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A NEW DIRECT TIME DOMAIN IDENTIFICATION METHOD FOR LINEAR SYSTEMS USING SINGULAR VALUE DECOMPOSITION.

ir. J. Swevers
Res. Ass.

ir. M. Adams
Res. Ass.

Katholieke Universiteit Leuven
Department of Mechanical Engineering
Celestijnenlaan 300B B-3030 Leuven Belgium

ir. B. De Moor
Res. Ass.

Katholieke Universiteit Leuven
Department of Electrical Engineering
Kard. Mercierlaan 94 B-3030 Leuven Belgium

0. ABSTRACT.

This paper gives a survey of the different groups of time domain identification methods used for identification of dynamic systems such as mechanical structures, electrical systems,chemical processes,etc.... Also their properties and restrictions are discussed.
These considerations have led to a new direct identification strategy. This method is discussed in detail and illustrated with two applications.

1. INTRODUCTION.

The selection and identification of appropriate mathematical representations is of central importance in the analysis and control of dynamical systems. Mathematical models of a system enable investigation of the properties of a system and allow prediction of its behavior. Also, for control applications mathematical models can be used to give a system any desired behavior.
Constructing a model and choosing its internal structure are highly non trivial, if only the input/output behavior is available for measurements. There are many types of models: lumped parameter or distributed parameter, time-variant or time-invariant, linear or non-linear, stochastic or deterministic, discrete time or continues time, and internal or external.

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The need for time domain discrete models increases rapidly as more and more digital controllers use linear system models. Possible time domain models for linear systems are: difference equations, unit sample response functions, and state space descriptions. Evidently all these models are related to each other.
Many algorithms have been developed to identify these models. They are classified into three groups: (1) parameter estimation methods, (2) multi step indirect identification methods, and (3) direct methods. However some identification methods may fit in more than one group.
In sections 3 and 4 a new identification method is explained and illustrated. The aim of this identification method is to avoid several drawbacks of the existing identification methods. The new identification method estimates the order and the matrices of the state space model, as well as the initial state. The starting point is a structured input/output description consisting of block Hankelmatrices. The singular value decomposition is the key tool in every step of the approach.

2. TIME DOMAIN IDENTIFICATION FOR LINEAR SYSTEMS.

Many time domain identification methods exist for linear systems. They are classified into three groups: parameter estimation methods, non parametric multi step identification methods, and direct methods.
This section gives a short survey of the principles of each group together with their respective advantages and drawbacks.

2.1. The parameter estimation methods.[4]

These identification methods propound some structure for the model. The parameters in this model are then estimated, using an optimization criterion. Normally this leads to solving an overdetermined set of linear equations,

$$A_{mxn} \cdot X_{nx1} = b_{mx1} \qquad \text{with } m>n \qquad [1]$$

The matrix A and the vector b contain measured input/output-signals. The vector X contains the unknown parameters of the model. This set of equations is solved using linear least squares or total linear least squares methods, depending on the noise that contaminates the measurements.
Most of the parameter identification methods used in experimental modal analysis can be classified in this family.
As these methods propound a certain model structure, it is assumed that the user has some knowledge of the system. As a result these methods are used in situations where one knows

the internal relations of the system (modeling) but not the actual parameters of these relations. These parameters are estimated based on measured input/output-data.
The major disadvantage of these methods is the large amount of a priori information about the system that is needed, such as the order of the system. Important errors may result if a wrong order is selected. Therefore reliable criteria to determine the order are indispensable. Typically order tests consist of calculating the model for increasing orders. For each model a model validation is carried out, using some criterion, an optimum is reached. Clearly these order tests require large computational efforts, especially in the case of complex models. In addition difficulties are encountered when the data are perturbated by noise.
The major advantage stems from the use of optimization criteria such as the least squares and maximum likelihood methods, because recursive methods exist for these criteria [5]. This means that the results of a previous identification can be updated with a minimum amount of calculation time and memory space as soon as new input/output-data are available. This makes on line identification of time variant systems feasible, on less expensive computers.
The derived models are input output models. They can be transformed into a state space model, such as controllable and observable canonical forms. These state space representations have a minimum number of parameters. The resulting system matrix is a sparse matrix. The number of non zero elements in this matrix is proportional to the order n of the system. This is in contrast with the "normal" system matrix which has n^2 non zero elements.

