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A GENERAL ANALOG-COMPUTER SIMULATION SCHEME FOR TRANSFER MATRICES OF LINEAR MULTIVARIABLE SYSTEMS

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Abstract

The main purpose of this paper is to present a more general analog-computer simulation scheme for transfer matrices in MFD. This scheme imposes no conditions like the column(row)-reducedness and the properness, on the transfer matrices. The minor requirement needed is that the right(left) MFD should be column(row) pseudo-proper. One can see, via this simulation scheme, how naturally the free impulsive behaviour arises with the inconsistent initial conditions.

1. Introduction

In the initial stage, the modelling of a linear time-invariant multivariable system very often ends up with the transfer matrix in matrix fraction description(MFD). If this MFD is proper and column-reduced(or row-reduced), the well-known analog-computer simulation configuration(first obtained by Wolovich(1971)) can be set up. This concrete analog-computer realizaton not only provides a practical way of simulating the system, but also has a great theoretical value, since it displays clearly the structure of the system.

However, it is often the case that the constructed MFD may neither be proper nor column-reduced. The conventional approach to handle this situation is to preprocess the transfer matrix so that the required conditions are met. In doing so, however, some important dynamical properties, for example, possible impulsive free-responses may be lost. These have been explained by Rosenbrock(1974) and Verghese et al.(1981), and have led to the development of the singular systems.

Our purpose in the paper is to show that if a given MFD satisfies the so-called pseudo-proper condition, which is considerably weaker than the properness and the column(row)-reducedness, a similar simulation scheme is still possible. In fact, we will present the detailed simulation configuration, explain some important properties and draw the comparision between the new and the existing scheme.

The paper is structured as follows. In the next section, we give the necessary definitions and notations. A step-by-step construction is given in the presentation of the simulation scheme in section 3. Section 4 is dedicated to show that if the outputs of all integrators are considered as a set of internal variables, our scheme will assume the singular system representation. Some properties of the scheme are also discussed. An illustrative example is provided in section 5 to show that the

impulsive behaviour may result when the system fails to be proper and/or column-reduced. Conclusions and remarks are in the final section.

2. Definitions and notations

A dynamical system described by

$$E\dot{x} = Ax + Bu$$
$$y = Cx \qquad (1)$$

is called a singular system (Rosenbrock(1974), Verghese et al.(1981)), where in (1), x is an n-vector of internal variables, u is an m-vector of inputs and y is a p-vector of outputs. E, A are square matrices, E may be singular; B, C are matrices of compatible sizes.

System (1) is said to be controllable (Rosenbrock(1974)) if rank $[Es-A \quad B] = n$ and rank $[E \quad B] = n$.

Let a transfer matrix be given in right MFD, i.e.,

$$G(s) = N(s)D^{-1}(s) \qquad (2)$$

where N(s) and D(s) are polynomial matrices of dimensions pxm and mxm, respectively. Denote by $n_i(s)$, $d_i(s)$ $(i = 1, \ldots, m)$ the i-th columns of N(s) and D(s), by $r_i = \deg n_i(s)$, $k_i = \deg d_i(s)$ the column degrees of the i-th columns of N(s) and D(s), respectively. Then, if $k_i > r_i$ $(i=1\ldots m)$, (2) will be called column pseudo-proper. The reader may notice that the column pseudo-properness does not necessarily imply the properness. However, the converse of the above statement is always true, which means that we are dealing with an even broader class of systems than just the proper ones.

Moreover, we define

$$f = \sum_{i=1}^{m} k_i$$

to be the generalized determinantal degree of the denominator of the given MFD (2). The algebraic significance of this definition is explained in Tan and Vandewalle(1986).

3. The simulation configuration

The task we set in this section is to construct an analog-computer simulation scheme from a given column pseudo-proper matrix in right MFD (2).

On introducing a so-called partial state z(s), (2) can be rewritten as

$$D(s)z(s) = u(s) \qquad (3a)$$
$$y(s) = N(s)z(s) \qquad (3b)$$

As usual, the strategy is to first implement Eq.(3a), (3b) will then follow as a simple extension.

Naturally, the initial step taken is to rearrange (3a) by separating the higher differential part from the lower one. This is done as follows

$$D(s) = D_{df}S(s) + D_{sf}T(s) \qquad (4)$$

where D_{df}, D_{sf} are two constant matrices, and

$$T^T(s) = \text{block diag}\ \{\lfloor s^{k_i-1} \quad s^{k_i-2} \quad \ldots \quad s \quad 1 \rfloor,\ i=1,\ldots,m\}$$

while

$$S(s) = s\,T(s)$$

It should be noted that D_{df} and D_{sf} in (4) are not unique.

Substituting (4) into (3a) and properly rearranging terms, we have

$$D_{df}S(s)z(s) = -D_{sf}T(s)z(s) + u(s) \tag{5}$$

The particular advantage in having (5) is that it is in a column-decoupled form. This special form makes the rest of the construction straightforward.

