Dynamic Bandplanning for Vectored DSL

Amir R. Forouzan, Marc Moonen, Jochen Maes, and Mamoun Guenach

Abstract

There are two types of crosstalk in digital subscriber line (DSL) systems, namely near-end crosstalk (NEXT) and far-end crosstalk (FEXT). NEXT is usually much stronger than FEXT. Therefore, high-speed DSL systems transmit in preplanned disjoint downstream (DS) and upstream (US) frequency bands to avoid NEXT. Although easy for implementation, such a fixed bandplan can lead to inefficient bandwidth usage, depending on the DS and US bit rate requirements and the loop topology, particularly in so-called vectored DSL systems, which include signal coordination for FEXT cancellation. In dynamic bandplanning (DBP), each frequency band is allocated to either DS, US, or to both directions depending on the aforementioned parameters. In this paper, we consider optimal DBP for vectored DSL with linear as well as with nonlinear transmitter/receiver structures. We propose an optimal DBP algorithm for systems with disjoint DS and US bands. For systems with overlapping bands, the problem of finding the optimal transmitter/receiver filters is nonconvex and we propose two iterative algorithms based on recent optimum

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spectrum balancing schemes from the literature. We also study the effect of echo and US NEXT cancellation on the system performance. Finally, simulation results are provided to demonstrate that our algorithms can indeed significantly increase the achievable bit rates.

Index Terms—Bandplanning, crosstalk cancellation, digital subscriber line (DSL), echo cancellation, FEXT, NEXT, resource allocation, vectoring.

I. Introduction

One of the main impairments of digital subscriber line (DSL) systems is crosstalk. Due to crosstalk, i.e., the electromagnetic induction between the twisted wire pairs inside a telephone cable, the signal transmitted by one DSL modem is also received by another (victim) modems. There are two types of crosstalk in DSL systems, namely, near-end crosstalk (NEXT) and far-end crosstalk (FEXT). NEXT is the crosstalk received by a (victim) modem from a transmitting modem located at the same cable end, while FEXT is the crosstalk received from a transmitting modem located at the other end of the cable (Fig. 1). NEXT is usually much stronger than FEXT, because FEXT travels a much longer distance in the cable and hence undergoes a much larger attenuation.

In recent years, the bit rate requirement of broadband services has experienced a significant boost mainly because of the increasing popularity of video-on-demand, live streaming, and on-line gaming. It has been estimated that the sum DS and US bit rate requirement of a residential customer could easily reach a few hundred Mbps up to one Gbps in a few years. The capacity of the twisted wire pair is not sufficient to deliver these bit rates over very long distances. Therefore, future DSL systems will connect consumers to the nearby cabinets in the streets located no farther than a few hundred meters. The DSL network is then connected to the backbone network using fiber-to-the curb technology. Such a topology is still much more cost efficient than a complete fiber-to-the home topology.

Earlier DSL systems used single-carrier modulation and overlapping frequency bands for the DS and US transmission. For example, SHDSL [1] used a frequency band from near DC to about 280 kHz (the 3-dB bandwidth) in both directions. However, as NEXT coupling is small at these low frequencies, NEXT did not have a catastrophic impact on the performance. Advanced high-speed DSL systems, however, use much higher frequencies. For example, VDSL2 E30 [2, 3] uses up to 30 MHz of bandwidth. At these frequencies, NEXT can easily cease the communication. In full duplex transmission, echo can even be much stronger than NEXT and hence should also be avoided or canceled. Therefore, current DSL systems use disjoint DS and US bands to avoid NEXT and echo. This well-known technique is called frequency-division duplexing (FDD)[1].

[1]A similar technique is time-division duplexing (TDD) in which DS and US transmission take place

When NEXT and echo are avoided, FEXT becomes the dominant impairment. Recently, MIMO crosstalk cancellation techniques, a.k.a. vectoring techniques, have attracted a lot of attention for FEXT cancellation [4, 5]. Using state-of-the art vectoring techniques [6–10], FEXT can be canceled almost perfectly and it is possible to closely approximate FEXT-free performance.

When FEXT is effectively canceled, the next promising step towards increasing the achievable bit rates in DSL systems can be dynamic bandplanning (DBP). In DBP, each frequency band is allocated to either US, DS, or both directions depending on the channel conditions and instant bit rate requirements.

In this paper, we consider optimal DBP for vectored DSL with linear as well as with nonlinear transmitter/receiver structures. We consider DBP for systems with disjoint DS and US bands as well as for systems with overlapping bands. Moreover, since the transmitted symbols in the DS direction are available at the line-termination (LT) side, it is possible to remove US NEXT using vectoring, and so we will also consider echo and US NEXT cancellation for systems with overlapping bands. We assume analog echo reduction (e.g., using hybrid circuits [11]) as well as echo cancellation in the digital domain. Perfect echo cancellation is possible in the digital domain, however, analog-to-digital converters with a higher resolution are needed.

We propose an optimal DBP algorithm for systems with disjoint bands. For systems with overlapping bands, finding the optimal vectoring scheme is a non-convex problem. We propose two iterative algorithms to solve this problem. In the first algorithm, the DS and US optimization are separated. The US optimization is based on the MAC-OSB algorithm [9] and the DS optimization is based on the BC-OSB algorithm [10]. In the second algorithm, the DS and US optimization are not completely separated. When the US parameters (US transmit powers and receiver filters) are optimized, DS transmit powers are also optimized using the IF/MAC-OSB algorithm [12]. Similarly, when the DS parameters (DS transmit powers and transmitter filters) are optimized, US transmit powers are also optimized using the IF/BC-OSB algorithm [13]. We compare the performance and

at different time slots. Echo can be avoided using TDD but in order to also avoid NEXT, the users have to be synchronized such that all use the same time slots for DS and US transmission. For this reason and a number of other reasons such as a smaller latency, current DSL systems use FDD.

computational complexity of these algorithms using computer simulations and point out a few techniques for their computational complexity reduction. Our simulation results show that these algorithms can increase the achievable bit rates significantly, particularly when echo and US NEXT cancellation are used.

Although in this paper we focus on DBP for vectored DSL, it should be noted that benefits of DBP are not merely limited to vectored DSL. Basically, DBP can be used for systems in which the PSD of the users is static or systems in which spectrum balancing is feasible among users [14].

The paper is organized as follows. In Sec. II, we describe the DSL transmission model and review some results on vectoring. Then, we focus on the DBP problem and propose DBP algorithms for systems with disjoint and overlapping DS/US bands in Sec. III. Simulation results are provided in Sec. IV. Finally the paper is concluded in Sec. V.

A note on math style: In this paper, all parameters are typeset in italics except those in Greek letters which are always in regular form. Functions such as $\min\{\cdot\}$ and operators such as $(\cdot)^{\mathrm{H}}$ are in regular form. Text and text abbreviations, such as DS in $\boldsymbol{H}_{\mathrm{DS}}^{k}$, are in regular form too. Matrices and vectors are bold and italic, where matrices are shown by capital letters and vectors by lower case letters. The element at row n and column m of matrix $\boldsymbol{A}$ is denoted by $a_{n,m}$ or $[\boldsymbol{A}]_{n,m}$ and $\boldsymbol{a}_n$ denotes the n-th column of $\boldsymbol{A}$. Similarly, v_n denotes the n-th element of vector $\boldsymbol{v}$. Matrices $\boldsymbol{I}_N$ and $\boldsymbol{0}_N$ are the identity and all zero matrix of size N and $\boldsymbol{e}_{N,n} \equiv (\overbrace{0,\ldots,0}^{n-1 \text{ times}},1,\overbrace{0,\ldots,0}^{N-n \text{ times}})^{\mathrm{T}}$ (or simply $\boldsymbol{e}_n$) is the n-th basis vector of size N (or of appropriate size). Integer k denotes the tone index and is always used as a superscript, whereas n and m indicate users and are used as subscripts. The transpose and conjugate transpose of a matrix or a vector are denoted by $(\cdot)^{\mathrm{T}}$ and $(\cdot)^{\mathrm{H}}$, respectively. If $\boldsymbol{v}$ is a vector of size N, then $\mathrm{diag}\{\boldsymbol{v}\}$ denotes an $N \times N$ matrix with diagonal elements equal to $\boldsymbol{v}$ and $\|\boldsymbol{v}\|_2$ denotes its ℓ_2 norm. Often we make similar statements for parameters defined in DS and US. For brevity, we use "DS/US" to refer to those parameters. For example instead of "$\boldsymbol{x}_{\mathrm{DS}}$ is the DS transmitted vector and $\boldsymbol{x}_{\mathrm{US}}$ is the US transmitted vector" we state "$\boldsymbol{x}_{\mathrm{DS/US}}$ is the DS/US transmitted vector".

II. System Description

We assume discrete multi-tone (DMT) modulation with K tones and synchronous transmission among N users. We also assume that the cyclic prefix and suffix are long enough to ensure orthogonality between the received signals on different tones for any two users in both directions [15]. Due to inconsistency of an optimized bandplan with standard bandplans, DBP cannot be practiced together with local-loop unbundling. Therefore, we assume that all loops in the cable are operated by the same service provider.

