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CANONICAL FORMS FOR SINGULAR SYSTEMS

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Abstract

We extend the well-known canonical froms for regular systems to singular systems. In particular, the generalized controller canonical form and the generalized observer canonical form are derived in detail.

1. Introduction

Throughout the paper we consider singular systems of the form

$$\begin{aligned} E\dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \qquad (1)$$

where E and A are real nxn matrices, B is a real nxq matrix and C is a real pxn matrix. In particular, E can be singular. In the past few years, the study of such systems has been largely devoted to its controllability and observability. Many results of this kind have been established (see, for example, Rosenbrock [1], Verghese et al. [3], Yip and Sincovec [5], Cobb [2]). However, the results concerning the extension of state feedback, observer design and many other important concepts from regular systems are relatively rare and incomplete. Among recent works in this direction, we mention Cobb [6], El-Tohami et al. [7], [8].

As is easily recognized, the main difficulty, compared with the regular system theory, is the lack of so-called canonical forms for singular systems. More specifically, for regular systems we have the controllable canonical form as in Kalman [11], the canonical forms in Luenberger [10], and a more extensive and coherent collection given in Kailath [4]. However, these canonical forms can not be so obviously generalized for singular systems. This is largely due to the fact that if E in (1) is singular, the pencil Es-A is not a natural basis for applying the conventional companion form of the matrix.

Fortunately, the recent development [12,13] of a so-called generalized companion form has overcome the aforementioned difficulty. This form provides a surprisingly natural structure for pencil Es-A with singular E, thus enables us to generalize directly and satisfactorily the canonical forms well-known in the regular system theory to singular systems. Here we shall, however, not go into the detailed exposition of the theory of the generalized companion form. The interested readers are referred to [12] and [13]. Our goal in this paper is to develop the canonical forms for singular systems. We shall also show in two companion papers how the state feedback and observer design problems can be solved in a simple and straightforward way by applying the canonical forms developed here.

We now give the outline of the paper. For better exposition, we shall give separate developments for singlevariable systems as well as for multivariable systems. Starting with the singlevariable systems, the notions of controllability and observability are introduced. Then we proceed to show that any controllable system can be reduced by the restricted system equivalence operations to the generalized controller canonical form. As an algebraically dual result, an observable system can be reduced by r.s.e. operations to the generalized observer canonical form. For the multivariable systems, we follow the same line of development, thus similar results can be obtained.

2. Canonical forms for singlevariable systems

Following Rosenbrock [1] (see also Cobb [2]), we define (1) to be controllable if

$$\begin{aligned} &\text{rank } [E \quad B] = n \\ &\text{and} \\ &\text{rank } [Es-A \; B] = n \end{aligned} \qquad (2)$$

Here we adopt the convention of Cobb [2] to distinguish (2) from the definition proposed by Verghese et al. [3]. The latter is termed by Cobb the impulse controllability.

Let two systems be given in system matrix form (Rosenbrock [9])

$$\begin{bmatrix} Es-A & B \\ -C & 0 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} E_1 s-A_1 & B_1 \\ -C_1 & 0 \end{bmatrix} \qquad (3)$$

They are said to be restricted system equivalent (or r.s.e.) if they are related by

$$\begin{bmatrix} M & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} Es-A & B \\ -C & 0 \end{bmatrix} \begin{bmatrix} N & 0 \\ 0 & I \end{bmatrix} = \begin{bmatrix} E_1 s-A_1 & B_1 \\ -C_1 & 0 \end{bmatrix} \qquad (4)$$

where M and N are nonsingular matrices. In the regular system theory, the equivalence corresponds to a similarity transformation. There, by a proper choice of the similarity transformation the

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controllable (or observable) system can be put into certain canonical forms. We shall show that the same holds for singular systems. Before doing so, we remark that essentially all canonical forms for regular systems have their generalized forms. For better exposition of the main idea, however, we shall only generalize two of them to singular systems, namely, controller canonical form and observer canonical form (Kailath [4]), since these two forms are of special significance in state feedback and observer design problems. The generalization of other forms can follow the same derivation. But this will not be done here.

