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A methodology for assessing eco-cruise control for passenger vehicles

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ABSTRACT

This paper compares and assesses fuel consumption models, cost functions, and solution methods, as they all have an influence on the resulting profile and associated fuel savings of an eco-cruise control system for passenger vehicles. An eco-cruise control system uses road topographical data obtained from a high-resolution digital map to control the vehicle velocity to optimize its fuel consumption. The optimal velocity profile is the result of an optimal control problem.

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1. Introduction

Adapting the vehicle velocity on hilly roads can lower the fuel consumption. This approach, used in heavy duty truck driving and known as predictive cruise control (PCC), is a system that uses the downstream road profile to actively change the vehicle velocity through a cruise controller. The approach can be adopted in driving a passenger vehicle (Ahn et al., 2011), where it is called eco-cruise control (ECC). This paper focuses on the underlying calculation methods of an eco-cruise control system to find an optimal velocity profile on a hilly road.

2. Eco-cruise control system components

Fig. 1 shows the scheme of an eco-cruise control system. The driver chooses the reference velocity v_{ref} [m/s] and a velocity band in which the vehicle velocity v [m/s] can vary ($v_{min} \leqslant v \leqslant v_{max}$). The eco-cruise control system also needs topographical information; the road slope θ [rad] can be obtained by a GPS system or a digital map. The optimal control unit calculates an optimal velocity profile that can be fed to a conventional cruise controller as the target velocity v_{cc} [m/s]. The optimal controller has three main components, choice of which will have a significant influence on the calculation time, ease of implementation, and potential consumption savings.

- *vehicle* mode: This contains a model of the longitudinal dynamics of the vehicle, powertrain, and fuel consumption.
- *cost function*: This function optimized; it includes the fuel consumption and e.g. a penalty for deviating from the reference velocity.
- *solution method*: Several solution methods are available.

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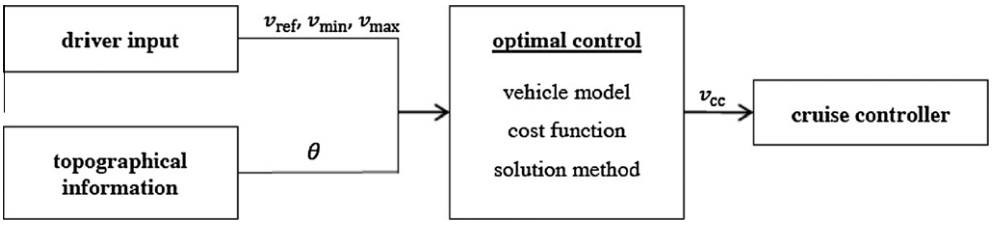


Fig. 1. Scheme of an eco-cruise system.

2.1. Vehicle model

The vehicle is considered a point mass and its motion the result of Newton's second law. Basic longitudinal dynamics that include inertia, road friction, aerodynamic drag, and road slope resistance are used. We assume there are no gear shifts, and that in case of an automatic transmission the power converter uses a lock-up. Given a velocity v and acceleration dv/dt, the engine power needed; P [W] is:

$$P = v \cdot \left(I \frac{dv}{dt} + c_0(\theta) + c_1(\theta) \cdot v + c_2 \cdot v^2 \right), \tag{1}$$

where I [kg] is inertia (vehicle mass and rotational inertia of the engine and driveline) and c_k [$\sim$] are lumped coefficients resulting from the longitudinal dynamics that can also include driveline friction. c_0 and c_1 can be a function of the road slope θ.