A. Channel Model

With disjoint bands, the transmission model depends on the transmission direction. In DS, vectoring is applied at the transmitter side. In US, vectoring is applied at the receiver side. Assuming linear encoding/decoding, the DS and US transmission on tone k are described as follows [6]

$$\boldsymbol{y}_{\mathrm{DS}}^k = \boldsymbol{H}_{\mathrm{DS}}^k \boldsymbol{Q}_{\mathrm{DS}}^k \boldsymbol{x}_{\mathrm{DS}}^k + \boldsymbol{z}_{\mathrm{DS}}^k, \qquad \text{if } k \in \mathcal{K}_{\mathrm{DS}} \text{ and} \tag{2.1a}$$

$$\boldsymbol{y}_{\mathrm{US}}^k = \left[\boldsymbol{Q}_{\mathrm{US}}^k\right]^{\mathrm{H}} \boldsymbol{H}_{\mathrm{US}}^k \boldsymbol{x}_{\mathrm{US}}^k + \left[\boldsymbol{Q}_{\mathrm{US}}^k\right]^{\mathrm{H}} \boldsymbol{z}_{\mathrm{US}}^k, \qquad \text{if } k \in \mathcal{K}_{\mathrm{US}}, \tag{2.1b}$$

where $\boldsymbol{x}_{\mathrm{DS/US}}^k$, $\boldsymbol{y}_{\mathrm{DS/US}}^k$, and $\boldsymbol{z}_{\mathrm{DS/US}}^k$ are the $N \times 1$ DS/US transmitted, received, and noise vector respectively, $\boldsymbol{H}_{\mathrm{DS/US}}^k$ is the DS/US (direct and) FEXT channel matrix, $\boldsymbol{Q}_{\mathrm{DS/US}}^k$ is the transmitter/receiver filter matrix,and $\mathcal{K}_{\mathrm{DS/US}}$ is the set of DS/US tones. We assume that the elements of $\boldsymbol{z}_{\mathrm{DS/US}}^k$ are independent and $\boldsymbol{H}_{\mathrm{DS/US}}^k$ is normalized such that

$$\mathrm{E}\left\{\boldsymbol{z}_{\mathrm{DS/US}}^k \left[\boldsymbol{z}_{\mathrm{DS/US}}^k\right]^{\mathrm{H}}\right\} = \boldsymbol{I}_N. \tag{2.2}$$

Assuming overlapping bands, the DS and US transmission on tone k are written as

$$\boldsymbol{y}_{\mathrm{DS}}^k = \boldsymbol{H}_{\mathrm{DS}}^k \boldsymbol{Q}_{\mathrm{DS}}^k \boldsymbol{x}_{\mathrm{DS}}^k + \boldsymbol{G}_{\mathrm{DS}}^k \boldsymbol{x}_{\mathrm{US}}^k + \boldsymbol{z}_{\mathrm{DS}}^k, \text{ and} \tag{2.3a}$$

$$\boldsymbol{y}_{\mathrm{US}}^k = \left[\boldsymbol{Q}_{\mathrm{US}}^k\right]^{\mathrm{H}} \boldsymbol{H}_{\mathrm{US}}^k \boldsymbol{x}_{\mathrm{US}}^k + \left[\boldsymbol{Q}_{\mathrm{US}}^k\right]^{\mathrm{H}} \boldsymbol{G}_{\mathrm{US}}^k \boldsymbol{Q}_{\mathrm{DS}}^k \boldsymbol{x}_{\mathrm{DS}}^k + \left[\boldsymbol{Q}_{\mathrm{US}}^k\right]^{\mathrm{H}} \boldsymbol{z}_{\mathrm{US}}^k, \tag{2.3b}$$

where $\boldsymbol{G}_{\mathrm{DS/US}}^k$ is the DS/US (echo and) NEXT channel matrix with its diagonal and off-diagonal elements denoting the echo and the NEXT couplings respectively. Figure 2.(a) shows a schematic diagram of a DSL network with overlapping bands. Rearranging (2.3) we obtain

$$\begin{bmatrix} \boldsymbol{y}_{\mathrm{DS}}^k \\ \boldsymbol{y}_{\mathrm{US}}^k \end{bmatrix} = \begin{bmatrix} \boldsymbol{I}_N & \boldsymbol{0}_N \\ \boldsymbol{0}_N & \boldsymbol{Q}_{\mathrm{US}}^k \end{bmatrix}^{\mathrm{H}} \left(\begin{bmatrix} \boldsymbol{H}_{\mathrm{DS}}^k & \boldsymbol{G}_{\mathrm{DS}}^k \\ \boldsymbol{G}_{\mathrm{US}}^k & \boldsymbol{H}_{\mathrm{US}}^k \end{bmatrix} \begin{bmatrix} \boldsymbol{Q}_{\mathrm{DS}}^k & \boldsymbol{0}_N \\ \boldsymbol{0}_N & \boldsymbol{I}_N \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_{\mathrm{DS}}^k \\ \boldsymbol{x}_{\mathrm{US}}^k \end{bmatrix} + \begin{bmatrix} \boldsymbol{z}_{\mathrm{DS}}^k \\ \boldsymbol{z}_{\mathrm{US}}^k \end{bmatrix} \right). \tag{2.4}$$

When echo is canceled the diagonal elements of $\boldsymbol{G}^k_{\mathrm{DS/US}}$ are set to zero. When echo and US NEXT are canceled (Fig. 2.(b)) $\boldsymbol{G}^k_{\mathrm{US}}$ and the diagonal elements of $\boldsymbol{G}^k_{\mathrm{DS}}$ are set to zero and equations (2.3b) and (2.4) are rewritten as

$$\boldsymbol{y}^k_{\mathrm{US}} = \left[\boldsymbol{Q}^k_{\mathrm{US}}\right]^{\mathrm{H}} \boldsymbol{H}^k_{\mathrm{US}} \boldsymbol{x}^k_{\mathrm{US}} + \left[\boldsymbol{Q}^k_{\mathrm{US}}\right]^{\mathrm{H}} \boldsymbol{z}^k_{\mathrm{US}} \tag{2.5}$$

and

$$\begin{bmatrix} \boldsymbol{y}^k_{\mathrm{DS}} \\ \boldsymbol{y}^k_{\mathrm{US}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{I}_N & \boldsymbol{0}_N \\ \boldsymbol{0}_N & \boldsymbol{Q}^k_{\mathrm{US}} \end{bmatrix}^{\mathrm{H}} \left(\begin{bmatrix} \boldsymbol{H}^k_{\mathrm{DS}} & \boldsymbol{G}^k_{\mathrm{DS}} \\ \boldsymbol{0}_N & \boldsymbol{H}^k_{\mathrm{US}} \end{bmatrix} \begin{bmatrix} \boldsymbol{Q}^k_{\mathrm{DS}} & \boldsymbol{0}_N \\ \boldsymbol{0}_N & \boldsymbol{I}_N \end{bmatrix} \begin{bmatrix} \boldsymbol{x}^k_{\mathrm{DS}} \\ \boldsymbol{x}^k_{\mathrm{US}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{z}^k_{\mathrm{DS}} \\ \boldsymbol{z}^k_{\mathrm{US}} \end{bmatrix} \right). \tag{2.6}$$

B. Power, Bit Rate, and SNR

Let $\boldsymbol{q}^k_{n,\mathrm{DS/US}}$ denote the n-th column of $\boldsymbol{Q}^k_{\mathrm{DS/US}}$, let $s^k_{n,\mathrm{DS/US}}$ and $p^k_{n,\mathrm{DS/US}}$ denote DS/US the transmit power on tone k for user n and the transmit power on tone k for line n, respectively, and let $u^k_{n,\mathrm{DS/US}} \equiv \mathrm{E}\left\{\left|x^k_{n,\mathrm{DS/US}}\right|^2\right\}$, where $x^k_{n,\mathrm{DS/US}}$ is the n-th element of $\boldsymbol{x}^k_{\mathrm{DS/US}}$. In US we have $s^k_{n,\mathrm{US}} = p^k_{n,\mathrm{US}} = u^k_{n,\mathrm{US}}$. However, in DS, we have $s^k_{n,\mathrm{DS}} \equiv u^k_{n,\mathrm{DS}} \left\|\boldsymbol{q}^k_{n,\mathrm{DS}}\right\|_2^2$ and $p^k_{n,\mathrm{DS}} \equiv \left[\boldsymbol{Q}^k_{\mathrm{DS}} \operatorname{diag}\left\{\boldsymbol{u}^k_{\mathrm{DS}}\right\} \left[\boldsymbol{Q}^k_{\mathrm{DS}}\right]^{\mathrm{H}}\right]_{n,n}$ where in the general case $s^k_{n,\mathrm{DS}} \neq p^k_{n,\mathrm{DS}} \neq u^k_{n,\mathrm{DS}}$. The total transmit power on line n in DS/US direction is