The following definition extends, in an interesting manner, the well-known controller canonical form to the singular systems.

Definition 1.

A singlevariable system (1) is said to be in the generalized controller canonical form if

$$E = \begin{bmatrix} a_0 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ . & . & & . \\ . & . & & . \\ 0 & 0 & \dots & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} -a_1 & -a_2 & \dots & -a_{n-1} & -a_n \\ 1 & 0 & \dots & 0 & 0 \\ . & . & & . & . \\ . & . & & . & . \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix}$$

$$B^T = [1 \quad 0 \dots 0] \tag{5}$$

where a_0 is either 1 or zero. We remark that the structure of C is irrelevant. Moreover, for obvious reasons, Es-A in (5) will be called the generalized companion form. The case that a_0 is zero corresponds to a singular E. Not surprisingly, we have the following

Theorem 1.

The system (1) is controllable if and only if it is r.s.e. to a system in the generalized controller canonical form (5).

Proof.

Sufficiency. Since, by assumption, (1) is r.s.e. to a system in the generalized controller form, there exist nonsingular M and N such that [MENs-MAN MB] is in structure (5). Therefore

$$\text{rank}[MENs-MAN\ MB] = \text{rank}\begin{bmatrix} a_0 s+a_1 & a_2 & \dots & a_{n-1} & a_n & 1 \\ -1 & s & \dots & 0 & 0 & 0 \\ 0 & -1 & \dots & 0 & 0 & 0 \\ . & . & & . & . & . \\ . & . & & . & . & . \\ 0 & 0 & \dots & -1 & s & 0 \end{bmatrix} = n$$

and

$$\text{rank}\,[MEN\ MB] = \text{rank}\begin{bmatrix} a_0 & 0 & \dots & 0 & 1 \\ 0 & 1 & \dots & 0 & 0 \\ . & . & & . & . \\ . & . & & . & . \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix} = n$$

Of course, the inverse of the r.s.e. transformation preserves this property and hence it is controllable.

Necessity. Let (1) be controllable. Then rank [E B] = n. Since B is a column vector, we must have $n-1 \le$ rank $E \le n$. Hence we can choose nonsingular M_1 and N_1 such that

$$M_1 E N_1 = \begin{bmatrix} a_0 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ . & . & & . \\ . & . & & . \\ 0 & 0 & \dots & 1 \end{bmatrix}$$

a_0 will be 1 if rank E=n, and zero otherwise. Now, apply N_1 together with M_1 to [E B] in the way below

$$M_1[E\ B]\begin{bmatrix} N_1 & 0 \\ 0 & I \end{bmatrix} = [M_1 E N_1\ M_1 B] = \begin{bmatrix} a_0 & 0 & \dots & 0 & b_1 \\ 0 & 1 & \dots & 0 & b_2 \\ . & . & & . & . \\ . & . & & . & . \\ 0 & 0 & \dots & 1 & b_n \end{bmatrix} \tag{6}$$

If $a_0=1$, (1) reduces to a regular system, it then can obviously be brought to (5) by similarity transformations. If $a_0=0$, we will surely have b_1 in (6) nonzero because the rank of (6) is n. Multiplying the first row of (6) by b_1^{-1}, then annihilating $b_2 \dots b_n$ with the first row, we get

$$\begin{bmatrix} 0 & 0 & \dots & 0 & 1 \\ 0 & 1 & \dots & 0 & 0 \\ . & . & & . & . \\ . & . & & . & . \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix} \tag{7}$$

Notice that the above transformations are r.s.e. operations applied to (6), or equivalently, we can find M_2 and N_2 such that $[M_2 M_1 E N_1 N_2\ M_2 M_1 B]$ assumes the form (7).