We use the VT-CPFM1 model (Rakha et al., 2011) to minimize fuel consumption;

$$\dot{m}_f = \begin{cases} \alpha_0 + \alpha_1 P + \alpha_2 P^2, & \text{if } P \geqslant 0, \\ \alpha_0, & \text{if } P < 0. \end{cases} \tag{2}$$

with α_k, model parameters. This model is able to estimate vehicle consumption rates consistent with in-field measurements ($R^2 > 0.9$) and can be calibrated using publicly available data. Low-degree polynomial fuel consumption models are commonly used for this type of application (Saerens et al., in press). For comparison and validation, a more detailed six-parameter polynomial fuel consumption model is also considered:

$$\dot{m}_f = \alpha_1 \omega + \alpha_2 \omega^2 + \alpha_3 \omega^3 + \alpha_4 \omega \tau + \alpha_5 \omega^2 \tau + \alpha_6 \omega \tau^2, \tag{3}$$

with $\tau = T/T_{max}(\omega)$ [–] the engine load,T[Nm] the engine torque, and ω the engine rotation speed. This model yields a good fit with experimental fuel consumption measurements ($R^2 = 0.99$), but the calibration requires the gathering of in-field vehicle fuel consumption data.

A combination of dynamics and consumption model specifies the optimal velocity v_{opt}:

$$\left. \frac{\partial \frac{\dot{m}_f(\omega, P_{ss}(v,\theta))}{v}}{\partial v} \right|_{v = v_{opt}} = 0, \tag{4}$$

where $P_{ss}(v, \theta)$ [W] is the necessary power to maintain a steady-state (constant) velocity v on a road with slope θ. The optimal velocity minimizes the fuel consumption per traveled distance.

2.2. Cost function

An eco-cruise control system allows a deviation from the reference velocity v_{ref} in a specified velocity band ($v_{min} \leqslant v \leqslant v_{max}$) to minimize the fuel consumption levels. Given a travel distance of s_e [m], this results in the optimization:

$$\min_{v(\cdot), s(\cdot), P(\cdot)} \int_0^{s_e} L \cdot ds, \quad L = \frac{\dot{m}_f}{v}, \tag{5}$$

with L [kg/m] the integral cost function. The cost function of Eq. (5) will ensure that the cruise controller drives the vehicle around the optimal velocity (bounded by v_{min} and v_{max}). This may result in an average velocity that can differ strongly from the reference velocity. For some drivers, this can be considered as unwanted behavior. One could prefer that the cruise controller allows the velocity to fluctuate around the reference velocity, resulting in an average velocity that more or less equals the reference velocity. Different cost functions are used in the literature to tackle the above-mentioned problem. For example Latteman et al. (2004) penalize a deviation from the reference velocity:

$$L = \frac{\dot{m}_f + \phi \cdot (v_{ref} - v)^2}{v}. \tag{6}$$

One can also use a time constraint to make the average velocity equal the set velocity:

$$L = \frac{\dot{m}_f + \mathcal{M}}{v}, \tag{7}$$

where $\mathcal{M}$ [kg/s] can be determined iteratively (Hellström et al., 2010). This would be a computationally demanding task. An easier solution is to use an indirect time constraint and determine $\mathcal{M}$ such that the steady-state velocity on a level road equals the reference velocity:

$$\left.\frac{\partial \frac{\dot{m}_f(\omega, P_{ss}(v,\theta)) + \mathcal{M}}{v}}{\partial v}\right|_{v=v_{\text{ref}},\theta=0} = 0. \tag{8}$$

This can be interpreted as an alteration of the fuel consumption model, such that the new optimal velocity equals the reference velocity on a level road. This new optimal velocity is defined as the steady-state cruising velocity $v_{ss}(\theta)$ [m/s] and depends on the road slope θ. On a level road this velocity equals the reference velocity: $v_{ss}(0) = v_{\text{ref}}$.

2.3. Solution method

The optimal velocity profile is the result of the following optimal control problem:

$$\begin{aligned} \min_{v(\cdot),s(\cdot),P(\cdot)} \quad & \int_0^{Se} L \cdot ds, \\ \text{s.t.}: \quad & \frac{dv}{ds} = \frac{\frac{P}{v} - c_0 - c_1 v - c_2 v^2}{Iv}, \\ & v_{\min} \leqslant v \leqslant v_{\max}. \end{aligned} \tag{9}$$

Generally speaking, there are three basic approaches to address optimal control problems: (1) dynamic programming, (2) direct methods, and (3) indirect methods (Pontryagin's maximum principle). Dynamic programming (DP), using the principle of optimality of subarcs, is deployed as a recursive feedback control (Bellman, 1957), with a discretized state grid being used.

