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Optimal Robot Path Following for Minimal Time versus Energy Loss Trade-Off using Sequential Convex Programming

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***Abstract*—This paper focusses on the time-energy optimal path following for robots. This considers the problem of moving along a predetermined geometric path with a minimal trade-off between the motion time and the two major thermal energy losses in electric actuators. Theses losses consist of resistive electrical losses and mechanical friction losses. When only taking into account the electrical losses for a simplified robotic manipulator, a convex reformulation has been derived previously [1]. In this paper we include the dynamic joint friction losses into the objective. This also implies that we have to include the dynamic joint friction into the robot equations of motion, which appear in the torque constraints. Both the resulting objective and torque constraints are non-convex. The present paper proposes an efficient sequential convex programming (SCP) approach to solve the resulting optimal control problem. A key step here is to decompose the non-convex functions involved as a difference of convex functions. Numerical simulations illustrate the fast convergence of the proposed method in only a few SCP iterations, confirming the efficiency of the proposed framework. This high efficiency allows for an efficient tool to investigate the trade off between time-optimality and energy-optimality.**

I. Introduction

Path following deals with the problem of following a geometric path without any preassigned timing information. Many industrial robot tasks, such as welding, glueing, laser cutting and milling can be cast as path following problems. In addition, path following is often considered to be the low level stage in a decoupled motion planning approach [2], [3]. First, a high level planner determines a geometric path ignoring the system dynamics but taking into account geometric path constraints. Second, an optimal trajectory along the geometric path is determined that accounts for the system dynamics and limitations. Since the dynamics along a geometric path can be described by a scalar path coordinate s and its time derivatives [2], [3], the decoupled approach simplifies the motion planning problem to great extent. Recently it was shown that the time-optimal path following problem for a robotic manipulator with simplified system dynamic, e.g. in the absence of dynamic joint friction, can be cast as a *convex* optimization problem [1]. This guarantees the efficient computation of globally optimal solutions.

Since true time-optimal trajectories result in significant thermal energy losses [1], it is interesting to look at optimal trajectories that minimize a trade-off between thermal energy losses and motion time. In this paper we consider two major causes of thermal energy losses, i.e. resistive electrical energy losses and dynamic friction energy losses in the robot joints. Considering only the electrical losses results in a convex optimization problem. However, many practical robotic systems suffer from significant dynamic friction in the joints. Including the dynamic friction energy losses result in a non-convex optimization problem, since the dynamic joint friction introduces non-convexity in the torque constraints and in the energy loss term in objective function. To account for this non-convexity we propose an extension of the convex framework of [1] using sequential convex programming (SCP) since these non-convex constraints and objective function can easily be written as a difference of two convex (DC) functions [4], [5]. In SCP the concave parts of the objective function or constraints are sequentially linearised while preserving the convex parts [6]–[8]. Fast convergence of this method in only four to five iterations is observed in the numerical examples discussed in this paper, making it a highly efficient framework.

The paper is organized as follows, Section 2 reviews the convex reformulation of the time-optimal path following problem of [1] and states the non-convex time-energy optimal path following problem for robots with dynamic joint friction. Section 3 describes DC decompositions of the non-convex constraints and objective function and reviews a sequential convex programming approach [6]–[8] to efficiently solve the non-convex time-energy optimal path following problem. To illustrate the practical use and efficiency of the proposed SCP approach, Section 4 discusses some numerical simulations.

Throughout the paper we will use the following shorthand notations for the derivatives of a function $f(s(t))$: $\dot{f} = \frac{df}{dt}, \ddot{f} = \frac{d^2f}{dt^2}, f' = \frac{\partial f}{\partial s}$ and $f'' = \frac{\partial^2 f}{\partial s^2}$ where t indicates time and s the path coordinate. Furthermore, we indicate scalars with a lower-case letter, e.g. n, vectors with a bold lower-case letter, e.g. $\boldsymbol{q}$, and matrices with an upper-case letter, e.g. M. $\boldsymbol{q}_i$ denotes the i-th element of $\boldsymbol{q}$. To indicate an all-ones vector of size n we use $\mathbf{1}_n \in \mathbb{R}^n$.