$$P_{n,\mathrm{DS/US}} = \Delta_{\mathrm{f}} \sum_{k \in \mathcal{K}_{\mathrm{DS/US}}} p^k_{n,\mathrm{DS/US}}, \tag{2.7}$$

where Δ_{f} is the DMT tone spacing. The bit rate for user n in DS/US direction is

$$R_{n,\mathrm{DS/US}} = f_{\mathrm{s}} \sum_{k \in \mathcal{K}_{\mathrm{DS/US}}} b^k_{n,\mathrm{DS/US}}, \tag{2.8}$$

where f_{s} is the DMT symbol rate and $b^k_{n,\mathrm{DS/US}}$ is the bit loading defined as

$$b^k_{n,\mathrm{DS/US}} = \begin{cases} 0; & \log_2\left(1 + \frac{1}{\Gamma}\mathrm{SNR}^k_{n,\mathrm{DS/US}}\right) < b_{\min}, \\ \min\left\{b_{\max}, \left\lfloor \log_2\left(1 + \frac{1}{\Gamma}\mathrm{SNR}^k_{n,\mathrm{DS/US}}\right)\right\rfloor\right\}; & \text{otherwise}, \end{cases} \tag{2.9}$$

where $\Gamma = 9.75$ dB is the SNR gap and $b_{\min} = 2$ and $b_{\max} = 15$ are the minimum and maximum number of bits that can be loaded to a tone [2]. The SNR depends on the transmitter/receiver structure. With linear encoders/decoders and disjoint bands, the SNR is

$$\mathrm{SNR}^k_{n,\mathrm{DS}} = \frac{u^k_{n,\mathrm{DS}} \left|\boldsymbol{e}_n^{\mathrm{T}} \boldsymbol{H}^k_{\mathrm{DS}} \boldsymbol{q}^k_{n,\mathrm{DS}}\right|^2}{1 + \sum_{m \neq n} u^k_{m,\mathrm{DS}} \left|\boldsymbol{e}_n^{\mathrm{T}} \boldsymbol{H}^k_{\mathrm{DS}} \boldsymbol{q}^k_{m,\mathrm{DS}}\right|^2}, \qquad \text{if } k \in \mathcal{K}_{\mathrm{DS}} \tag{2.10}$$

and

$$\mathrm{SNR}^k_{n,\mathrm{US}} = \frac{u^k_{n,\mathrm{US}} \left|\left[\boldsymbol{q}^k_{n,\mathrm{US}}\right]^{\mathrm{H}} \boldsymbol{H}^k_{\mathrm{US}} \boldsymbol{e}_n\right|^2}{\left\|\boldsymbol{q}^k_{n,\mathrm{US}}\right\|_2^2 + \sum_{m \neq n} u^k_{m,\mathrm{US}} \left|\left[\boldsymbol{q}^k_{n,\mathrm{US}}\right]^{\mathrm{H}} \boldsymbol{H}^k_{\mathrm{US}} \boldsymbol{e}_m\right|^2}, \qquad \text{if } k \in \mathcal{K}_{\mathrm{US}}. \tag{2.11}$$

If a nonlinear vector Tomlinson-Harashima precoder (THP) is used for the DS trasmission, the SNR is

$$\mathrm{SNR}_{n,\mathrm{DS}}^{k} = \frac{u_{n,\mathrm{DS}}^{k} \left| \boldsymbol{e}_n^{\mathrm{T}} \boldsymbol{H}_{\mathrm{DS}}^{k} \boldsymbol{q}_{n,\mathrm{DS}}^{k} \right|^2}{1 + \sum_{m \in \ \mathcal{I}_{n,\mathrm{DS}}} u_{m,\mathrm{DS}}^{k} \left| \boldsymbol{e}_n^{\mathrm{T}} \boldsymbol{H}_{\mathrm{DS}}^{k} \boldsymbol{q}_{m,\mathrm{DS}}^{k} \right|^2}, \qquad \text{if } k \in \mathcal{K}_{\mathrm{DS}} \tag{2.12}$$

where $\mathcal{I}_{n,\mathrm{DS}}$ is the set of interfering users for user n in DS, i.e. the users that are encoded after user n.

Similarly, if a generalized decision feedback equalizer (GDFE) is used for the US reception, the SNR is

$$\mathrm{SNR}_{n,\mathrm{US}}^{k} = \frac{u_{n,\mathrm{US}}^{k} \left| \left[\boldsymbol{q}_{n,\mathrm{US}}^{k} \right]^{\mathrm{H}} \boldsymbol{H}_{\mathrm{US}}^{k} \boldsymbol{e}_n \right|^2}{\left\| \boldsymbol{q}_{n,\mathrm{US}}^{k} \right\|_2^2 + \sum_{m \in \ \mathcal{I}_{n,\mathrm{US}}} u_{m,\mathrm{US}}^{k} \left| \left[\boldsymbol{q}_{n,\mathrm{US}}^{k} \right]^{\mathrm{H}} \boldsymbol{H}_{\mathrm{US}}^{k} \boldsymbol{e}_m \right|^2}, \qquad \text{if } k \in \mathcal{K}_{\mathrm{US}} \tag{2.13}$$

where $\mathcal{I}_{n,\mathrm{US}}$ is the set of interfering users for user n in US, i.e., the users that are decoded after user n.

When overlapping bands are used, NEXT and echo contribute to the received interference as well. In this case, the following term has to be added to the denominator of (2.10) and (2.12):

$$\sum_{m=1}^{N} u_{m,\mathrm{US}}^{k} \left| \boldsymbol{e}_n^{\mathrm{T}} \boldsymbol{G}_{\mathrm{DS}}^{k} \boldsymbol{e}_m \right|^2 \tag{2.14}$$

and the following term has to be added to the denominator of (2.11) and (2.13) (unless NEXT and echo are canceled in the US direction):

$$\sum_{m=1}^{N} u_{m,\mathrm{DS}}^{k} \left| \left[\boldsymbol{q}_{n,\mathrm{US}}^{k} \right]^{\mathrm{H}} \boldsymbol{G}_{\mathrm{US}}^{k} \boldsymbol{q}_{m,\mathrm{DS}}^{k} \right|^2. \tag{2.15}$$

III. Dynamic Bandplanning Algorithms

In this paper, we consider the DBP problem in the following weighted sum rate maximization form:

$$\text{maximize} \quad \sum_{n=1}^{N} w_{n,\mathrm{DS}} R_{n,\mathrm{DS}} + \sum_{n=1}^{N} w_{n,\mathrm{US}} R_{n,\mathrm{US}} \tag{3.1a}$$

$$\text{subject to} \quad P_{n,\mathrm{DS}} \le P_{n,\mathrm{DS}}^{\max}; \quad 1 \le n \le N, \tag{3.1b}$$

$$P_{n,\mathrm{US}} \le P_{n,\mathrm{US}}^{\max}; \quad 1 \le n \le N, \tag{3.1c}$$

$$p_{n,\mathrm{DS}}^{k} \le p_{n,\mathrm{DS}}^{k,\mathrm{mask}}; \quad 1 \le n \le N, 1 \le k \le K, \quad \text{and} \tag{3.1d}$$

$$p_{n,\mathrm{US}}^{k} \le p_{n,\mathrm{US}}^{k,\mathrm{mask}}; \quad 1 \le n \le N, 1 \le k \le K, \tag{3.1e}$$

where $w_{n,\mathrm{DS/US}}$, $R_{n,\mathrm{DS/US}}$, $P_{n,\mathrm{DS/US}}$, and $P^{\max}_{n,\mathrm{DS/US}}$ are the DS/US bit rate weight factor and bit rate for user n, and the total transmit power and total transmit power budget for line n, respectively, and $p^k_{n,\mathrm{DS/US}}$ and $p^{k,\mathrm{mask}}_{n,\mathrm{DS/US}}$ are the DS/US transmit power and the nominal PSD mask for line n on tone k.

For a system with disjoint DS and US bands the maximization takes place over $\mathcal{K}_{\mathrm{DS/US}}$, $\boldsymbol{Q}^k_{\mathrm{DS/US}}$, and DS/US transmit powers. For a system with overlapping DS and US bands, the maximization takes place over $\boldsymbol{Q}^k_{\mathrm{DS/US}}$ and DS/US transmit powers. If a nonlinear receiver/transmitter structure (GDFE or vector THP) is used, the decoding/encoding order of the users is also optimized.

The Lagrange dual decomposition approach has been shown to be optimal for MIMO-OFDM problems when the number of tones is large enough [16–19]. Using dual decomposition techniques, (3.1) is decoupled into the following K independent problems

$$\begin{array}{ll} \text{maximize} & \mathcal{L}^k, \\ \text{subject to} & \text{(3.1d) and (3.1e)}, \end{array} \qquad \text{for } k = 1 \ldots K \tag{3.2}$$

where

$$\mathcal{L}^k \equiv \sum_{n=1}^{N} w_{n,\mathrm{DS}} b^k_{n,\mathrm{DS}} + \sum_{n=1}^{N} w_{n,\mathrm{US}} b^k_{n,\mathrm{US}} - \sum \lambda_{n,\mathrm{DS}} p^k_{n,\mathrm{DS}} - \sum_{n=1}^{N} \lambda_{n,\mathrm{US}} p^k_{n,\mathrm{US}}, \tag{3.3}$$

and $\lambda_{n,\mathrm{DS/US}}$ are the Lagrange multipliers corresponding to the DS/US per line total power constraints (3.1b) and (3.1c). The Lagrange multipliers should be optimized to satisfy the corresponding constraints. For more details see [20].