To solve the eco-cruise control problem, the distance is divided into N steps of Δs [m] and the permitted velocity band is discretized with a grid size of Δv [m/s]. The basis of DP is a backward or forward iteration. In the case of a backward iteration we have $k = N-1, N-2, \ldots$:

$$J(s_k, v_l) = \min_{dv}(l(s_k, v_l, dv) + J(s_{k+1}, v_l + dv)), \tag{10}$$

where J [kg] is the cost-to-go function that gives the remaining minimum cost to go from a given distance s_k and velocity v_l to the end. The transition cost $l(s_k, v_l, dv)$ [kg] gives the cost to go from v_l at s_k to $v_l + dv$ at s_{k+1}, with dv a multiple of Δv. Here, this transition is calculated using the integral cost function L and Eq. (1) with the trapezoidal rule for integration.

Direct multiple shooting (DMS) transforms the optimal control problem of Eq. (9) into a new optimization problem (Bock and Plitt, 1984). The control P is discretized on a distance grid:

$$P(s) = P(k) \quad \text{for} \quad s \in [s_k, s_{k+1}]. \tag{11}$$

Values of the velocity at these grid points are defined as v_k. Although the controls are discretized, the cost function and the evolution of the velocity are evaluated with a continuous integration between grid points. The new optimization problem then looks as follows:

$$\begin{aligned} \min_{v(\cdot),s(\cdot),P(\cdot)} \quad & \sum_{k=0}^{N-1}\left(\int_{s_k}^{s_{k+1}} L(v, s, P_k) ds\right), \\ \text{s.t.}: \quad & v_0 = v_N = v_{\text{ref}}, \\ & v_{\min} \leqslant v \leqslant v_{\max}, \\ & v_{k+1} = v_k + \int_{s_k}^{s_{k+1}} \frac{\frac{P}{v} - c_0 - c_1 v - c_2 v^2}{Iv} ds. \end{aligned} \tag{12}$$

This is a non-linear and non-convex problem. Thus, this method does not guarantee that the global optimal solution is obtained. Therefore, it is necessary to start the sequential quadratic program that solves problem (12) with a good initial solution. A good candidate is the level road solution: $v_k = v_{ss}(0)$ and $P_k = P_{ss}(v_{\text{ref}})$.

Pontryagin's maximum principle (PMP) (Bryson and Ho, 1975) is an analytical method to solve optimal control problems. To solve the problem, first the Hamiltonian function $\mathcal{H}$ is defined. For illustration, the VT-CPFM1 model is used for the cost function in combination with an indirect time constraint. When using the explicit approach presented by Schwarzkopf and Leipnik (1977), one can express the optimal power P^* analytically:

$$P^* = P_{ss} \pm \sqrt{\frac{\alpha_0 + \alpha_1 P_{ss} + \alpha_2 P_{ss}^2 + \mathcal{M} + \lambda_s v}{\alpha_2}}. \tag{13}$$

With λ_s [kg/m] an adjoint variable. Now, λ_s has to be determined. This can be done in two ways: reactive and predictive.

For the reactive method, no knowledge of the upcoming road slope is used. The reactive eco-cruise control system (RECC) just reacts to the changing road slope and realizes an evolution towards $v_{ss}(\theta)$. For the current road slope θ, $P^*(v_{ss},\theta)$ should equal $P_{ss}(v_{ss},\theta)$. This yields:

$$\lambda_s(\theta) = -\frac{\alpha_0 + \alpha_1 P_{ss}(v_{ss},\theta) + \alpha_2 P_{ss}(v_{ss},\theta)^2 + \mathcal{M}}{v_{ss}}. \tag{14}$$