2.2. The non parametric multi step identification methods.[6]

In most cases a priori information about the system is rather limited, but input/output-measurements are abundantly available. Therefore non parametric models, such as the impulse response (Markov parameters) for linear time invariant systems is closer to reality.
The purpose of this identification method is to derive a state space model:

$$\underset{nx1}{x_{k+1}} = \underset{nxn}{A} \cdot \underset{nx1}{x_k} + \underset{nxm}{B} \cdot \underset{mx1}{u_k}$$

$$\underset{lx1}{y_k} = \underset{lxn}{C} \cdot \underset{nx1}{x_k} + \underset{lxm}{B} \cdot \underset{mx1}{u_k} \qquad [2]$$

with u_k the sequence of input vectors, y_k the sequence of output vectors and x_k the sequence of state vectors

The identification method estimates the matrices A, B, C, D and the order of the system n, using measured input/output-data.
In a first step, the deconvolution step, the Markov parameters H_k are calculated. The basic equation for this step is derived from the state space model [2]. This equation is in fact the convolution of the impulse response with the input response which leads to the output sequence:

$$y_k = \sum_{i=0}^{k} \underset{lxm}{H_i} \cdot u_{k-i} + A^k \cdot x_0 \qquad [3]$$

$$\text{with } H_k = C.A^{k-1}.B \text{ for } k>0$$
$$H_0 = D$$

The existing methods consider the initial state x_0 equal to zero. This simplifies the convolution equation. From this simplified equation a set of equations is derived from which the Markov parameters can be calculated.
In a second step the matrices A, B, C, D, and the order of the system are calculated. This step is called the realization step.

There are many problems associated with these identification methods. Some of them are;
-The deconvolution problem remains unsolved for multi-input/multi-output systems(mimo systems). These systems lead to underdeterminded sets of linear equations. But one can perhaps use the structure of the matrices to create additional equations. This is a more theoretical problem. Only for finite impulse responses their exist an exact solution (in the noise-free case). For stable systems a good approximation can be found.
-The deconvolution requires a large amount of calculation time and memory space in the case of mimo systems.
-The deconvolution problem is converted into solving a set of static equations. This only applies to stable systems, because only for stable systems the number of unknowns is restricted or can be restricted. Nevertheless in many situations the system under study is unstable, and a model is needed to design a regulator for it to stabilize the system.
-The assumption that the initial state is equal to zero is another problem when one has to deal with a system changing slowly in time, which has to be identified on line.In such a situation the system is considered to be time invariant during certain periods of time. After every period the model is changed. This new model then represents the system for the following period. The calculation is accelerated if this new model is derived in an adaptive way. The old model is changed, used as a first approximation, to become the new model using the new measured data. The new calculated Markov

parameters are not connected in a continuous way with the previous set, if in every new calculation the "initial" state is considered to be equal to zero. This method can not be used in an adaptive scene.

Most of the mentioned disadvantages of the parameter estimation method and the multi step methods are avoided by the third class of methods: the direct identification methods.

2.3. The direct identification methods.[7]

These identification methods calculate directly, without first calculating the Markov parameters, the state space model. A priori information about the system is very restricted. The starting point of these methods is a structured input/output-relationship. They are derived from the state space model.
Most of these direct identification methods select linear independent rows of the structured matrices of the input/output-relations. This has two important drawbacks. First this takes many rank tests of large matrices. Second, noise perturbated data leads always to a situation where all the rows are linear independent, although this would not be the case in the noise free situation.
Besides, these direct identification methods very quickly degenerate in pure algebraic manipulation of formulas, without getting insight into the problem of identification.