II. Problem formulation

A. Time-optimal path following problem

Consider a robotic manipulator with n degrees of freedom and joint angles $\boldsymbol{q} \in \mathbb{R}^n$. The equations of motion are given by

$$\boldsymbol{\tau} = M(\boldsymbol{q})\ddot{\boldsymbol{q}} + C(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + H(\boldsymbol{q})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) \tag{1}$$

where $\boldsymbol{\tau} \in \mathbb{R}^n$ are the joint torques, $M \in \mathbb{R}^{n\times n}$ is the mass matrix, $C \in \mathbb{R}^{n\times n}$ is a matrix, linear in $\dot{\boldsymbol{q}}$, accounting for Coriolis and centrifugal effects and, $H \in \mathbb{R}^{n\times n}$ is a diagonal matrix accounting for dynamic friction in the joints, $\boldsymbol{g}$ is a vector accounting for gravity and other position-dependent torques.

Consider a prescribed geometric path $\boldsymbol{q}(s)$ as a function of a scalar path coordinate s, given in joint space coordinates. The time dependence of the path is determined through $s(t)$. Without loss of generality it is assumed that the trajectory starts at $t = 0$, ends at $t = T$, and $0 = s(0) \leq s(t) \leq s(T) = 1$. It is furthermore assumed that we always move forward along the path, i.e. $\dot{s}(t) \geq 0, \forall t \in [0, T]$.

Using the chain-rule we write joint velocities and accelerations as

$$\dot{\boldsymbol{q}}(s) = \boldsymbol{q}'(s)\dot{s} \text{ and, } \ddot{\boldsymbol{q}}(s) = \boldsymbol{q}''(s)\dot{s}^2 + \boldsymbol{q}'(s)\ddot{s}, \tag{2}$$

where $\boldsymbol{q}' = d\boldsymbol{q}(s)/ds$ and, $\boldsymbol{q}'' = d^2\boldsymbol{q}(s)/ds^2$. Substitution of the above equations in (1) projects the equations of motion onto the path [1]:

$$\boldsymbol{\tau}(s) = \boldsymbol{m}(s)\ddot{s} + \boldsymbol{c}(s)\dot{s}^2 + \boldsymbol{h}(s)\dot{s} + \boldsymbol{g}(s). \tag{3}$$

where $\boldsymbol{h}(s) = H(\boldsymbol{q}(s))\boldsymbol{q}(s)'$. By introducing the variable

$$b(s) = \dot{s}^2, \tag{4}$$

where $b'(s) = 2\ddot{s}$, it was shown in [1] that the time-optimal path following problem without dynamic friction in the joints is transformed into a *convex* optimization problem. By using (3) and (4), a torque constraint $\boldsymbol{\tau}_- \leq \boldsymbol{\tau} \leq \boldsymbol{\tau}_+$ can be written in the form

$$\boldsymbol{\tau}_- \leq \boldsymbol{m}(s)\frac{b'(s)}{2} + \boldsymbol{c}(s)b(s) + \boldsymbol{h}(s)\sqrt{b(s)} + \boldsymbol{g}(s) \leq \boldsymbol{\tau}_+ \tag{5}$$

In the case of no dynamic joint friction ($\boldsymbol{h}(s) = 0$), this constraint is linear in $b(s)$ and $b'(s)$, and hence convex. Furthermore, since $b(s) \geq 0$, the total motion time

$$T = \int_0^T 1dt = \int_0^1 \frac{1}{\dot{s}}ds = \int_0^1 \frac{1}{\sqrt{b(s)}}ds, \tag{6}$$

is a convex function. To conclude, the time-optimal path following problem is then reformulated as

$$\begin{aligned} \underset{b(\cdot),\boldsymbol{\tau}(\cdot)}{\text{minimize}} \quad & \int_0^1 \frac{1}{\sqrt{b(s)}}ds \\ \text{subject to} \quad & b(0) = \dot{s}_0^2, b(1) = \dot{s}_T^2 \\ & b(s) \geq 0 \\ & \boldsymbol{\tau}_- \leq \boldsymbol{m}(s)\frac{b'(s)}{2} + \boldsymbol{c}(s)b(s) \\ & \quad + \boldsymbol{h}(s)\sqrt{b(s)} + \boldsymbol{g}(s) \leq \boldsymbol{\tau}_+ \\ & \text{for } s \in [0, 1]. \end{aligned} \tag{7}$$

Here, the dynamic friction term introduces a non-convexity in the torque constraints depending on the sign of $\boldsymbol{h}_i(s)$.