A. Disjoint Bands

For a system with disjoint bands, depending on $k \in \mathcal{K}_{\mathrm{DS}}$ or $k \in \mathcal{K}_{\mathrm{US}}$, $\mathcal{L}^k$ is simplified to

$$\mathcal{L}^k_{\mathrm{DS}} \equiv \sum_{n=1}^{N} w_{n,\mathrm{DS}} b^k_{n,\mathrm{DS}} - \sum_{n=1}^{N} \lambda_{n,\mathrm{DS}} p^k_{n,\mathrm{DS}} \tag{3.4}$$

or

$$\mathcal{L}^k_{\mathrm{US}} \equiv \sum_{n=1}^{N} w_{n,\mathrm{US}} b^k_{n,\mathrm{US}} - \sum_{n=1}^{N} \lambda_{n,\mathrm{US}} p^k_{n,\mathrm{US}}. \tag{3.5}$$

Thus, in order to determine the transmission direction on tone k for a particular set of Lagrange multipliers, we solve the following two problems

$$\text{maximize} \quad \mathcal{L}^k_{\mathrm{DS}}, \tag{3.6}$$

$$\text{maximize} \quad \mathcal{L}_{\text{US}}^{k}. \tag{3.7}$$

Let $\hat{\mathcal{L}}_{\text{DS}}^{k}$ and $\hat{\mathcal{L}}_{\text{US}}^{k}$ denote the corresponding solutions. If $\hat{\mathcal{L}}_{\text{DS}}^{k} > \hat{\mathcal{L}}_{\text{US}}^{k}$ then $k \in \mathcal{K}_{\text{DS}}$, otherwise $k \in \mathcal{K}_{\text{US}}$. The optimal solutions to (3.6) and (3.7) can be found using the BC-OSB and MAC-OSB algorithms proposed in [10] and [9], respectively. A pseudocode for this algorithm is presented in Alg. 1, where we assume that vector THP and GDFE are used in the DS transmitter and US receiver respectively. The calculation of the optimal transmitter/receiver filters and powers for the DS and US is explained in Sec. III.A.1.

A.1 Optimal Transmitter/Receiver Filters: One-Sided Coordination

For the US, consider the following multiple access channel (MAC) (with simplified notation):

$$\boldsymbol{y} = \boldsymbol{Q}^{\text{H}}\boldsymbol{H}\boldsymbol{x} + \boldsymbol{Q}^{\text{H}}\boldsymbol{z} \tag{3.8}$$

where the elements of $\boldsymbol{z}$ are Gaussian random variables with covariance matrix $\boldsymbol{Z} \equiv \text{E}\left\{\boldsymbol{z}\boldsymbol{z}^{\text{H}}\right\}$. Note that in DBP with overlapping bands, the contribution of echo and NEXT can be included in $\boldsymbol{z}$. The elements of $\boldsymbol{x}$ are i.i.d Gaussian variables with power $u_n = \text{E}\left\{|x_n|^2\right\}$ and $\boldsymbol{u} \equiv (u_1, u_2, \ldots, u_N)^{\text{T}}$. The n-th column of $\boldsymbol{Q}$, $\boldsymbol{q}_n$, denotes the receiver filter for user n. The SNR maximizing receive filter is the MMSE filter which is obtained by

$$\boldsymbol{q}_n = \boldsymbol{C}_n^{-1}\boldsymbol{h}_n. \tag{3.9}$$

where $\boldsymbol{C}_n$ is the covariance matrix of the total noise and interference for user n. For a linear receiver we have

$$\boldsymbol{C}_n = \boldsymbol{Z} + \sum_{m \neq n} u_m \boldsymbol{h}_m \boldsymbol{h}_m^{\text{H}}. \tag{3.10}$$

where $\boldsymbol{h}_m$ is the m-th column of $\boldsymbol{H}$. For a nonlinear GDFE receiver, we have

$$\boldsymbol{C}_n = \boldsymbol{Z} + \sum_{m \in \mathcal{I}_n} u_m \boldsymbol{h}_m \boldsymbol{h}_m^{\text{H}}. \tag{3.11}$$

where $\mathcal{I}_n$ is the set of interfering users for user n. Using the above equations the optimal receiver filters can be calculated from the transmit powers. With the MMSE filter calculated in (3.9), the SNR for user n for both a linear and a nonlinear receiver as defined in (2.11) and (2.13) reduce to

$$\text{SNR}_n = u_n \boldsymbol{q}_n^{\text{H}} \boldsymbol{h}_n. \tag{3.12}$$

The bit loading then follows from (2.9).

Reversely, given the bit loadings, it is possible to calculate the required transmit powers and the optimal receiver filters. For the GDFE receiver, these can be explicitly calculated one by one starting from the last decoded user to the first decoded user. For each user n, $\boldsymbol{C}_n$ is calculated using (3.11) given the transmit power for all users that are decoded later. Then $\boldsymbol{q}_n$ and $u_n = \mathrm{SNR}_n/\left(\boldsymbol{q}_n^{\mathrm{H}}\boldsymbol{h}_n\right)$ are calculated using (3.9) and (3.12), respectively. For linear receivers, the transmit powers can be calculated from the bit-loadings using an iterative algorithm. In the first iteration, the transmit power of all users is assumed to be zero. The transmit powers are updated in each iteration by calculating $\boldsymbol{C}_n$ from (3.10) using the latest values of u_m $(m \neq n)$. Then $\boldsymbol{q}_n$ and u_n are calculated using (3.9) and (3.12), respectively. Simulation results show that this algorithm converges to the solution (if feasible) in 3 to 4 iterations [12].

For the DS, the BC-MAC duality theory [17, 21–23] can be used to calculate the optimal transmit filters and powers. Consider the following broadcast channel (BC) (with simplified notation) and assume first that linear encoding is used:

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{Q}\boldsymbol{x} + \boldsymbol{z}, \tag{3.13}$$

where the channel is normalized such that $\boldsymbol{z}$ has unit power i.i.d Gaussian elements and so $\mathrm{E}\left\{\boldsymbol{z}\boldsymbol{z}^{\mathrm{H}}\right\} = \boldsymbol{I}$ (the correlation between the elements of $\boldsymbol{z}$ is ignored because it cannot be exploited to improve the performance [10, 13]). The dual MAC of (3.13) is defined by

$$\tilde{\boldsymbol{y}} = \boldsymbol{Q}^{\mathrm{H}}\boldsymbol{H}^{\mathrm{H}}\tilde{\boldsymbol{x}} + \boldsymbol{Q}^{\mathrm{H}}\tilde{\boldsymbol{z}}, \tag{3.14}$$

where $\tilde{\boldsymbol{z}}$ again has unit power i.i.d Gaussian elements. Let $\tilde{u}_n = \mathrm{E}\left\{|\tilde{x}_n|^2\right\}$ and $\tilde{\boldsymbol{u}} \equiv (\tilde{u}_1, \tilde{u}_2, \ldots, \tilde{u}_N)^{\mathrm{T}}$. The MMSE filters are SNR maximizing in the dual MAC and can be calculated using (3.9). Let $\boldsymbol{\gamma} \equiv (\gamma_1, \gamma_2, \ldots, \gamma_N)^{\mathrm{T}}$ denote a set of achievable SNR's in the dual MAC corresponding to a set of bit loadings $\boldsymbol{\beta} = (b_1, \ldots, b_N)^{\mathrm{T}}$ based on (2.9). The BC-MAC duality states that the same set of SNR's (and hence bit loadings) can be achieved in the primal BC. The required transmit powers to reach $\boldsymbol{\gamma}$ in the primal BC is calculated by

$$\boldsymbol{u} = \boldsymbol{X}^{-1}\boldsymbol{1}_N \tag{3.15}$$

and the required transmit powers to reach $\boldsymbol{\gamma}$ in the dual MAC is calculated by [23]

$$\tilde{\boldsymbol{u}} = \boldsymbol{X}^{-\mathrm{T}}\boldsymbol{\sigma}, \tag{3.16}$$

where $\boldsymbol{\sigma}$ is a column vector of size N containing the diagonal elements of $\boldsymbol{Q}^{\mathrm{H}}\boldsymbol{Q}$, and $\boldsymbol{X}$ depends on the transmitter structure. For a linear encoder $\boldsymbol{X}$ is defined as

$$x_{n,m} = \begin{cases} \gamma_n^{-1} \left|\hat{h}_{n,n}\right|^2, & \text{if } m = n \\ -\left|\hat{h}_{n,m}\right|^2, & \text{if } m \neq n, \end{cases} \tag{3.17}$$

where $\hat{\boldsymbol{H}} \equiv \boldsymbol{H}\boldsymbol{Q}$. For a nonlinear encoder, i.e. when vector THP is used, we have to use the dual nonlinear receiver structure, i.e., the GDFE, in the dual MAC [21]. The decoding order in the GDFE must be the reverse of the encoding order in the THP. That is, assuming the decoding order in the GDFE is $1, 2, \ldots, N$, then the encoding order in the THP is $N, N-1, \ldots, 1$. Equation (3.15) can still be used to calculate the transmit powers from the SNR's where $\boldsymbol{X}$ is redefined as:

$$x_{n,m} = \begin{cases} \gamma_n^{-1} \left|\hat{h}_{n,n}\right|^2, & \text{if } m = n \\ -\left|\hat{h}_{n,m}\right|^2, & \text{if } m \in \mathcal{I}_n \\ 0, & \text{otherwise,} \end{cases} \tag{3.18}$$

where $\mathcal{I}_n$ is the set of interfering users for user n in the primal BC, i.e., the set of users encoded after user n in the primal BC or equivalently those decoded before user n in the dual MAC.