The reactive eco-cruise control law is then given by:

$$P^* = \begin{cases} P_{ss} + \sqrt{\frac{\alpha_0+\alpha_1 P_{ss}+\alpha_2 P_{ss}^2+\mathcal{M}+\lambda_s v}{\alpha_2}}, & \text{if } v < v_{ss}, \\ P_{ss}, & \text{if } v = v_{ss}, \\ P_{ss} - \sqrt{\frac{\alpha_0+\alpha_1 P_{ss}+\alpha_2 P_{ss}^2+\mathcal{M}+\lambda_s v}{\alpha_2}}, & \text{if } v > v_{ss}, \end{cases} \tag{15}$$

For the predictive method, knowledge of the upcoming road slope is used. The predictive eco-cruise control system (PECC) can thus take into account future road slope changes. Schwarzkopf and Leipnik (1977) present a method to calculate v_θ, the optimal velocity at the change of road grade. This method assumes that the length of the road section with the new slope is long enough that the new cruising velocity can be reached.

The predictive eco-cruise control law can be summarized as follows: Determine v_{ss} and λ_s for the next road slope, and v_θ for the road slope change. Calculate the distance that would be traveled to reach v_θ under the following control law:

$$P^* = \begin{cases} P_{ss}(v',\theta_1) + \sqrt{\frac{\alpha_0+\alpha_1 P_{ss}(v',\theta_1)+\alpha_2 P_{ss}(v',\theta_1)^2+\mathcal{M}+\lambda_s v'}{\alpha_2}}, & \text{if } v' < v_\theta, \\ P_{ss}(v',\theta_1) - \sqrt{\frac{\alpha_0+\alpha_1 P_{ss}(v',\theta_1)+\alpha_2 P_{ss}(v',\theta_1)^2+\mathcal{M}+\lambda_s v'}{\alpha_2}}, & \text{if } v' > v_\theta, \end{cases} \tag{16}$$

$$v' = \begin{cases} \max(v, (1+\epsilon)\cdot v_{ss}), & \text{if } v > v_{ss}, \\ \max(v, (1-\epsilon)\cdot v_{ss}), & \text{if } v \leqslant v_{ss}, \end{cases}$$

With ϵ a small value, e.g. $\epsilon = 0.01$. If this distance is larger than the distance to the actual road slope change, follow the control law of Eq. (16). Otherwise, follow the control law of Eq. (15) until this distance equals the real distance to the road grade change, then follow (16).

3. Eco-cruise control system assessment

The optimal velocity profile and the fuel consumption are calculated over a road section, with fuel consumption evaluated using the detailed six-parameter model of Eq. (3). The simulated vehicle is a 2004 Toyota Corolla Verso with a l gasoline engine, and a fuel consumption of 9.5 and 6.41/100 km on the European city and highway cycles.

Four fuel consumption models are taken into account:

- detailed six-parameter model of Eq. (3),
- VT-CPFM1 model of Eq. (2),

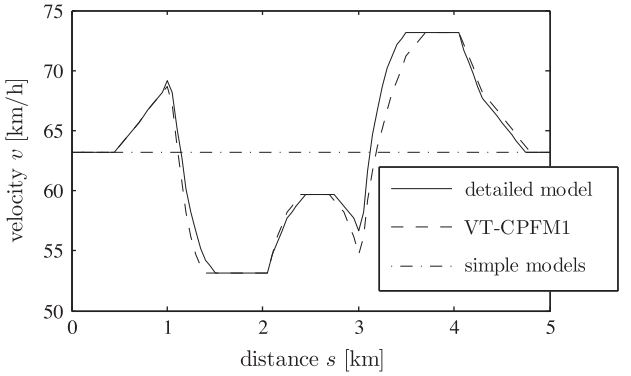


Fig. 2. Comparison of velocity on a simple road profile.

- single-parameter consumption model: $\dot{m}_f = \alpha P$ (Chang and Morlok, 2005),
- two-parameter consumption model: $\dot{m}_f = \alpha_0 \omega + \alpha_1 P$ (Hellström et al., 2010).