Now we turn to [Es-A B]. Denote $M_2 M_1$ and $N_1 N_2$ by M_3 and N_3 respectively, apply them to [Es-A B]

$$M_3[Es-A\ B]\begin{bmatrix} N_3 & 0 \\ 0 & I \end{bmatrix} = [M_3 E N_3 s - M_3 A N_3\ M_3 B] = \begin{bmatrix} g_1 & g^T & 1 \\ p & Is-A_1 & 0 \end{bmatrix} \tag{8}$$

where g and p are column (n-1)-vectors, g_1 is a scalar and A_1 is an $(n-1)\times(n-1)$ matrix. By observing rank [p Is-A_1] = n-1, we know that (A_1, p) is a controllable pair. Hence a transformation T can be found such that

$$T^{-1}\ [p \quad Is-A_1]\begin{bmatrix} I & 0 \\ 0 & T \end{bmatrix}$$

$$= [T^{-1}p \quad Is-T^{-1}A_1 T]$$

$$= \begin{bmatrix} 1 & s+c_1 & c_2 & \dots & c_{n-2} & c_{n-1} \\ 0 & -1 & s & \dots & 0 & 0 \\ . & . & . & \dots & . & . \\ . & . & . & \dots & . & . \\ 0 & 0 & 0 & \dots & -1 & s \end{bmatrix} \tag{9}$$

construct $\begin{bmatrix} I & 0 \\ 0 & T^{-1} \end{bmatrix}$ and $\begin{bmatrix} I & 0 & 0 \\ 0 & T & 0 \\ 0 & 0 & 1 \end{bmatrix}$, apply them to (8),

$$\begin{bmatrix} I & 0 \\ 0 & T^{-1} \end{bmatrix} \begin{bmatrix} g_1 & g^T & 1 \\ p & Is-A_1 & 0 \end{bmatrix} \begin{bmatrix} I & 0 & 0 \\ 0 & T & 0 \\ 0 & 0 & I \end{bmatrix}$$

$$= \begin{bmatrix} g_1 & d_1 & d_2 & \cdots & d_{n-2} & d_{n-1} & 1 \\ 1 & s+c_1 & c_2 & \cdots & c_{n-2} & c_{n-1} & 0 \\ 0 & -1 & s & \cdots & 0 & 0 & 0 \\ . & . & . & & . & . & . \\ . & . & . & & . & . & . \\ 0 & 0 & 0 & \cdots & -1 & s & 0 \end{bmatrix} \qquad (10)$$

(we shall always use different Roman letters to indicate possible nonzero elements resulting from the transformation). Note that the above used operations are r.s.e. transformations. Using the first column of (10) to zero out c_1, c_2, ..., c_{n-1}, multiplying the first column of the resulting matrix by -1, (10) is converted into

$$\begin{bmatrix} r_1 & r_2 & r_3 & \cdots & r_{n-1} & r_n & 1 \\ -1 & s & 0 & \cdots & 0 & 0 & 0 \\ 0 & -1 & s & \cdots & 0 & 0 & 0 \\ . & . & . & & . & . & . \\ . & . & . & & . & . & . \\ 0 & 0 & 0 & \cdots & -1 & s & 0 \end{bmatrix} \qquad (11)$$

(11) is obviously r.s.e. to (1), and it assumes the structure (5) with a_0=0. The theorem is proven.

We now proceed to develop the generalized observer canonical form for (1). Again from [1], system (1) is observable if

$$\text{rank} \begin{bmatrix} E \\ -C \end{bmatrix} = n \quad \text{and} \quad \text{rank} \begin{bmatrix} Es-A \\ -C \end{bmatrix} = n \qquad (12)$$

(12) is apparently an algebraically dual statement of the controllability (2). Therefore, the preceding results can be carried over easily by duality.

Definition 2.

System (1) is said to be in the generalized observer canonical form if

$$E = \begin{bmatrix} a_0 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ . & . & & . \\ . & . & & . \\ . & . & & . \\ 0 & 0 & \cdots & 1 \end{bmatrix} \qquad A = \begin{bmatrix} -a_1 & 1 & \cdots & 0 \\ -a_2 & 0 & \cdots & 0 \\ . & . & & . \\ . & . & & . \\ -a_{n-1} & 0 & \cdots & 1 \\ -a_n & 0 & \cdots & 0 \end{bmatrix}$$

$$C = [1 \quad 0 \quad \cdots \quad 0] \qquad (13)$$

where a_0 is either 1 or zero.

Similar to Theorem 1, we can state

Theorem 2.