B. Time-energy optimal path following problem

We now consider the optimal control problem that minimizes a trade-off between the total motion time and the two most important thermal energy losses. We consider electrical resistive losses and dynamic friction losses.

The electrical resistive energy loss for joint i is equal to

$$\boldsymbol{e}_{\boldsymbol{e}i} = \int_0^T R_i\lambda_i^2\boldsymbol{\tau}_i(s)^2dt = \int_0^1 \frac{R_i\lambda_i^2\boldsymbol{\tau}_i(s)^2}{\sqrt{b(s)}}ds, \tag{8}$$

where R_i is the electrical resistance and λ_i is the constant that relates the motor current to the joint torque. It can be shown [9] that $x^2/\sqrt{y}$ is a convex function of (x, y), for $y \geq 0$, hence the electrical energy losses are convex.

The mechanical friction energy loss due to dynamic friction for joint i is given by

$$\boldsymbol{e}_{\boldsymbol{m}i} = \int_0^T H_{ii}(\boldsymbol{q_i}(s))\dot{\boldsymbol{q}}_i(s)^2dt = \int_0^1 \boldsymbol{h}_i(s)\boldsymbol{q'}_i(s)\sqrt{b(s)}ds. \tag{9}$$

Since $H_{ii}(\boldsymbol{q_i}(s)) \geq 0 \ \forall s \in [0, 1]$, and hence $H_{ii}(\boldsymbol{q_i}(s))\boldsymbol{q'}_i(s)^2 = \boldsymbol{h}(s)\boldsymbol{q'}_i(s) \geq 0 \ \forall s \in [0, 1]$, the mechanical friction loss is non-convex.

The total thermal energy loss can be written as the sum of the mechanical friction losses and electrical resistive losses for all joints

$$\begin{aligned} e &= \sum_{i=1}^n \boldsymbol{e}_{\boldsymbol{e}i} + \sum_{i=1}^n \boldsymbol{e}_{\boldsymbol{m}i} \\ &= \sum_{i=1}^n \int_0^1 \frac{R_i\lambda_i^2\boldsymbol{\tau}_i(s)^2}{\sqrt{b(s)}}ds + \sum_{i=1}^n \int_0^1 \boldsymbol{h}_i(s)\boldsymbol{q'}_i(s)\sqrt{b(s)}. \end{aligned}$$

Hence, the time-energy optimal path following problem can be written as

$$\begin{aligned} \underset{b(\cdot)}{\text{minimize}} \quad & \int_0^1 \frac{1}{\sqrt{b(s)}}ds + \gamma\int_0^1 \sum_{i=1}^n \frac{R_i\lambda_i^2\boldsymbol{\tau}_i(s)^2}{\sqrt{b(s)}}ds \\ & + \gamma\int_0^1 \sum_{i=1}^n \boldsymbol{h}_i(s)\boldsymbol{q'}_i(s)\sqrt{b(s)}ds \\ \text{subject to} \quad & b(0) = \dot{s}_0^2, b(1) = \dot{s}_T^2 \\ & b(s) \geq 0 \\ & \boldsymbol{\tau}_- \leq \boldsymbol{m}(s)\frac{b'(s)}{2} + \boldsymbol{c}(s)b(s) \\ & \quad + \boldsymbol{h}(s)\sqrt{b(s)} + \boldsymbol{g}(s) \leq \boldsymbol{\tau}_+ \\ & \text{for } s \in [0, 1]. \end{aligned} \tag{10}$$

where γ is the trade-off between time-optimality and energy-optimality. The electric losses are convex in the variable $\boldsymbol{\tau}_i$ which can be found from the linearised torque constraints according to Section 3.2. The dynamic friction energy losses introduce non-convexity into the objective function in addition to the non-convexity introduced by the dynamic friction in the torque constraints. This non-convexity in the objective function is always non-convex since $\boldsymbol{h}(s)\boldsymbol{q'}_i(s) \geq 0 \ \forall s \in [0, 1]$.