First, a simple road profile is considered consisting of five 1 km sections: level, uphill (3°), level, downhill (−3°), and level. The detailed model results in an optimal velocity of 63 km/h, the VT-CPFM1 model 60 km/h. The simple models (single- and two-parameter) both show an optimal velocity of 31 km/h. Optimal velocity profiles are seen in Fig. 2, calculated using discrete dynamic programming, a reference velocity of $v_{\text{ref}} = v_{\text{opt}} = 63$ km/h, $v_{\min} = 53$ km/h, $v_{\max} = 73$ km/h, and an indirect time constraint. The profile of the detailed model and the VT-CPFM1 model are almost the same, while the simple power-based models result in constant velocity profiles. Only on steep slopes will the latter result in an optimal velocity profile that is not constant (Fröberg et al., 2006). A steep downhill slope sees a vehicle accelerate when the engine does not produce power, while a steep uphill slope results in a deceleration with the engine at maximum power. Passenger vehicles have a higher engine power to mass ratio than heavy duty trucks with steep slopes rarely occurring and, in consequently, there are benefits in using fuel consumption models that capture optimal velocity changes on slopes that are not steep.

To do a representative assessment with regards to the fuel consumption, a simulation is done based on a section of Interstate 81 from Roanoke to Blacksburg in Virginia, US. The section has grades between −4% and 4% and does not include steep slopes. The simple models result in a constant velocity and a fuel consumption of 2186.8 g. The VT-CPFM1 and detailed model results in consumptions of 2116.0 g and 2115.7 g: negligible differences.

From the simulations it can be concluded that the VT-CPFM1 model is appropriate for use in ECC calculations. An extra validation is done on a universal engine dynamometer. A simple road profile is considered: 1 km uphill (2°), 1 km downhill (−2°), 1 km level. Two velocity profiles are compared: (1) conventional cruise control ($v_{\text{ref}} = 70$ km/h) and (2) eco-cruise control (VT-CPFM1, PMP reactive, $v_{\text{ref}} = 70$ km, indirect time constraint). The velocity profiles are tracked with a PI-controller that controls the throttle valve of the engine. Fig. 3 shows the results. The conventional cruise controller (CCC) results in a consumption of 172.1 g and a travel time of 205.5 s. The ECC results in a consumption of 167.0 g and a travel time of 205.4 s. This indicates that the VT-CPFM1 fuel consumption model is appropriate for use in ECC calculations.

We take a cost function that includes fuel consumption $\dot{m}_f$, a deviation penalty ϕ [kg s/m^2], and an indirect time constraint $\mathcal{M}$ [kg/s]:

$$L = \frac{\dot{m}_f + \phi \cdot (v_{\text{ref}} - v)^2 + \mathcal{M}}{v}. \tag{17}$$

First, $v_{\text{ref}} = v_{\text{opt}} = 63$ km/h is considered ($v_{\min} = 53$ km/h and $v_{\max} = 73$ km/h). A small indirect time constraint is needed ($\mathcal{M} = 0.048$ kg/s) to compensate for the difference in optimal velocity between the control model (VT-CPFM1) and the detailed model used for fuel consumption calculations. Yet, in what follows this will be considered as no time constraint, since $v_{\text{ref}} = v_{\text{opt}}$. Fig. 4 shows a comparison of two velocity profiles. The optimal velocity profile is calculated with DP for the same road profile as before. It is clear that when the deviation from the reference velocity is too large, a deviation penalty can be used. Numerical results are given in Table 1.

Next, we consider the case where $v_{\text{opt}} > v_{\text{ref}} = 50$ km/h; ($v_{\min} = 40$ km/h and $v_{\max} = 60$ km/h). Fig. 5 shows a comparison of three cost function. When no deviation penalty or time constraint is used, the velocities are relatively high. With a deviation penalty, the velocities will be lower, but are on average still higher than v_{ref}. With an indirect time constraint, the velocities will vary around v_{ref}.