System (1) is observable if and only if it is r.s.e. to a system in the generalized observer canonical form (13).

This theorem can be proved either by duality or by spelling out the proof of Theorem 1 with proper changes.

3. Canonical forms for multivariable singular systems

Unlike singlevariable systems, even for multivariable regular systems, the canonical form is generally not unique. This results in many different canonical forms existing for the multivariable regular systems. Although the extension of other forms can be done in a similar way, our aim in this section is to generalize two canonical forms to singular systems.

First we recall that a regular system [Is-A B] is controllable if and only if rank [B AB A^2B ... $A^{n-1}B$] = n. Or write it down columnwise

$$\text{rank } [b_1,..,b_q, Ab_1,..,Ab_q,..,A^{n-1}b_1,..,A^{n-1}b_q] = n \qquad (14)$$

Consequently, there are n linearly independent columns in (14). We now select from left to right in (14) n linearly independent columns. That is, start from b_1, b_2, ..., b_q and then Ab_1, Ab_2, ..., Ab_q till a linearly dependent column is found. We skip it and continue the search until n linearly independent columns are obtained. Conventionally this way of searching is called the scheme II search [4].

As a standard result, any controllable regular system is similar to the scheme II controller canonical form (see Kailath [4], Luenberger [10]). Here we use a special case to show the structure

$$\left[\begin{array}{ccc|ccc|cccc|cccc} s+X & X & X & X & X & X & X & X & X & X & 1 & X & X & X \\ -1 & s & & & & & & & & & & & & X \\ & -1 & s & & & & & & & & & & & X \\ \hline X & X & X & s+X & X & X & X & X & X & X & & 1 & X & X \\ & & & -1 & s & & & & & & & & & X \\ & & & & -1 & s & & & & & & & & X \\ \hline X & X & X & X & X & X & s+X & X & X & X & & & 1 & X \\ & & & & & & -1 & s & & & & & & X \\ & & & & & & & -1 & s & & & & & X \\ & & & & & & & & -1 & s & & & & X \end{array}\right] \qquad (15)$$

Where X's denote possible nonzero values, the entries not shown are zeros. Intuitively one would expect that the singular controllable system takes the form (special case for illustration)

$$\left[\begin{array}{ccc|ccc|cccc|cccc} a_0s+X & X & X & X & X & X & X & X & X & X & 1 & X & X & X \\ -1 & s & & & & & & & & & & & & X \\ & -1 & s & & & & & & & & & & & X \\ \hline X & X & X & a_1s+X & X & X & X & X & X & X & & 1 & X & X \\ & & & -1 & s & & & & & & & & & X \\ & & & & -1 & s & & & & & & & & X \\ \hline X & X & X & X & X & X & a_2s+X & X & X & X & & & 1 & X \\ & & & & & & -1 & s & & & & & & X \\ & & & & & & & -1 & s & & & & & X \\ & & & & & & & & -1 & s & & & & X \end{array}\right] \qquad (16)$$

ere a_i (i=0,1,2) are either 1 or zero. This tuition is indeed true as proved in the following eorem.

orem 3.

Every controllable singular system [Es-A B] is .e. to the form

$$\left[\begin{array}{cccc|cc}
s+X & X..X & X & X & X..X & X \\
-1 & s..0 & 0 & 0 & & \\
 & \ddots & & & & \\
 & & -1 & s & & \\
\hline
X & X..X & X & a_1 s+X & X..X & X \\
 & & & -1 & s..0 & 0 \\
 & & & & \ddots & \\
 & & & & 0 & -1\ s \\
\hline
 & & & & & \\
 & & & & & \\
X & X & \cdots & \cdots & a_k s+X & X..X \\
 & & & & -1 & s..0 \\
 & & & & & \ddots \\
 & & & & 0 & -1\ s
\end{array}\right.
\left.\begin{array}{cc}
X..X & X..X \\
 & \\
 & \\
 & \\
\hline
 & \\
 & \\
 & \\
 & \\
\hline
 & \\
 & \\
 & \\
 & \\