As conventionally done, we temporarily assume that each $z_i^{(k_i)}$ is available (where z_i is the i-th variable of the partial state z in the time domain). Then, by integrating each $z_i^{(k_i)}$ k_i times, we obtain m chains of all required lower derivatives. A simple assembling of all derivatives according to Eq.(5) will give the whole scheme. Fig.1 shows the schematic of the connections at the matrix level. Where in Fig.1 $z^{(k)} = \lfloor z_1^{(k_1)} \ \ldots \ z_m^{(k_m)} \rfloor^T$.

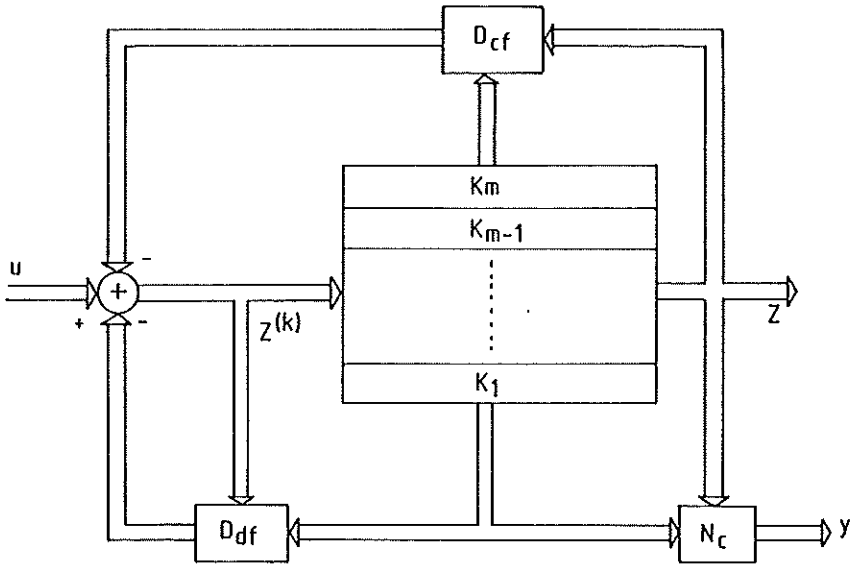


Fig.1. A schematic of the simulation scheme.

Whenever (3a) is available, we simply rewrite (3b) into the form

$$y(s) = N_c T(s)z(s) \tag{6}$$

so that it is obtained from the existing derivatives. This part is also incorporated into Fig.1.

It is clear from Fig.1 that apart from the familiar static feedback designated by D_{sf}, there also exists a feedback designated by D_{df}. Since the latter feedback only involves derivatives, we shall call it the differential feedback. This term will further be justified in the next section, when we start to build up the dynamical representation of Fig.1.

The second point we wish to mention is that the number of integrators used in Fig.1 is evidently equal to the generalized determinantal degree (defined in Sec.2) of the matrix D(s). This is analogous to one of the features of the conventional scheme (though in a generalized sense).

4. A singular system representation of the simulation scheme

Due to the presence of the differential feedback, one can imagine that the underlying system is not necessarily representable, depending on whether it is column-reduced or not, in state-space form. It can, however, be described more generally by the singular system representation. This is shown below.

As usual, the outputs of all integrators are considered to be the set of internal variables. Denoting by x this set of internal variables, it is easily seen that when the feedbacks (static and differential) are not incorporated, the m integrator chains can readily be seen to be linked by

$$\dot{x} = A_c x + B_c u \tag{7}$$

where

$$A_c = \text{block diag} \left\{ \begin{bmatrix} 0 & 0 & \dots & 0 & 0 \\ 1 & 0 & \dots & 0 & 0 \\ . & . & & . & . \\ . & . & & . & . \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix}, k_i x k_i,\ i=1, \dots, m \right\}$$

$$B_c = \text{block diag} \left\{ \lfloor 1 \quad 0 \dots 0 \quad 0 \rfloor,\ 1 x k_i,\ i=1, \dots, m \right\}$$

Since each output of the integrator is both an internal variable and the derivative of the output of the succeeding integrator (except, of course, for each of the last outputs of m chains), when considering it as an internal variable the static feedback in Fig.1 can be thought in terms of an additional input given by $D_{sf}x$; while considering it as the derivative, all these derivatives together with $z^{(k)}$ form yet another input given by $(D_{df}-B_c^T)\dot{x}$. We note that the term $B_c^T\dot{x}$ should be substracted out from the latter input in order to make $z^{(k)}$ available.

By including the aforementioned two additional inputs into (7), we have

$$\dot{x} = A_c x + B_c\lfloor -D_{sf}x - (D_{df} - B_c^T)\dot{x} + u \rfloor \tag{8}$$

rearranging terms in (8)

$$(I - B_c B_c^T + B_c D_{df})\dot{x} = (A_c - B_c D_{sf})x + B_c u \tag{9a}$$

For the output equation, it can simply be read out from Fig.1

$$y = N_c x \tag{9b}$$

Now (9) is in the singular system representation (compared to (1)), as we have expected. It should be noted that whenever the given right MFD is column-reduced, the matrix $E=I-B_cB_c^T+B_cD_{df}$ is nonsingular, which implies that the underlying system is proper.