Obviously, the achievable rates for the GDFE and vector THP depend on the order of decoding and encoding. In [12] it has been shown that setting the decoding order according to the order of US weight factors $\{w_{1,\mathrm{US}}, \ldots, w_{N,\mathrm{US}}\}$ such that the user with the smallest weight factor is decoded first and the user with the largest weight factor is decoded last is near-optimal in US DSL. Also, setting the encoding order in the vector THP such that the user with the largest DS weight factor is encoded first and the user with the smallest DS weight factor is encoded last is near-optimal in DS DSL [13].

From the BC-MAC duality, we also know that

$$\sum s_n = \sum \tilde{u}_n \tag{3.19a}$$

$$\text{and} \qquad \sum p_n = \sum \tilde{u}_n, \tag{3.19b}$$

where

$$p_n \equiv \left[\boldsymbol{Q}\mathrm{diag}\left\{\boldsymbol{u}\right\}\boldsymbol{Q}^{\mathrm{H}}\right]_{n,n} \tag{3.20}$$

is the transmit power on line n. Equation (3.19a) is very helpful in solving optimization problems for which the utility function and the constraints are functions of the (per-user)

bit rates together with the sum transmit power for all the users. For example the weighted sum rate maximization problem with sum power constraint in the BC can be solved in the dual MAC directly using (3.19a).

However, in DSL, e.g. as in (3.2), we usually have constraints on the per-line rather than the per-user transmit powers. This problem has been addressed in [23], where the BC-MAC duality is shown to be a special case of Lagrange duality. Briefly, if we scale the noise vector in (3.14) such that $\mathrm{E}\left\{|\tilde{z}_n|^2\right\} = \lambda_n$ and calculate the optimal filters and the corresponding transmit powers in the primal and the dual channel with scaled noise using (3.9) to (3.15), we obtain $\sum \lambda_n p_n = \sum \tilde{u}_n$. This result can be used to solve optimization problems with constraints on per-line transmit powers, where λ_n plays the role of the Lagrange multiplier corresponding to the power constraint on line n.

By increasing λ_n the transmit power on line n decreases and vice versa. The optimal value of λ_1 to λ_N, satisfying all power constraints and leading to the maximum weighted sum rate, is usually calculated in an iterative process. However, in [13] it has been shown that due to the the row-wise diagonal dominance (RWDD) property [6] of the DS channel, setting $\lambda_1 = \lambda_2 = \ldots = \lambda_N = 1$ is near optimal in DSL.

B. Overlapping Bands

In DBP with overlapping bands, full signal coordination is not available. However vectoring is applied at both sides. Consider the following channel with (partial) vectoring at both sides

$$\boldsymbol{y} = \boldsymbol{Q}_{\mathrm{rx}}^{\mathrm{H}} \boldsymbol{H} \boldsymbol{Q}_{\mathrm{tx}} \boldsymbol{x} + \boldsymbol{Q}_{\mathrm{rx}}^{\mathrm{H}} \boldsymbol{z}. \tag{3.21}$$

Here $\boldsymbol{y}$ is a $2N \times 1$ vector, which is consistent with (2.4) and (2.6). In this section, we will then speak of $2N$ users (rather than N users), namely N DS users together with N US users. The dual of (3.21) is

$$\tilde{\boldsymbol{y}} = \boldsymbol{Q}_{\mathrm{tx}}^{\mathrm{H}} \boldsymbol{H}^{\mathrm{H}} \boldsymbol{Q}_{\mathrm{rx}} \tilde{\boldsymbol{x}} + \boldsymbol{Q}_{\mathrm{tx}}^{\mathrm{H}} \tilde{\boldsymbol{z}}. \tag{3.22}$$

Using the theory reviewed in Sec. III.A.1, from (3.21), we can find the optimal value of $\boldsymbol{Q}_{\mathrm{rx}}$ (maximizing (3.3)) provided that $\boldsymbol{Q}_{\mathrm{tx}}$ and the transmit powers are known. On the other hand from (3.22), we can find the optimal value of $\boldsymbol{Q}_{\mathrm{tx}}$ provided that $\boldsymbol{Q}_{\mathrm{rx}}$ and the transmit powers in the dual channel are known. However, the optimal transmitter

and receiver filters cannot be calculated jointly because the Lagrangian is non-convex in either case. For (3.21), it is non-convex in terms of $\boldsymbol{Q}_{\text{tx}}$ and for (3.22) it is non-convex in terms of $\boldsymbol{Q}_{\text{rx}}$. It is known that when full signal coordination exists among the users at both the receiver and transmitter side, waterfilling against the singular values of the channel matrix is capacity achieving [24]. Unfortunately, full signal coordination is not available for the DBP case with overlapping bands as can be seen in (2.4) and (2.6). For this reason, we consider iterative techniques for solving (3.2). In the following, we propose two algorithms to solve this problem.

B.1 DBP Algorithms for Systems with Overlapping Bands

The **first algorithm** is based on the MAC-OSB [9] and BC-OSB [10] and is essentially an extension of Alg. 1 to systems with overlapping bands where we again assume that vectored THP and GDFE are used in the DS and US respectively. We call this algorithm, DBP with overlapping bands and separate DS/US optimization. A pseudocode for this algorithm is presented in Alg. 2.

The DS and US transmission are iteratively optimized. For US the optimization, the DS transmitter filters and transmit powers, i.e., $\boldsymbol{Q}_{\text{DS}}^k$ and $\boldsymbol{u}_{\text{DS}}^k$, are fixed to those of the best solution found so far. Then for all US bit loading combinations, $\boldsymbol{Q}_{\text{US}}^k$ and $\boldsymbol{u}_{\text{US}}^k$ are calculated using the formulas given in Sec. III.A.1 with term (2.15) added in the SNR formula. By changing $\boldsymbol{u}_{\text{US}}^k$, the NEXT power and consequently the SNR for the DS transmission will change (eqn. (2.14)). Therefore, for each set of US bit loadings, DS bit loadings and the Lagrangian are recalculated. If the Lagrangian is the largest Lagrangian so far, the solution is updated.

Similarly for the DS optimization, $\boldsymbol{u}_{\text{US}}^k$ and $\boldsymbol{Q}_{\text{US}}^k$ are fixed to those of the best solution found so far. DS optimization is carried out in the dual channel. For all DS bit loading vectors in the range, the optimal transmit filters and required transmit powers in the dual channel are calculated. Using (3.15), the transmit powers are mapped back to the primal channel. Then, the achievable US bit loadings and the Lagrangian are recalculated. If the Lagrangian is the largest Lagrangian so far, the solution is updated.

The DS and US optimization are repeated iteratively until convergence. The convergence is detected when the Lagrangian is within 0.1 % of the Lagrangian in the previous

iteration. Usually 3 to 4 iterations are enough for convergence.

The **second algorithm** (Alg. 3) is based on the IF/MAC-OSB [12] and IF/BC-OSB [13] algorithm. Unlike in Alg. 2, the exhaustive search over the DS and US bit loadings is not separated. The algorithm is executed for each combination of DS and US bit loadings. Using an iterative scheme, it is checked if the combination is reachable and if this is the case, the solution is updated provided that the corresponding Lagrangian is larger than the largest Lagrangian found so far.

Similar to Alg. 2, the core of this algorithm consists of two main routines. However unlike Alg. 2, in the first routine, $\boldsymbol{Q}_{\mathrm{DS}}^{k}$ is fixed and $\boldsymbol{u}_{\mathrm{DS}}^{k}$, $\boldsymbol{u}_{\mathrm{US}}^{k}$, and $\boldsymbol{Q}_{\mathrm{US}}^{k}$ are optimized. In the second routine, $\boldsymbol{Q}_{\mathrm{US}}^{k}$ is fixed and $\boldsymbol{u}_{\mathrm{US}}^{k}$, $\boldsymbol{u}_{\mathrm{DS}}^{k}$, and $\boldsymbol{Q}_{\mathrm{DS}}^{k}$ are optimized in the dual channel. The two routines are iterated until the procedure converges, diverges, or the maximum number of iterations is reached. Joint calculation of the optimal transmitter/receiver filters and DS and US transmit powers when the receiver/transmitter filters are fixed is explained in more details in Sec. III.B.2.