Where $v_{\text{ref}} = v_{\text{opt}}$, having no deviation penalty gives the lowest fuel consumption. If $v_{\text{ref}} \neq v_{\text{opt}}$, no deviation penalty and no time constraint gives the lowest fuel consumption. No deviation penalty and a time constraint implies the highest fuel consumption. One could argue that the appropriate cost function to use is the first, but this results in an average velocity $\bar{v}$ km/h that differs the most from the reference velocity, and the gain in fuel economy mainly comes from the fact that the average velocity is closer to the optimal velocity. This is illustrated quantitatively on the I81 section. If the section is driven at a

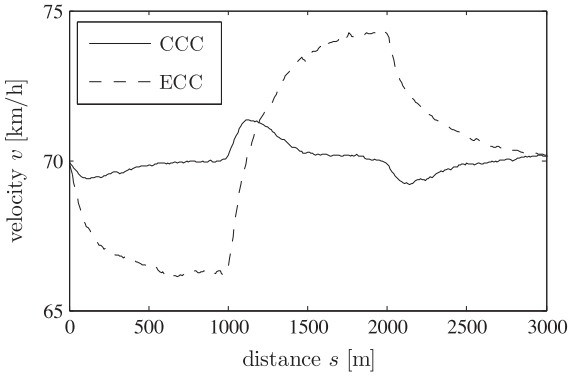


Fig. 3. Comparison of a conventional cruise controller and an eco-cruise controller on a universal dynamometer.

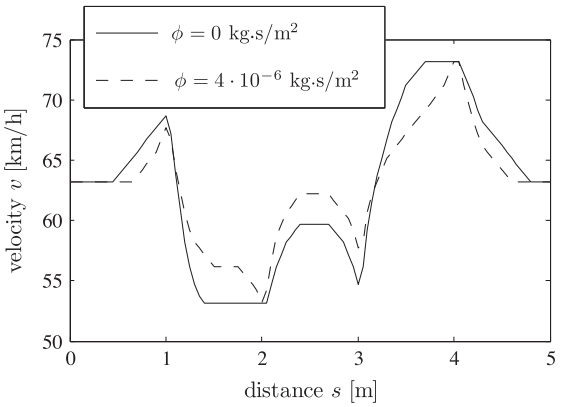


Fig. 4. Comparison of the velocity profile with different cost functions, with $v_{ref} = v_{opt}$.

Table 1
Comparison of cost functions.

v_{ref} [km/h]	ϕ [kg s/m^2]	$\mathcal{M}$ [g/s]	m_f [g]	$\bar{v}$ [km/h]
63	0×10^{-6}	0.0477	222.21	62.10
	4×10^{-6}	0.0477	223.34	62.57
50	0×10^{-6}	−0.1477	233.04	48.66
	0×10^{-6}	0	227.20	55.40
	4×10^{-6}	0	228.66	53.82
80	0×10^{-6}	0.5428	235.71	79.01
	0×10^{-6}	0	229.68	73.07
	4×10^{-6}	0	230.75	74.72

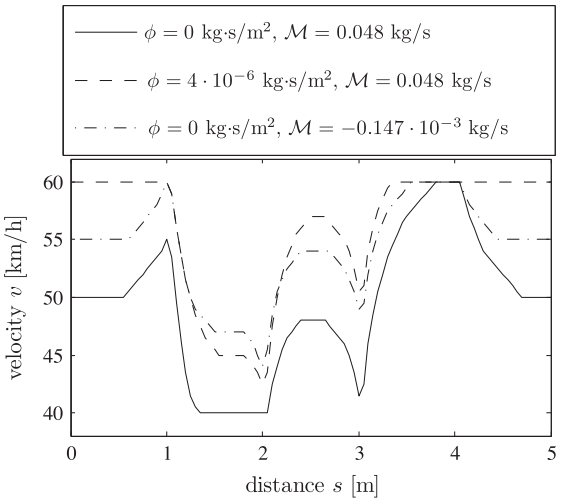


Fig. 5. Comparison of the velocity profile with different cost functions, with $v_{ref} < v_{opt}$.