The non-convexity in the torque constraints however depends on the sign of $\boldsymbol{h}(s)$. Both the non-convex objective and the torque constraints, when non-convex, can easily be decomposed into a difference of convex functions. The following section illustrates this decomposition and reviews a sequential convex programming approach to efficiently solve the resulting non-convex optimization problem with DC objective and DC constraints.

Since the resulting time-optimal path following problem has an infinite number of optimization variables and an infinite number of constraints, it is discretized by adopting the direct transcription method from [1].

III. Sequential convex programming algorithm

A. SCP algorithm

Consider the optimization problem with DC (difference of two convex functions) constraints and objective:

$$\begin{aligned} \underset{\boldsymbol{x}\in\mathbb{R}^n}{\text{minimize}} \quad & f(\boldsymbol{x}) - g(\boldsymbol{x}) \\ \text{subject to} \quad & \boldsymbol{u}_i(\boldsymbol{x}) - \boldsymbol{v}_i(\boldsymbol{x}) \leq 0,\ i = 1, \dots, l \\ & \boldsymbol{x} \in \Omega, \end{aligned} \tag{11}$$

where $f : \mathbb{R}^n \mapsto \mathbb{R}$ and $g : \mathbb{R}^n \mapsto \mathbb{R}$ are convex, $\Omega \subseteq \mathbb{R}^n$ is a nonempty, closed convex set, and $\boldsymbol{u}_i$ and $\boldsymbol{v}_i$ are convex functions. The feasible set is denoted by

$$\mathcal{D} = \{\boldsymbol{x} \in \Omega \mid \boldsymbol{u}_i(\boldsymbol{x}) - \boldsymbol{v}_i(\boldsymbol{x}) \leq 0, i = 1, \dots, l\}$$

and its relative interior by

$$\mathrm{ri}(\mathcal{D}) = \{\boldsymbol{x} \in \mathrm{ri}(\Omega) \mid \boldsymbol{u}_i(\boldsymbol{x}) - \boldsymbol{v}_i(\boldsymbol{x}) < 0, i = 1, \dots, l\}.$$

The main idea of the algorithm is to iteratively linearise the concave part of the DC objective and DC inequality constraint, to transform the problem into a convex optimization problem, which can be solved efficiently.

Suppose that $\boldsymbol{x}^k \in \Omega$ is a given point, then the linearized problem of (11) around $\boldsymbol{x}^k$ is

$$\begin{aligned} \underset{\boldsymbol{x}\in\mathbb{R}^n}{\text{minimize}} \quad & f(\boldsymbol{x}) - g(\boldsymbol{x}^k) - \nabla_{\boldsymbol{x}} g(\boldsymbol{x}^k)(\boldsymbol{x} - \boldsymbol{x}^k) + \tfrac{\beta}{2}\|\boldsymbol{x} - \boldsymbol{x}^k\|_2^2 \\ \text{subject to} \quad & \boldsymbol{u}_i(\boldsymbol{x}) - \boldsymbol{v}_i(\boldsymbol{x}^k) - \nabla_{\boldsymbol{x}} \boldsymbol{v}_i(\boldsymbol{x}^k)(\boldsymbol{x} - \boldsymbol{x}^k) \leq 0, \\ & i = 1, \dots, l, \\ & \boldsymbol{x} \in \Omega, \end{aligned} \tag{12}$$

where a regularization term with parameter β is added to the objective function. The following algorithm for solving (11) is proposed in [4], [6], [7].

Sequential convex programming algorithm

Initialization: Choose a $\beta > 0$ and find an initial point $\boldsymbol{x}^0 \in \mathrm{ri}(\mathcal{D})$. Set $k = 0$.

Iteration k: For $k = 0, 1, \dots$ do

1) Solve the convex problem (12) to obtain a solution $\boldsymbol{x}^{k+1}$.
2) If $\|\boldsymbol{x}^{k+1} - \boldsymbol{x}^k\| \leq \varepsilon$ for a given tolerance $\varepsilon > 0$ then terminate. Otherwise, set $k = k + 1$ and go to step 1.

Note that when the initial point $\boldsymbol{x}^0 \in \mathrm{ri}(\mathcal{D})$, then the algorithm generates a sequence of points $\boldsymbol{x}^k$ which also belongs to $\mathcal{D}$. Geometrically, the algorithm can be seen as an inner approximation method. The feasible set of the linearized problem is an inner approximation of the true feasible set and, since a first order Taylor expansion of a concave function is always an overestimation, the objective is an overestimation of the true objective.