In Alg. 3 the two routines are executed for all DS and US bit loadings combinations (so the size of search space is $(b_{\max} - b_{\min} + 2)^{2N}$), while in Alg. 2, first a search over all US bit loading combinations is executed and then a search over all DS bit loading combinations is executed (i.e., two exhaustive searches of size $(b_{\max} - b_{\min} + 2)^{N}$). The benefit of this extra computational complexity is that the algorithm is less likely to converge to a local optimum compared to Alg. 2.

B.2 Optimal Transmitter/Receiver Filters: Two-Sided Coordination

In (3.21), assuming full coordination at the receiver side, the MMSE receiver filter for user n, $\boldsymbol{q}_{n,\mathrm{rx}}$ is obtained by

$$\boldsymbol{q}_{n,\mathrm{rx}} = \hat{\boldsymbol{C}}_{n}^{-1}\hat{\boldsymbol{h}}_{n}, \tag{3.23}$$

where $\hat{\boldsymbol{h}}_{n}$ is the n-th column of $\hat{\boldsymbol{H}} \equiv \boldsymbol{H}\boldsymbol{Q}_{\mathrm{tx}}$. The noise covariance matrix, $\hat{\boldsymbol{C}}_{n}$, depends on the transmitter and receiver structure. Assuming linear transmitter and receiver structures we have

$$\hat{\boldsymbol{C}}_{n} = \boldsymbol{Z} + \sum_{m \neq n} u_{m}\hat{\boldsymbol{h}}_{m}\hat{\boldsymbol{h}}_{m}^{\mathrm{H}}. \tag{3.24}$$

For a nonlinear vector THP transmitter and a nonlinear GDFE receiver, we have

$$\hat{\boldsymbol{C}}_n = \boldsymbol{Z} + \sum_{m \in \mathcal{I}_{n,\mathrm{tx}} \cap \mathcal{I}_{n,\mathrm{rx}}} u_m \hat{\boldsymbol{h}}_m \hat{\boldsymbol{h}}_m^{\mathrm{H}}. \tag{3.25}$$

where $\mathcal{I}_{n,\mathrm{tx/rx}}$ is the set of interfering users for user n at the transmitter/receiver side.

The above formulas hold for partial signal coordination among the users too. When two users n and m are not coordinated at the signal level, we have $q_{n,m,\mathrm{rx}} = 0$, however, the non-zero elements of $\boldsymbol{q}_{n,\mathrm{rx}}$ are still calculated optimally by the MMSE criterion using (3.23) after removing the rows and columns of $\hat{\boldsymbol{C}}_n$ and $\hat{\boldsymbol{h}}_n$ corresponding to the zero elements of $\boldsymbol{q}_{n,\mathrm{rx}}$ [13].

The transmit powers for the channel described by (3.21) and its dual (3.22) are obtained by

$$\boldsymbol{u} = \bar{\boldsymbol{X}}^{-1} \boldsymbol{\sigma}_{\mathrm{rx}}, \tag{3.26}$$

and

$$\tilde{\boldsymbol{u}} = \bar{\boldsymbol{X}}^{-\mathrm{T}} \boldsymbol{\sigma}_{\mathrm{tx}}, \tag{3.27}$$

where $u_n \equiv \mathrm{E}\left\{|x_n|^2\right\}$, $\tilde{u}_n \equiv \mathrm{E}\left\{|\tilde{x}_n|^2\right\}$, the column vectors $\boldsymbol{\sigma}_{\mathrm{tx}}$ and $\boldsymbol{\sigma}_{\mathrm{tx}}$ are defined as the diagonal elements of $[\boldsymbol{Q}_{\mathrm{rx}}]^{\mathrm{H}} \boldsymbol{Q}_{\mathrm{rx}}$ and $[\boldsymbol{Q}_{\mathrm{tx}}]^{\mathrm{H}} \boldsymbol{Q}_{\mathrm{tx}}$, respectively, and $\bar{\boldsymbol{X}}$ is a $2N \times 2N$ matrix which depends on the transmitter and receiver structure. Assuming linear transmitter and receiver structure, $\bar{\boldsymbol{X}}$ is defined as

$$\bar{x}_{n,m} = \begin{cases} \gamma_n^{-1} \left|\bar{h}_{n,m}\right|^2, & \text{if } m = n \\ -\left|\bar{h}_{n,m}\right|^2, & \text{otherwise.} \end{cases} \tag{3.28}$$

where γ_n is the SNR for user n and $\bar{\boldsymbol{H}} = \boldsymbol{Q}_{\mathrm{rx}}^{\mathrm{H}} \boldsymbol{H} \boldsymbol{Q}_{\mathrm{tx}}$. For a nonlinear vector THP transmitter and a nonlinear GDFE receiver, $\bar{\boldsymbol{X}}$ is defined as

$$\bar{x}_{n,m} = \begin{cases} \gamma_n^{-1} \left|\bar{h}_{n,m}\right|^2, & \text{if } m = n \\ -\left|\bar{h}_{n,m}\right|^2, & \text{if } m \in \mathcal{I}_{n,\mathrm{rx}} \cap \mathcal{I}_{n,\mathrm{tx}} \\ 0, & \text{otherwise,} \end{cases} \tag{3.29}$$

where $\mathcal{I}_{n,\mathrm{rx}}$ is the set of users of which the interference is not removed for user n at the receiver side (i.e., the users that are decoded after user n in the GDFE) and $\mathcal{I}_{n,\mathrm{tx}}$ is the set of users of which the interference is not removed for user n at the transmitter side (i.e., the users that are encoded after user n in the vector THP).

Equations (3.26) and (3.27) can be directly used for systems with overlapping bands, where

$$\boldsymbol{Q}_{\mathrm{tx}} \equiv \begin{bmatrix} \boldsymbol{Q}_{\mathrm{DS}}^k & \boldsymbol{0}_N \\ \boldsymbol{0}_N & \boldsymbol{I}_N \end{bmatrix}, \boldsymbol{Q}_{\mathrm{rx}} \equiv \begin{bmatrix} \boldsymbol{I}_N & \boldsymbol{0}_N \\ \boldsymbol{0}_N & \boldsymbol{Q}_{\mathrm{US}}^k \end{bmatrix} \tag{3.30}$$

and depending on whether US NEXT is canceled or not, respectively,

$$\boldsymbol{H} \equiv \begin{bmatrix} \boldsymbol{H}_{\rm DS}^{k} & \boldsymbol{G}_{\rm DS}^{k} \\ \boldsymbol{0}_{N} & \boldsymbol{H}_{\rm US}^{k} \end{bmatrix} \text{ or } \boldsymbol{H} \equiv \begin{bmatrix} \boldsymbol{H}_{\rm DS}^{k} & \boldsymbol{G}_{\rm DS}^{k} \\ \boldsymbol{G}_{\rm US}^{k} & \boldsymbol{H}_{\rm US}^{k} \end{bmatrix}. \tag{3.31}$$

The sets $\mathcal{I}_{n,\rm tx}$ and $\mathcal{I}_{n,\rm rx}$ are calculated as a function of $\mathcal{I}_{n,\rm DS}$ and $\mathcal{I}_{n,\rm US}$ as follows. Based on the definition of $\boldsymbol{Q}_{\rm tx}$ and $\boldsymbol{Q}_{\rm rx}$ in (3.30), if $1 \leq n \leq N$ then user n is the n-th DS user while if $N+1 \leq n \leq 2N$ then it is the $(n-N)$-th US user. For the n-th DS user, the interfering users are $\mathcal{I}_{n,\rm DS}$ plus all US users (which interfere through the US NEXT channel). Similarly for the n-th US user, the interfering users are $\{m+N\,|m \in \mathcal{I}_{n,\rm US}\}$ plus DS users (which interfere through the DS NEXT channel). Therefore

$$\mathcal{I}_{n,\rm tx} = \begin{cases} \mathcal{I}_{n,\rm DS} \cup \{N+1,\ldots,2N\} & 1 \leq n \leq N, \\ \{1,\ldots,2N\} \setminus \{n\} & N+1 \leq n \leq 2N \end{cases} \tag{3.32}$$

and

$$\mathcal{I}_{n,\rm rx} = \begin{cases} \{1,\ldots,2N\} \setminus \{n\} & 1 \leq n \leq N, \\ \{1,\ldots,N\} \cup \{m+N\,|m \in \mathcal{I}_{n,\rm US}\} & N+1 \leq n \leq 2N. \end{cases} \tag{3.33}$$

IV. Simulation Results

We have simulated the proposed DBP algorithms for a two-user VDSL2 997E17 [3] scenario. The scenario consists of a 800 m loop and a 400 m loop as depicted in Fig. 3. Simulation parameters are listed in Tab. I and simulation results are presented in Fig. 4. In this figure, the achievable sum bit rates of the users in US direction is plotted vs. the achievable sum bit rate of the users in DS direction for different algorithms and simulation assumptions.