The new identification method, which is discussed in the next section, is also a direct identification method. A structured input/output-relationship is the starting point, but the difficulties associated with the selection of linear independent rows are avoided. Instead the properties of block Hankelmatrices and the numerically very attractive technique of Singular Value Decomposition are used. The method is very simple and is geometrically interpretable.

3. A NEW IDENTIFICATION STRATEGY.[8]

A new direct identification method is explained in this section. This identification method is developed at the ESAT laboratory of the department of electrical engineering of the Catholic University of Leuven, and is now investigated at both the department of electrical engineering and the department of mechanical engineering.
A structured input/output relation is the starting point. It uses the numerical reliable technique of Singular Value Decomposition (SVD) of real matrices. First these two points are explained.

3.1. The input/output relation.

In this section the structured input/output-relation is derived as a first step in the identification approach presented further on. Linear, discrete time, n dimensional multi variable time invariant systems with m inputs and 1 outputs are considered. The state space model of such a system is:

$$\begin{matrix} x_{k+1} \\ n\ x\ 1 \end{matrix} = \begin{matrix} A \\ n\ x\ n \end{matrix} \cdot \begin{matrix} x_k \\ n\ x\ 1 \end{matrix} + \begin{matrix} B \\ n\ x\ m \end{matrix} \cdot \begin{matrix} u_k \\ m\ x\ 1 \end{matrix}$$

$$\begin{matrix} y_k \\ l\ x\ 1 \end{matrix} = \begin{matrix} C \\ l\ x\ n \end{matrix} \cdot \begin{matrix} x_k \\ n\ x\ 1 \end{matrix} + \begin{matrix} D \\ l\ x\ m \end{matrix} \cdot \begin{matrix} u_k \\ m\ x\ 1 \end{matrix} \qquad [4]$$

The matrices A, B, C and D are real. The vectors u_k, y_k, and x_k are the input/-, output/-, and the state vector at time k. Furthermore, the set of Markov parameters H_i of the linear system [4] are frequently used. These parameters are defined by:

$$H_0 = D$$
$$H_i = C.A^{i-1}.B \qquad i>0. \qquad [5]$$

The following input/output relation is derived from the manipulation of the system description [4].This relation is also found in [Liu,1977] and [Lamanna,1985].

$$Y_h(k,i,j) = \Gamma(i).X(k,j) + H_t(i).U_h(k,i,j) \qquad [6]$$

with;

$$Y_h(k,i,j) = \begin{bmatrix} Y_k & Y_{k+1} & Y_{k+2} & \cdot & \cdot & \cdot & Y_{k+j-1} \\ Y_{k+1} & Y_{k+2} & Y_{k+3} & \cdot & \cdot & \cdot & Y_{k+j} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ Y_{k+i-1} & Y_{k+i} & Y_{k+i+1} & \cdot & \cdot & \cdot & Y_{k+i+j-2} \end{bmatrix}$$

a l.i x j block Hankelmatrix constructed using i+j-1 consecutive output vectors,starting with y_k.

$$\Gamma(i) = \begin{bmatrix} C \\ C.A_2 \\ C.A^3 \\ \vdots \\ C.A^{i-1} \end{bmatrix}$$

is the i.l x n extended observability matrix.

$$X(k,j) = \begin{bmatrix} x_k & x_{k+1} & x_{k+2} & \cdot & \cdot & \cdot & x_{k+j-1} \end{bmatrix}$$

$$H_t(i) = \begin{bmatrix} H_0 & 0 & 0 & . & . & . & 0 \\ H_1 & H_0 & 0 & . & . & . & 0 \\ H_2 & H_1 & H_0 & . & . & . & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ H_{i-1} & H_{i-2} & . & . & . & . & H_0 \end{bmatrix}$$

, is a l.i x m.i block Toeplitzmatrix containing the first i Markov parameters.

$U_h(k,i,j)$ has the same structure as $Y_h(k,i,j)$ but is constructed using the input vectors.