be selected. Then we proceed to $\bar{b}_1$, $\bar{b}_2$, ... and select only those columns which are not linearly dependent on the previous vectors in the sequence. After having done this, as usual, we continue to select $\bar{A}_{22}\bar{a}_1$, $\bar{A}_{22}\bar{a}_2$, ..., $\bar{A}_{22}\bar{b}_1$, ... and so on. When all r linearly independent columns have been chosen, we arrange them so that chains relating to the columns of $\bar{B}_{22}$ precede chains relating to that of $\bar{A}_3$. This is shown as follows

$$[\bar{b}_1,\ \bar{A}_{22}\bar{b}_1,\ldots,\ \bar{b}_2,\ \bar{A}_{22}\bar{b}_2,\ldots,\bar{a}_1,\ \bar{A}_{22}\bar{a}_1,\ldots] \tag{24}$$

As a consequence of (24), we can construct a transformation T (as in [4] p.436, see also [10]) such that (special structure for illustration)

$$[\ T^{-1}\bar{A}_3\ \ 0\ \ Is-T^{-1}\bar{A}_{22}T\ \ 0\ \ \ T^{-1}\bar{B}_{22}\ ] =$$

$$\left(\begin{array}{cc|c|ccccccccc|ccc|cc}
X & X & 0 & s+d_1 & d_2 & d_3 & d_4 & d_5 & d_6 & d_7 & d_8 & 0 & 0 & 0 & 1 & X \\
 & & & -1 & s & 0 & & & & & & & & & & X \\
 & & & 0 & -1 & s & & & & & & & & & & X \\
1 & X & 0 & e_1 & & e_2 & e_3 & s+e_4 & e_5 & e_6 & e_7 & e_8 & 0 & 0 & 0 & 0 & X \\
 & & & & & & & -1 & s & & & & & & & & \\
0 & 1 & 0 & f_1 & & f_2 & f_3 & f_4 & f_5 & s+f_6 & f_7 & f_8 & 0 & 0 & 0 & 0 & X \\
 & & & & & & & & & -1 & s & 0 & & & & & X \\
 & & & & & & & & & 0 & -1 & s & & & & & X
\end{array}\right) \tag{25}$$

In what follows we shall w.l.o.g. work with a special structure for illustration. Returning to (22), Let

$$[\bar{A}_1\ \bar{A}_2\ -\bar{A}_{12}\ \bar{B}_{11}\ \bar{B}_{12}]$$

$$= \left(\begin{array}{cc|c|ccccccc|ccc|cc}
X & X & X & X & X & X & X & X & X & X & 1 & X & X & X & X \\
X & X & X & X & X & X & X & X & X & X & 0 & 1 & X & X & X \\
X & X & X & X & X & X & X & X & X & X & 0 & 0 & 1 & X & X
\end{array}\right) \tag{26}$$

and $[\bar{A}_3\ 0\ Is-\bar{A}_{22}\ 0\ \bar{B}_{22}]$ has already been brought to (25). Use 1 in the last column of $T^{-1}\bar{A}_3$ to zero out f_1, ..., f_8. Then do the same with 1 in the next to last column ... the first column of $T^{-1}\bar{A}_3$. When this is done, multiplying the first ... to the last column of $T^{-1}\bar{A}_3$ by -1, we get

$$\left(\begin{array}{cc|c|ccccccccc|ccc|cc}
X & X & X & X & X & X & X & X & X & X & 1 & X & X & X & X \\
X & X & X & X & X & X & X & X & X & X & 0 & 1 & X & X & X \\
X & X & X & X & X & X & X & X & X & X & 0 & 0 & 1 & X & X \\
\hline
X & X & 0 & s+X & X & X & X & X & X & X & X & & & 1 & X \\
 & & & -1 & s & 0 & & & & & & & & & X \\
 & & & 0 & -1 & s & & & & & & & & & X \\
-1 & 0 & 0 & 0 & 0 & 0 & s & 0 & 0 & 0 & 0 & & & 0 & X \\
 & & & & & & -1 & s & & & & & & & X \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & s & 0 & 0 & & & 0 & X \\
 & & & & & & & & -1 & s & 0 & & & & X \\
 & & & & & & & & 0 & -1 & 0 & & & & X
\end{array}\right) \tag{27}$$