The achievable sum bit rates by Alg. 2 when a) echo and b) echo and US NEXT is/are canceled and the achievable sum bit rates by Alg. 3 when a) echo b) US NEXT, and c) echo and US NEXT are canceled and when d) echo and US NEXT are not canceled are plotted in the figure. The achievable sum bit rates for the same vectoring schemes when a fixed VDSL2 997E17 bandplan is used is also plotted. A few simulation results are not included in Fig. 4 for the sake of higher readability. These include simulation results for Alg. 1 and simulation results for Alg. 2 (i.e., DBP with disjoint bands) when echo is not canceled (regardless of US NEXT being canceled or not). All of these results overlap with the results for Alg. 3 when echo is not canceled. The reason will be discussed shortly.

As expected, the best performance for Algs. 2 and 3 is achieved when echo and US NEXT are both canceled. The achievable sum bit rates in this case are significantly higher than the achievable sum bit rates for the same vectoring schemes [9, 10] using a fixed VDSL2 bandplan. In fact, they are almost equal to the achievable bit rates as if the entire band is fully available to both DS and US transmission (even though DS NEXT is not cancelled). When only echo is canceled but US NEXT is not canceled, the achievable sum bit rates are lower but still much higher than the achievable sum bit rates by the same vectoring schemes using a fixed VDSL2 bandplan.

When echo is not canceled (whether or not US NEXT is canceled), the rate region takes a right-triangular shape with its hypotenuse connecting the points at which the entire bandwidth is allocated to either DS or US. This is due to the fact that the echo signal is much stronger than the desired signal such that when transmitting power on a tone in US/DS, the SNR for DS/US is too small to justify DS/US transmission. In fact the performance of Alg. 1 for systems with disjoint bands is exactly the same as Alg. 2 and Alg. 3 when echo is present. Therefore, for higher readability of the figure, we have not plotted the results for Alg. 1 and for Alg. 2 in the presence of echo.

The rate region for vectoring with a fixed bandplan has a rectangular shape. This of course is an expected behavior as DS and US transmissions are then separated and do not affect each other.

Finally, we compare the performance of Alg. 2 with that of Alg. 3 for the cases with echo cancellation. It can be observed that Alg. 2 performs the same as Alg. 3 when higher bit rates are demanded in US direction. However, Alg. 2 does not achieve the same sum bit rates in DS direction as Alg. 3 when higher bit rates are demanded in the DS direction. This is because the DS and US optimization are separated in Alg. 2. Since the US optimization is carried out first (see Alg. 2) too many bits (requiring too much power) can be loaded to some tones in the US direction. In the subsequent DS optimization, the excessive crosstalk power from the US direction leads to a reduced power and bit loading on these tones in the DS direction. Consequently, the algorithm converges to a local optimum leading to a suboptimal performance.

V. Conclusion

In this paper, we have studied dynamic bandplanning (DBP) in vectored DSL. We have proposed an optimal algorithm for DBP in DSL with disjoint bands and two iterative sub-optimal algorithms for DBP in DSL with overlapping bands. We have studied the performance of the proposed algorithms by simulations which show that DBP can significantly increase the achievable bit rates of high speed DSL systems when vectoring is enabled. In systems with disjoint DS and US bands, DBP can increase the achievable bit rates by allocating the available bandwidth to the DS and US transmission much more efficiently, taking into account the topology and instant bit rate requirements. Much higher bit rates, however, can be achieved by letting DS and US bands overlap and by including echo cancellation. In this case, if US NEXT is also canceled, the bit rates approach those of a full-duplex system.

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Algorithm 1: DBP with disjoint bands

input : DS/US direct and FEXT channels, DS/US PSD masks, DS/US per-modem total power budgets

output: DS/US bandplans, DS/US bit loadings, DS/US transmit powers, DS transmit and US receive filters, DS encoding and US decoding orders

repeat
  Set/update $\lambda_{n,\mathrm{DS/US}},\ \forall\ n$;
  for $k = 1$ **to** K **do**
    Calculate the maximum achievable Lagrangian in DS direction, $\hat{\mathcal{L}}^{k}_{\mathrm{DS}}$, by solving (3.6) using BC-OSB Algorithm [10];
    Calculate the maximum achievable Lagrangian in US direction, $\hat{\mathcal{L}}^{k}_{\mathrm{US}}$, by solving (3.7) using MAC-OSB Algorithm [9];
    if $\hat{\mathcal{L}}^{k}_{\mathrm{DS}} > \hat{\mathcal{L}}^{k}_{\mathrm{US}}$ **then**
      $\mathcal{K}_{\mathrm{DS}} \leftarrow \mathcal{K}_{\mathrm{DS}} \cup \{k\}$;
      $\mathcal{K}_{\mathrm{US}} \leftarrow \mathcal{K}_{\mathrm{US}} \backslash \{k\}$;
      Store/update the optimal solution on tone k in DS direction;
    else
      $\mathcal{K}_{\mathrm{US}} \leftarrow \mathcal{K}_{\mathrm{US}} \cup \{k\}$;
      $\mathcal{K}_{\mathrm{DS}} \leftarrow \mathcal{K}_{\mathrm{DS}} \backslash \{k\}$;
      Store/update the optimal solution on tone k in US direction;
  $P_{n,\mathrm{DS/US}} \leftarrow \Delta_{\mathrm{f}} \sum_{k=1}^{K} p^{k,\ \mathrm{opt}}_{n,\mathrm{DS/US}},\ \forall\ n$;
until $P_{n,\mathrm{DS/US}} = P^{\max}_{n,\mathrm{DS/US}}$ or $(P_{n,\mathrm{DS/US}} < P^{\max}_{n,\mathrm{DS/US}}$ and $\lambda_{n,\mathrm{DS/US}} = 0),\ \forall\ n$;

Algorithm 2: DBP with overlapping bands and separate DS/US power optimization

input : DS/US direct, echo, FEXT and NEXT channels, DS/US PSD masks, DS/US per-modem total power budgets

output: DS/US bit loadings, DS/US transmit powers, DS transmit and US receive filters, DS encoding and US decoding orders

repeat
  Set/update $\lambda_{n,\mathrm{DS/US}}$, $\forall\ n$;
  for $k = 1$ **to** K **do**
    repeat
      `/* US Power and Receiver Filter Optimization (at fixed DS power and transmit filters) */`
      Set $\boldsymbol{u}^k_{\mathrm{DS}}$ and $\boldsymbol{Q}^k_{\mathrm{DS}}$ to the so-far optimal solution on tone k;
      for all US bit loading vectors $\boldsymbol{b}^k_{\mathrm{US}}$ in the range **do**
        for $n =$ *last decoded user in US direction* **to** *first decoded user* **do**
          $\gamma^k_{n,\mathrm{US,des}} \leftarrow \Gamma\left(2^{b_{n,\mathrm{US}}} - 1\right)$;
          Update US receiver filter and transmit power for user n, ($\boldsymbol{q}^k_{n,\mathrm{US}}$ and $u^k_{n,\mathrm{US}}$) to reach $\gamma^k_{n,\mathrm{US,des}}$;
        Update achievable SNRs and bit loadings (only in DS direction);
        if $p^k_{n,\mathrm{DS/US}} \leq p^{k,\mathrm{mask}}_{n,\mathrm{DS/US}}, \forall n$ and $\mathcal{L}^k > \mathcal{L}^k_{\max}$ **then**
          $\mathcal{L}^k_{\max} \leftarrow \mathcal{L}^k$;
          Store/update the optimal solution on tone k;
      `/* DS Power and Transmit Filter Optimization in Dual Space (at fixed US power and receiver filters) */`
      Set $\boldsymbol{u}^k_{\mathrm{DS/US}}$ and $\boldsymbol{Q}^k_{\mathrm{DS/US}}$ to the so-far optimal solution on tone k;
      Calculate transmit powers in dual channel using (3.27);
      for all DS bit loading vectors $\boldsymbol{b}^k_{\mathrm{DS}}$ in the range **do**
        for $n =$ *first encoded user in DS direction* **to** *last encoded user* **do**
          $\gamma^k_{n,\mathrm{DS,des}} \leftarrow \Gamma\left(2^{b_{n,\mathrm{DS}}} - 1\right)$;
          Update DS receiver filter and power for user n, ($\boldsymbol{q}^k_{\mathrm{DS},n}$ and $\tilde{u}^k_{n,\mathrm{DS}}$) to reach $\gamma^k_{n,\mathrm{DS,des}}$;
        Update achievable SNRs and bit loadings (only in US direction);
        Calculate transmit powers in primal channel, $\boldsymbol{u}^k_{\mathrm{DS/US}}$, using (3.26);
        if $p^k_{n,\mathrm{DS/US}} \leq p^{k,\mathrm{mask}}_{n,\mathrm{DS/US}}, \forall n$ and $\mathcal{L}^k > \mathcal{L}^k_{\max}$ **then**
          $\mathcal{L}^k_{\max} \leftarrow \mathcal{L}^k$;
          Store/update the optimal solution on tone k;
    until *convergence/divergence/maximum # of iterations*;
until $P_{n,\mathrm{DS/US}} = P^{\max}_{n,\mathrm{DS/US}}$ or ($P_{n,\mathrm{DS/US}} < P^{\max}_{n,\mathrm{DS/US}}$ and $\lambda_{n,\mathrm{DS/US}} = 0$), $\forall\ n$;