As a final point, the reader may wish to check that (9) is controllable in the sense given in Sec.2. This fact is also transparent from Fig.1, since the vector $z^{(k)}$ is directly controllable by the input u.

5. Illustrative example

We use an example in this section to clarify the ideas made above. The example is also helpful in exhibiting the so-called impulsive behaviour associated with the scheme.

Example. The transfer matrix of a system is given by

$$N(s) = \begin{bmatrix} s^2 + 1 & 0 \\ s & 1 \end{bmatrix} \qquad D(s) = \begin{bmatrix} s^3 + s & s + 2 \\ s^2 + s + 1 & 1 \end{bmatrix} \tag{10}$$

Note that (10) is column pseudo-proper. Since $k_1 = 3$, $k_2 = 1$, two chains of integrators can first be set up. This is shown in Fig.2.

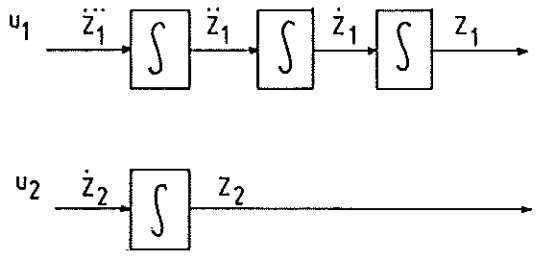


Fig.2. Integrator chains of the example to be the first step in setting up the whole scheme.

As the second step, we separate the differential feedback from the static one

$$D(s) = D_{df}S(s) + D_{sf}T(s)$$

$$= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} s^3 & 0 \\ s^2 & 0 \\ s & 0 \\ 0 & s \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} s^2 & 0 \\ s & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{11}$$

The next step is simply to add two types of feedbacks in Fig.2, according to (11). Fig.3 shows the final construction.

Once we have Fig.3, the outputs are straightforward to be generated. We shall omit this part of the drawing.

A particular kind of dynamical behaviour, known as impulsive free-response (first identified theoretically by Verghese et al. (1981)) can be readily seen in Fig.3.

For example, initially, we set $u_1 = u_2 = x_1(0) = x_2(0) = x_3(0) = 0$, but $x_4(0) \neq 0$. Since the differential feedback in the second chain in Fig.3 cancels the derivative of x_4, the nonzero $x_4(0)$ thus can not be balanced. Consequently, an impulse occurs in order to bring $x_4(0)$ to zero.

We note that the occurrence of the impulsive free-response is exclusively due to the set-up of the initial conditions. In fact, those initial conditions which cause the impulsive free-response had been termed inconsistent initial conditions prior to the work of Verghese et al. We refer the reader to Verghese et.al (1981) for a complete study of this new type of dynamical behaviour.

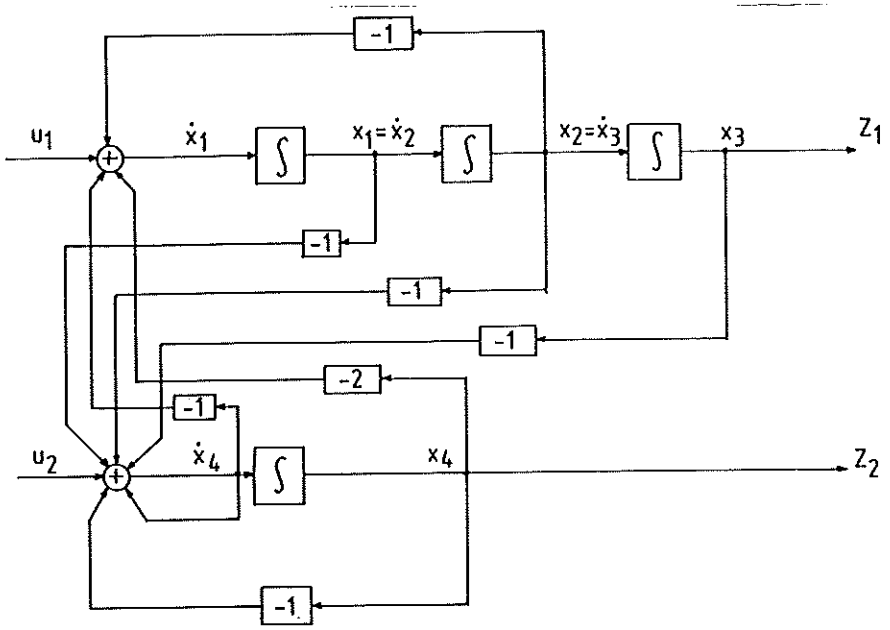


Fig.3. The complete simulation configuration for the example.

6. Conclusions

A general analog-computer simulation scheme for the pseudo-proper right MFD is presented. In fact, by a kind of dual argument, a scheme for the row pseudo-proper left MFD can be similarly obtained.

The importance of such a scheme is not confined to providing a simulation tool which solves the differential equation, it actually exhibits the fundamental nature of the system in a transparent way, and thus it enables to easily devise methods for both the analysis and synthesis problems of dynamical systems.

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