Under fairly mild assumptions, which in practice come down to the problem being well posed, it can be shown that the SCP algorithm converges[1] to a stationary point of (11) [7].

Furthermore, as we are interested in motions that optimize a trade-off between time and thermal energy losses, the stopping criterion may be changed to

$$||\text{objective}^{k+1} - \text{objective}^{k}|| \leq \varepsilon.$$

B. DC decomposition

To apply the algorithm to problem (10), we need to determine a DC decomposition of the torque constraint (5) and the objective by examinating the convexity. The torque constraint (5) is non-convex in the upper bound whenever $\boldsymbol{h}_i(s) > 0$ and in the lower bound whenever $\boldsymbol{h}_i(s) < 0$. Hence an obvious DC decomposition then is

$$\boldsymbol{u}_i(b(s)) = \boldsymbol{m}_i(s)\frac{b'(s)}{2} + \boldsymbol{c}_i(s)b(s) + \boldsymbol{g}_i(s) - \boldsymbol{\tau}_{+,i}$$

$$\boldsymbol{v}_i(b(s)) = -\boldsymbol{h}_i(s)\sqrt{b(s)}$$

for the non-convex upper bound and

$$\boldsymbol{u}_i(b(s)) = -\boldsymbol{m}_i(s)\frac{b'(s)}{2} - \boldsymbol{c}_i(s)b(s) - \boldsymbol{g}_i(s) + \boldsymbol{\tau}_{-,i}$$

$$\boldsymbol{v}_i(b(s)) = \boldsymbol{h}_i(s)\sqrt{b(s)}$$

for the non-convex lower bound.

Since $\boldsymbol{h}(s)\boldsymbol{q'}_i(s) \geq 0\ \forall s \in [0, 1]$, the dynamic friction loss (9) is concave and, since both the time (6) and the electrical losses (8) are convex, an obvious DC decomposition for the objective function is

$$f(b(s)) = \int_0^1 \frac{1}{\sqrt{b(s)}} ds + \gamma \int_0^1 \sum_{i=1}^{n} \frac{R_i \lambda_i^2 \boldsymbol{\tau}_i(s)^2}{\sqrt{b(s)}} ds,$$

and

$$g(b(s)) = -\gamma \int_0^1 \sum_{i=1}^{n} \boldsymbol{h}_i(s)\boldsymbol{q'}_i(s)\sqrt{b(s)} ds.$$

[1] In [7] they consider the convergence of (11) with convex objective ($g(\boldsymbol{x}) = 0$) and DC constraints. By introducing the concave part $-g(\boldsymbol{x})$ of the objective into the DC constraints through the introduction of a slack variable, it is shown that (11) with $g(\boldsymbol{x}) \neq 0$ also converges.

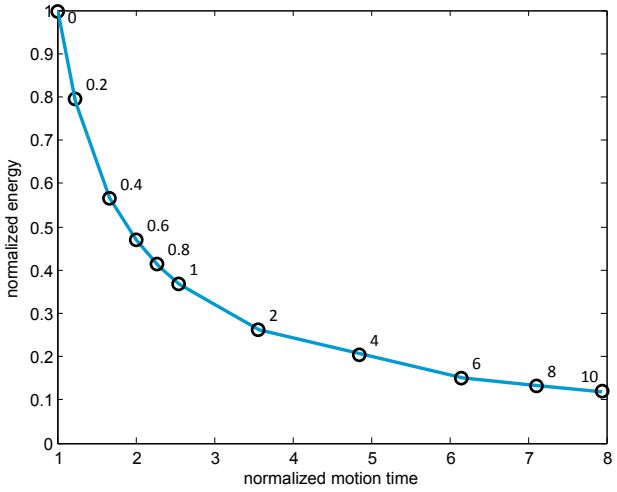


Fig. 1. Time versus energy loss for several values of γ.