3.2. The Singular Value Decomposition.[10]

The singular value decomposition (SVD) is a frequently used technique in this identification procedure. For completeness the definition and some properties are given. For an extensive survey we refer to the literature [Klema 1980] and [Golub 1983].

Definition: every real matrix A can be decomposed in a product of three matrices;

$$A_{mxn} = U_{mxm} \cdot \Sigma_{mxn} \cdot V^t_{nxn} \qquad [7]$$

with U, V orthonormal matrices; $U.U^t = I_m$
$V.V^t = I_n$
and Σ a real, pseudo diagonal matrix with non negative

elements.These are the singular values.

$$\Sigma = \begin{bmatrix} \sigma_1 & 0 & 0 & . & . & 0 & . & . & 0 \\ 0 & \sigma_2 & 0 & . & . & 0 & . & . & 0 \\ 0 & 0 & \sigma_3 & . & . & 0 & . & . & 0 \\ . & . & . & . & . & 0 & . & . & . \\ 0 & 0 & 0 & . & \sigma_r & 0 & . & . & . \\ 0 & 0 & 0 & . & 0 & 0 & . & . & . \\ . & . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

with $\sigma_1 \geq \sigma_2 \geq \ldots \sigma_r > 0$
and r is the algebraic rank of matrix A.

The columns of U are the left-, and those of V are the right-singular vectors. They represent an orthonormal base of the column-, respectively the row space of A.

Numerically reliable and efficient algorithms have been developed for the SVD. This makes rank estimation of a matrix relatively easy.
The best approximation of a matrix A of rank r with a matrix of rank r-1 is derived by reconstructing A from the SVD, but with σ_r equal to zero.The distance in Frobenius norm between these matrices is equal to σ_r.This is an important property when dealing with noise perturbated data.This property makes time domain filtering of noisy data possible.Due to noise on data the block Hankelmatrices constructed by these data are always of full rank,though theoretically, noise free, these matrices are of lower rank. SVD makes it possible to estimate the numerical rank by neglecting all the singular values which lie under a certain threshold. The resulting rank r matrix is then the best rank r approximation of the original matrix.

3.3. Interpretation of the input/output relation in the row space of the matrices.

The vector spaces spanned by the row vectors of the matrices in expression [6] are considered in this paragraph. Vector diagrams are used to represent these multidimensional spaces. A vector space V of dimension n is represented as

follows:

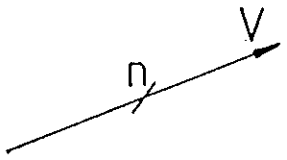


Figure 1:The vector representation of a n-dimensional vectorspace.

The expression $span_{row}(A)$ stands for the vector space spanned by the row vectors of matrix A.
Knowing this, the row space interpretation of expression [6] is represented as follows:

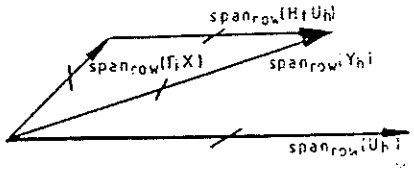


Figure 2:The vector representation of the row space interpretation of expression [9].

The matrix product $H_t.U_h$ results in a matrix with rows that are linear combinations of the rows of U_h. The space spanned by these rows is a subspace of $span_{row}(U_h)$. That's why $span_{row}(H_t.U_h)$ is presented parallel with the row space of U_h.