constant velocity $v = 104$ km/h, the fuel consumption is $m_f = 3145.9$ g. Eco-cruise control with an indirect time constraint on this section with $v_{ref} = 104$ km/h, $v_{min} = 94$ km/h, and $v_{max} = 114$ results in $m_f = 2992.3$ g and $v = 102.1$ km/h. Without a time constraint, $m_f = 2738.6$ g and $\bar{v} = 94.7$ km/h. Here, most of the fuel savings is a result of the lower velocity, knowing that driving at a constant velocity $v = 94$ km/h yields a consumption of $m_f = 2740.1$ g. If the driver does not mind driving at a lower average velocity, it would be better to set the cruise controller to $v_{ref} = 94$ km/h, $v_{min} = 84$ km/h, and $v_{max} = 104$ km/h. These settings result in $m_f = 2606.2$ g and $\bar{v} = 92.0$ km/h.

Thus an indirect time constraint should be used, keeping in mind that a reference velocity closer to the optimal velocity will result in a better fuel economy. If deviations from the reference velocity are considered to be too large, an additional deviation penalty can be used.

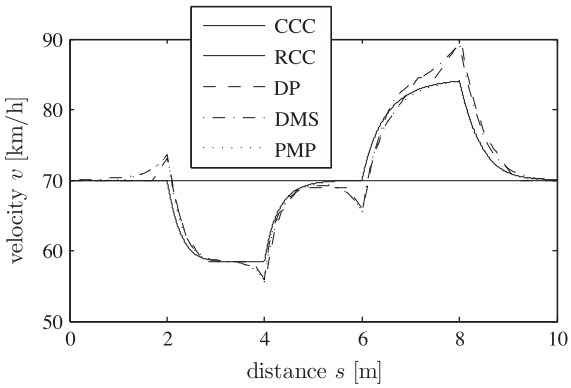


Fig. 6. Comparison of the velocity profile with different solution methods.

The following solution methods are compared:

- CCC: Perfect conventional cruise control that keeps the vehicle at a constant velocity, the reference case.
- RECC: Reactive eco-cruise control, calculated with the maximum principle.
- PECC-DP: Predictive eco-cruise control, calculated with dynamic programming.
- PECC-DMS: Predictive eco-cruise control, calculated with direct multiple shooting.
- PECC-PMP: Predictive eco-cruise control, calculated with the maximum principle.

First, consider the same simple road profile, yet with 2 km sections instead of 1 km. Fig. 6 shows a comparison of the velocity profiles for different methods. No velocity constraints are applied. Up to 6 km, PECC-DMS and PECC-PMP have the same profile, except for some wrinkles on the DMS profile caused by numerical errors. After 6 km, the profiles diverge. This is on the downhill section between 6 and 8 km. The section is too short to reach the new cruising velocity. The PECC-DP profile is a rough approximation of the PECC-DMS profile, due to the rough discretization in the DP algorithm. The RECC profile does not anticipate future road slopes. The fuel consumptions do not differ a lot for the ECC methods, thus when it is used on a road without steep slopes, anticipating the future road slope induces little fuel economy.

4. Conclusions

We have discussed calculation methodologies for finding optimal velocity profiles for eco-cruise control for passenger vehicles; the main components of an eco-cruise control system being fuel consumption, cost function, and solution method. The VT-CPFM1 fuel consumption model is appropriate. This finding is based on a comparison with a more detailed polynomial consumption model and a validation on a universal engine dynamometer. Besides the fuel consumption, an indirect time constraint should be added to the cost function. The driver should take into account that the closer the reference velocity is to the optimal velocity, the better the fuel economy will be. For vehicles that drive on roads without steep slopes, anticipating the road slope does not enhance the vehicle fuel economy significantly. Because of its simplicity and fast calculation, a reactive method based on Pontryagin's maximum principle is a good method for calculating the optimal velocity profile. In a simulation of a 45 km section of the I81 in Virginia, an eco-cruise controller is able to save 5% of fuel, compared to a conventional cruise controller with a reference velocity of 104 km/h.

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