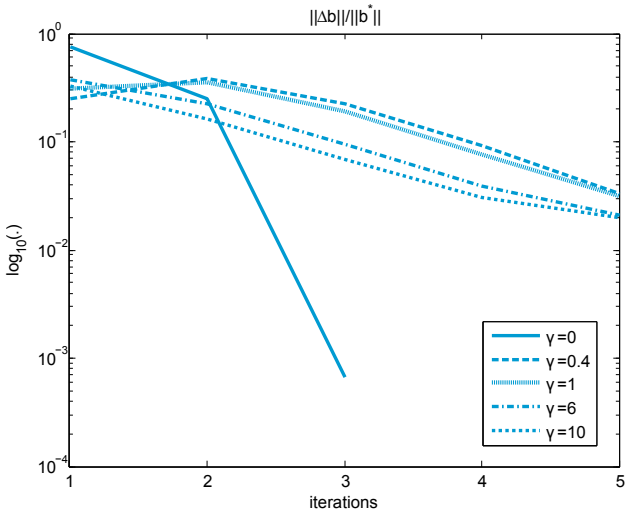


Fig. 2. Convergence of the objective for several values of γ

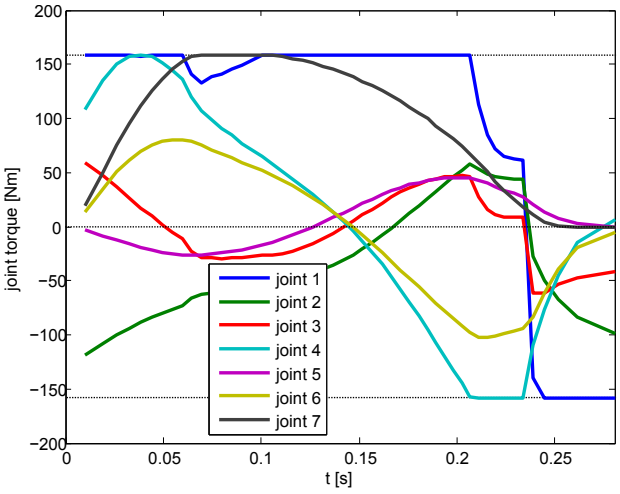


Fig. 3. Joint torque in function of time for $\gamma = 0$.

IV. Numerical Examples

In this section we give numerical simulations which illustrate the efficiency of the proposed SCP framework. We consider a seven-dof robot executing a linear Cartesian path. Furthermore, since the electrical losses are already treated in [1] and to illustrate the SCP approach we only consider dynamic friction losses. So we neglect the thermal energy losses due to the electric resistance.

We consider the time-energy optimal path following problem for a robotic manipulator with dynamic friction in the joints (1 N m s/rad), which is solved for several values of γ using the proposed SCP approach with $\varepsilon = 10^{-2.5}$.

Figure 1 shows the motion time versus the mechanical energy loss for several values of γ. Both objectives are normalized by division by their value for $\gamma = 0$. The shallow slope of the curve on the right indicates that energy-optimal motions are very slow but initially a large reduction in motion time can be achieved with a minor increase in energy consumption. Vice versa, the steep slope on the left indicates that time-optimal motions come a significant energy cost. Although a minor reduction in energy consumption can already be achieved with a minor increase in motion time.

The convergence of the SCP algorithm is given in Figure 2 for several values of γ with $\varepsilon = 10^{-2.5}$. Here the normalised residual ($||\Delta \boldsymbol{b}||/||\boldsymbol{b}^*||$ with $\Delta \boldsymbol{b} = \boldsymbol{b}^k - \boldsymbol{b}^{k-1}$) is shown for each iteration k. It can be concluded that the proposed SCP approach converges fast to an accurate solution. On average we see a convergence of four to five iterations for varying values of dynamic friction and values for the trade-off parameter γ.

To see the impact of γ on the joint torques and velocities we compare two situations: $\gamma = 0$ and $\gamma = 0.2$. Figure 3 and Figure 4 show the joint torques and velocities in function of time for the joints of the robot for $\gamma = 0$. Due to the 100% time optimality we see a bang-bang behaviour of the joint torques, at any time instance one of the joint torques hits one of the torque bounds. Figure 5 and Figure 6 show the joint torques and velocities in function of time for $\gamma = 0.2$. Here, due to the trade-off between time-optimality and energy-optimality, we can see that the torques do not hit the bounds everywhere, hence resulting in a longer motion time ($\sim 20\%$). However, due to lower joint speeds, there is less dynamic friction loss ($\sim 30\%$).