3.4. First step of the algorithm: determination of the column space of Γ(i).

The column space of Γ(i) has an unique relation with the matrices A and C. To separate this column space the term $H_t.U_h$ is eliminated from expression [6]. This is done by left multiplying the expression with $U_h^{\perp}$. $U_h^{\perp}$ is the orthonormal complement of U_h,i.e.:

$$\underset{m.i \times j}{U_h} \cdot \underset{j \times t}{U_h^{\perp}} = 0 \qquad \text{with } t = \dim(\text{kern}(U_h)) \qquad [8]$$

Expression [6] then becomes:

$$Y_h \cdot U_h^{\perp} = \Gamma(i) \cdot X \cdot U_h^{\perp} \qquad [9]$$

The left singular vectors of the matrix $Y_h.U_h^{\perp}$ span the column space of Γ(i). Only these left singular vectors and

the singular values of $Y_h.U_h^{\perp}$ are of our interest. These are not affected if the expression is right multiplied by an orthonormal matrix K^t. The expression then becomes:

$$Y_h \; . \; U_h^{\perp} \; . \; K^t \;=\; \Gamma(i) \; . \; X \; . \; U_h^{\perp} \; . \; K^t \qquad [10]$$

The expression is geometrically interpretable if K^t is equal to $U_h^{\perp t}$. The multiplication of Y_h with the matrix $U_h^{\perp}.(U_h^{\perp})^t$ projects the rows of Y_h in the vector space orthogonal to the row space of U_h.

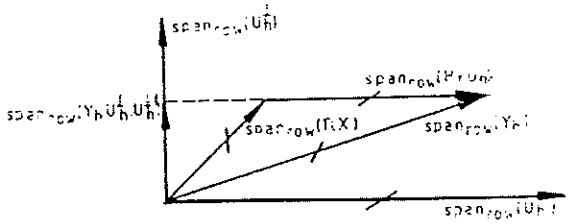


Figure 3:The vector representation of expression [13] with $K=U_h^{\perp}$.

Remark that the projection of $\Gamma(i).X$ in the row space of $U_h^{\perp}$ is the same as the projection of Y_h in that space.

3.5. Second step of the algorithm: determination of the order of the system.

The order of the system is one of the most important variables of the system. It determines the complexity of the process, the dimensions of the matrices and the state vector. The order of the system is the dimension of the vector space that can be reached by the state vector. This consideration leads to the criterion to calculate the order of the system.
The order of the system is denoted by n. It's assumed that the system is completely observable. This results that the observability matrix $\Gamma(n)$ is of full column rank. Then if i is chosen such that:

$$i \;\geq\; i_0 = \min_i(\text{rank } \Gamma(i) = n), \qquad [11]$$

the column space of $\Gamma(i)$ is of rank n.

Applying the expression for the rank of the product of two matrices to the product of $\Gamma(i)$ with an arbitrary matrix $M \in R^{nxt}$, then one obtains:

$$\begin{aligned} \text{rank}(\Gamma(i).M) &= \text{rank}(M) - \dim\{\text{span}_{col}(M) \cap \text{span}_{row}(\Gamma(i)^{\perp})\} \\ &= \text{rank}(M) \quad \text{because } \text{span}_{row}(\Gamma(i)^{\perp})=0 \end{aligned} \qquad [12]$$

in our case M is equal to $X.U_h^{\perp}$.

So

$$\text{rank}(Y_h.U_h^{\perp}) = \text{rank}(\Gamma(i).X.U_h^{\perp})$$

$$= \text{rank}(X.U_h^{\perp}) \qquad [13]$$

This last expression can be rewritten using an analog expression for the rank of the product of two matrices.

$$\text{rank}(X.U_h^{\perp}) = \text{rank}(X) - \dim\{\text{span}_{row}(X) \cap (\text{span}_{col}(U_h^{\perp})^{\perp}\}$$

$$= \text{rank}(X) - \dim\{\text{span}_{row}(X) \cap \text{span}_{row}(U_h)\}$$

$$\text{with } X = [\ x_0\ x_1\ .\ .\ .\ x_{j-1}\ ] \qquad [14]$$

The rank of matrix X is equal to the order of the system, if the input signals excite the structure such that the state vector reaches the whole state space within j time steps. This is condition of persistent excitation. This condition is assumed to be satisfied.
So if the last term of expression [14] is equal to zero, the following equation results:

$$\text{rank}(Y_h.U_h^{\perp}) = \text{rank}(X) = \text{order of the system.} \qquad [15]$$