Algorithm 3: DBP with overlapping bands and joint DS/US power optimization

input : DS/US direct, echo, FEXT and NEXT channels, DS/US PSD masks, DS/US per-modem total power budgets

output: DS/US bit loadings, DS/US transmit powers, DS transmit and US receive filters, DS encoding and US decoding orders

repeat
  Set/update $\lambda_{n,\mathrm{DS/US}},\ \forall\ n$;
  for $k = 1$ **to** K **do**
    for all bit loading vectors $\left(\boldsymbol{b}^k_{\mathrm{DS}}, \boldsymbol{b}^k_{\mathrm{US}}\right)$ in the range **do**
      $\boldsymbol{u}^k_{\mathrm{DS}} \leftarrow \mathbf{0}$;
      $\boldsymbol{Q}^k_{\mathrm{DS}} \leftarrow \boldsymbol{I}_N$;
      $\gamma^k_{n,\mathrm{DS/US,des}} \leftarrow \Gamma\left(2^{b_{n,\mathrm{DS/US}}} - 1\right), \forall n$;
      repeat
        `/* US Power and receiver Filter Optimization (at fixed DS power and transmit filters) */`
        for $n =$ *last decoded user in US direction* **to** *first decoded user* **do**
          Update US receiver filter and transmit power for user n, ($\boldsymbol{q}^k_{n,\mathrm{US}}$ and $u^k_{n,\mathrm{US}}$) to reach $\gamma^k_{n,\mathrm{US,des}}$;
        Update achievable SNRs (only in DS direction);
        `/* DS Power and Transmit Filter Optimization in Dual Space (at fixed US power and receiver filters) */`
        Calculate transmit powers in dual channel using (3.27);
        for $n =$ *first encoded user in DS direction* **to** *last encoded user* **do**
          Update DS receiver filter and power for user n, ($\boldsymbol{q}^k_{\mathrm{DS},n}$ and $\tilde{u}^k_{n,\mathrm{DS}}$) to reach $\gamma^k_{n,\mathrm{DS,des}}$;
        Update achievable SNRs (only in US direction);
        Calculate transmit powers in primal channel, $\boldsymbol{u}^k_{\mathrm{DS/US}}$, using (3.26);
      until *convergence/divergence/maximum # of iterations*;
      if *convergence,* $p^k_{n,\mathrm{DS/US}} \leq p^{k,\mathrm{mask}}_{n,\mathrm{DS/US}}, \forall n$ and $\mathcal{L}^k > \mathcal{L}^k_{\max}$ **then**
        $\mathcal{L}^k_{\max} \leftarrow \mathcal{L}^k$;
        Store/update the optimal solution on tone k;
until $P_{n,\mathrm{DS/US}} = P^{\max}_{n,\mathrm{DS/US}}$ or ($P_{n,\mathrm{DS/US}} < P^{\max}_{n,\mathrm{DS/US}}$ and $\lambda_{n,\mathrm{DS/US}} = 0$), $\forall\ n$;

Table I: Simulation Parameters

PARAMETER	VALUE
Bandplan and PSD mask	DS VDSL2E17 B7-9 bandplan [2,3]
Cable type	26 AWG [25]
Noise	White noise, -140 dBm/Hz
Tone spacing, Δ_{f}	4.3125 kHz
Symbol rate, f_{s}	4 kHz
$b_{\max}$	15
$b_{\min}$	2
SNR Gap	9.45 dB
Echo signal's gain	-15 dB

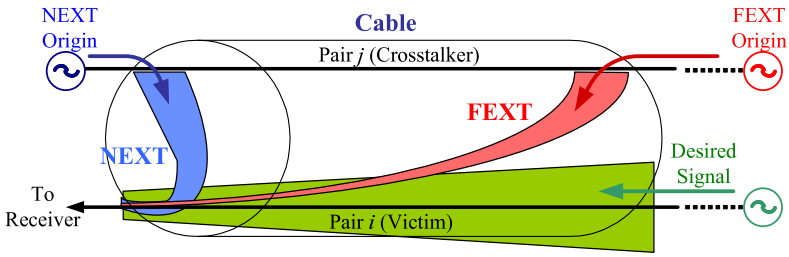


Figure 1: Near-end and far-end crosstalk in DSL.

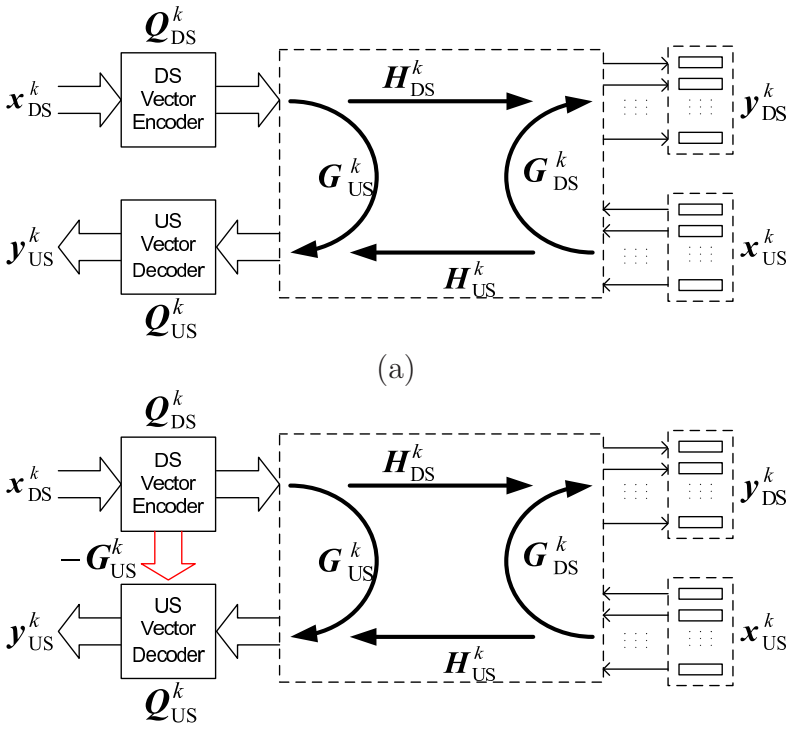


Figure 2: (Schematic) Block diagram of dynamic bandplanning (a) without US NEXT cancellation and (b) with US NEXT cancellation.

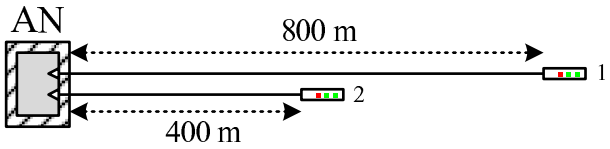


Figure 3: Simulation scenario.

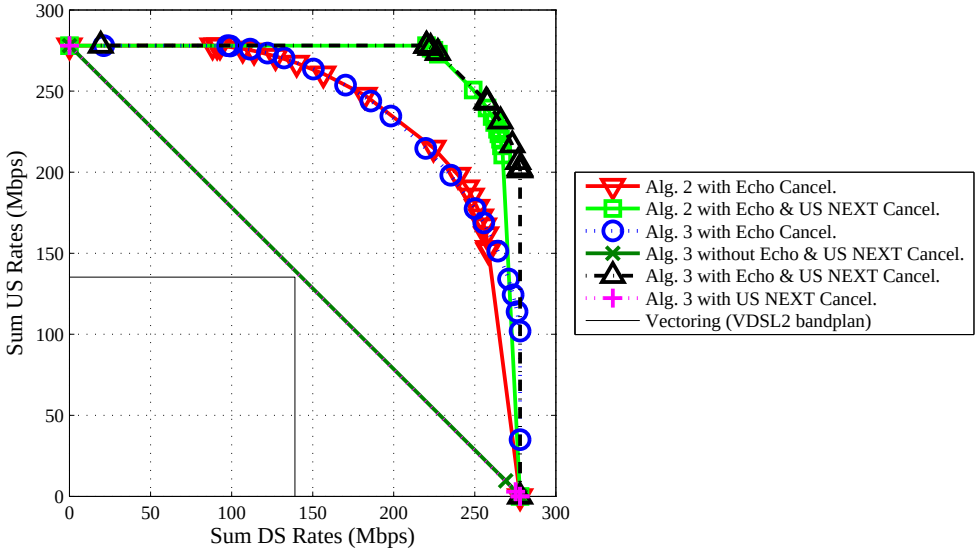


Figure 4: Achievable sum bit rate of the users in US direction vs. achievable sum bit rate of the users in DS direction for the scenario in Fig. 3 using different vector DBP schemes.