V. Discussion

The SCP approach given in section 3 is very appealing from a theoretical and practical point of view. Theoretically it ensures that a stationary point of the non-convex optimal path following problem (10) is attained, Furthermore, the solution is found with high accuracy in only a few iterations, which is important from a practical point of view. All numerical examples given here were performed off-line, but thanks to its high efficiency, the proposed SCP approach has potential to be used in on-line applications.

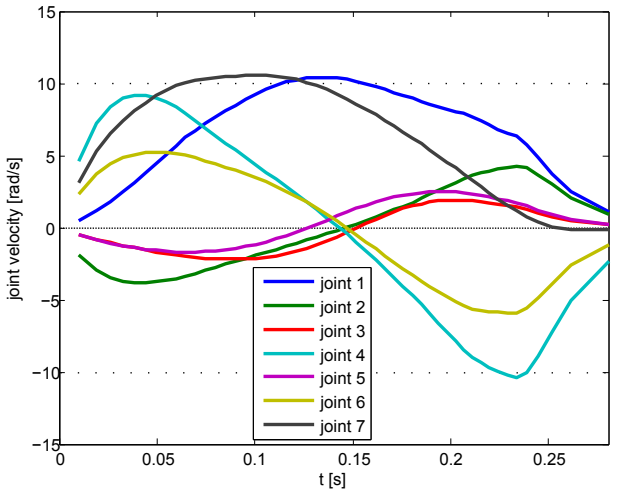


Fig. 4. Joint velocity in function of time for $\gamma = 0$.

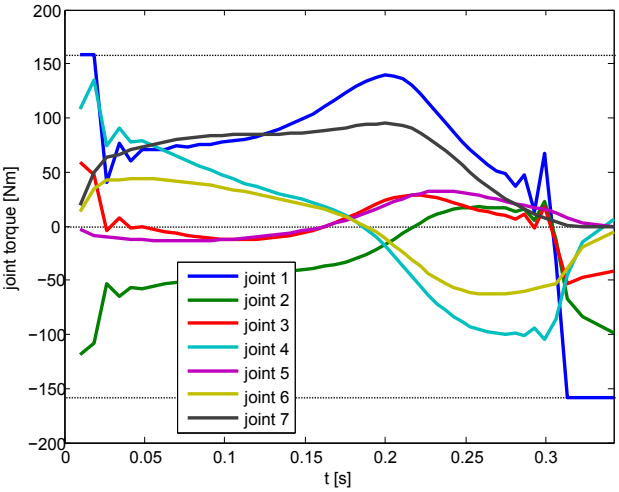


Fig. 5. Joint torque in function of time for $\gamma = 0.2$.

Also, the SCP approach can easily be implemented. The algorithm can be built around the convex optimal path following framework from [1] with minimal extra effort. The DC decompositions involve only a few lines of code.

Furthermore, the SCP framework proposed in this paper counters a major downside of the convex framework [1]. In [1] the dynamic friction in the joints cannot be taken into account into the robot dynamics and thermal losses. The proposed SCP approach however is able to deal with the introduced non-convexity in an efficient way and converges in only four to five iterations.

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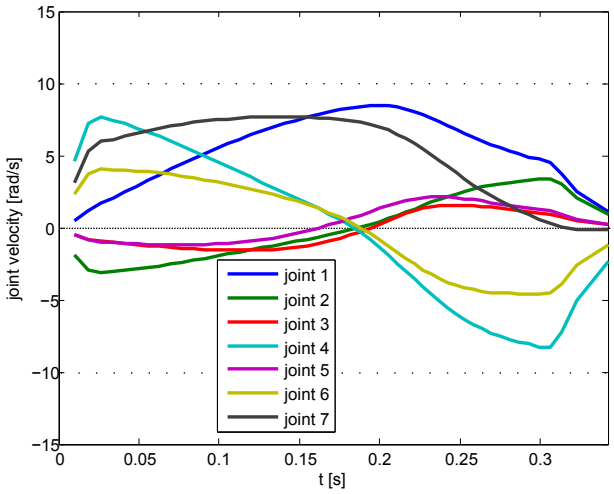


Fig. 6. Joint velocity in function of time for $\gamma = 0.2$.

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