(27) can then be easily transformed, by r.s.e. operations, to

$$\left(\begin{array}{cccccccccc|ccccc}
X & X & X & X & X & X & X & X & X & X & 1 & X & X & X & X \\
\hline
X & X & X & X & X & X & X & X & X & X & 0 & 1 & X & X & X \\
 & -1 & s & 0 & & & & & & & & & & & \\
 & 0 & -1 & s & & & & & & & & & & & \\
X & X & X & X & X & X & X & X & X & X & 0 & 0 & 1 & X & X \\
 & & & & -1 & s & 0 & 0 & & & & & & & \\
 & & & & 0 & -1 & s & 0 & & & & & & & \\
 & & & & 0 & 0 & -1 & s & & & & & & & \\
X & X & X & X & X & X & X & X & s+X & X & X & 0 & 0 & 0 & 1 & X \\
 & & & & & & & & -1 & s & 0 & & & & & \\
 & & & & & & & & 0 & -1 & s & & & & &
\end{array}\right) \tag{28}$$

(28) is of structure (17). The theorem is proved.

Like Theorem 1, Theorem 3 also has a converse.

Theorem 4.

If a singular system is r.s.e. to the generalized controller canoncial form, it is controllable.

This theorem can be proved by first converting the system into the canonical form, its controllability is obvious. Then by recalling the fact that r.s.e. operations preserves the controllability.

As in the singlevariable case, the observer canonical form can similarly be generalized as a simple algebraic dual of Theorem 3. It is stated as follows (again only search scheme II observer form is considered).

Theorem 5.

A multivariable singular system (1) is observable if and only if its pencil $[E^Ts-A^T\ -C^T]^T$ is r.s.e. to the generalized observer canonical form shown below where a_i (i = 0, ..., k) are either 1 or zero

$$\left(\begin{array}{ccccccccccc}
a_0s+X & -1 & \ldots & 0 & X & & & X & & & \\
X & s & \ldots & 0 & X & & & X & & & \\
. & . & & . & . & & & . & & & \\
. & . & & . & . & & & . & & & \\
X & 0 & \ldots & -1 & X & & & X & & & \\
X & 0 & \ldots & s & X & & & X & & & \\
X & & & & a_1s+X & -1 & \ldots & 0 & X & & \\
X & & & & X & s & \ldots & 0 & X & & \\
. & & & & . & . & & . & . & & \\
. & & & & . & . & & . & . & & \\
X & & & & X & 0 & \ldots & -1 & X & & \\
X & & & & X & 0 & \ldots & s & X & & \\
. & & & & . & & & . & . & & \\
. & & & & . & & & . & . & & \\
X & & & & X & & & a_ks+X & -1 & \ldots & 0 \\
X & & & & X & & & X & s & \ldots & 0 \\
. & & & & . & & & . & . & & . \\
. & & & & . & & & . & . & & . \\
X & & & & X & & & X & 0 & \ldots & -1 \\
X & & & & X & & & X & 0 & \ldots & s \\
\hline
1 & & & & 0 & & & & & & \\
X & & & & 1 & & & & & & \\
. & & & & . & & & & & & \\
. & & & & . & & & & & & \\
X & & & & X & & & 1 & & & \\
X & X & \ldots & X & X & X & \ldots & X & \ldots & X & X \ldots X \\
. & . & & . & . & . & & . & & . & . \\
. & . & & . & . & . & & . & & . & . \\
X & X & \ldots & X & X & X & \ldots & X & \ldots & X & X \ldots X
\end{array}\right) \tag{29}$$

Other well-known canonical forms of regular systems can also be generalized in the same way. However, proper care must be taken in the process of derivation.

4. Conclusions

It has been shown that the canonical forms widely used in the regular system theory can be extended naturally to singular systems. We demonstrated in detail how the extension can be accomplished with four special forms. As in the regular theory, canonical forms usually serve as convenient means in deriving certain general results for the systems. We shall show in two companion papers how the canonical forms developed here are utilized to tackle the state feedback and observer design problems for singular systems.

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