}}^{*}_{r(J)}[l_m]) & (1-\tilde{\mathbf{Q}}^{*}_{r(J)}[l_m]\mathbf{Q}_{r(J)}[l])\end{pmatrix}$$

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