This is the criterion to calculate the order of the system. The rank of matrix $Y_h.U_h^{\perp}$ results from the SVD of that matrix, as explained earlier.
If the last term of expression [14] differs from zero, there is rank cancellation between matrices X and $U_h^{\perp}$. The criterion then results in an underestimation of the order. Rank cancellation is not very likely if one applies inputs to an unknown system. But it's possible that rank cancellation nearly occurs. Then the rank of matrix $Y_h.U_h^{\perp}$ is still n but the n^{th} singular value is very small (the matrix is nearly of rank n-1). This is no problem if the data are noise free. But if there is noise on the data, all the singular values of $Y_h.U_h^{\perp}$ are different from zero. Then it's possible that the n^{th} singular value has the same magnitude as the singular values due to the noise. So only the first n-1 singular values are considered different from zero. This results in an underestimation of the order of the system. This phenomenon determines the noise sensibility of the order estimation. It's already established that the order estimation , with noisy data, becomes easier when the number of block rows (i) and the number of columns (j) of the input/output hankelmatrices is increased such that j/i is constant.

3.6. Third step: the calculation of the matrices A and C.
The algorithm to calculate the matrices A and C is based on the following property, which is proven in ref [8].

Given: $Y = \Gamma(i).M$
with $\Gamma(i)$ an extended observability matrix with
$i > i_0 = \min_i(\text{rank}(\Gamma(i)) = 0)$
with $M \in R^{nxt}$ and rank(M) = n
with the SVd of Y = $P_{l.ixn} \cdot S_{nxn} \cdot Q^t_{nxt}$

Then there exists only one matrix T_{nxn} such that:

$\underline{P}.T=\overline{P}$ with $\overline{P}$ the last i-1 block rows of P
with $\underline{P}$ the first i-1 block rows of P

T and A are equivalent matrices,i.e.: there exists a non singular matrix R such that:
$T=R^{-1}.A.R$

This leads to the equation $P=\Gamma(i).T$.
Applied to our case this means that under the conditions of persistent excitation (X of full rank) and absence of rank cancellation, the column space of $Y_h.U_h^{\perp} = \Gamma(i).X.U_h^{\perp}$ and the column space of $\Gamma(i)$ are identical. In fact any base in the column space of $Y_h.U_h^{\perp}$ may be chosen as a realization of $\Gamma(i)$. This because the matrices A and C of the state space model, which construct $\Gamma(i)$, are not unique. They depend on the chose of base in the state space.
Besides orthonormal matrices have numerical advantages. It requires no extra calculation to get $\Gamma(i)$ orthonormal, because the SVD of $Y_h.U_h^{\perp}$ is already necessary to estimate the order.
Once P is chosen as $\Gamma(i)$, matrices A and C are easily calculated.
C is the first block row of P.
A is calculated by solving the equation $\underline{P}.A = \overline{P}$

3.7. Fourth step: Calculation of the matrices B and D and the initial state x_0.
The calculation of the matrices B and D also starts from the structured input/output equation [6]. This equation must be right and left multiplied by matrices P and U_h^{-I},
with P such that $P.\Gamma(i)=0$,
and U_h^{-I} the pseudo inverse matrix of U_h.
The equation then becomes:

$$P.Y_h.U_h^{-I} = P.\Gamma(i).X.U_h^{-I} + P.H_t.U_h.U_h^{-I} \qquad [16]$$

$$= P.H_t$$

Define matrix $P.Y_h.U_h^{-I}$ as matrix K. The equation then becomes:

$$K = P.H_t \qquad \text{with } \dim(K) = r \times m.i \quad \dim(P) = r \times l.i \quad \dim(H_t) = l.i \times m.i \quad r = \dim(\text{kern}(\Gamma(i)^t) = l.i-n$$

Matrices K and P are divided into i block columns. This leads to the following structure:

$$\begin{bmatrix} K_1 & K_2 & . & .. & K_i \end{bmatrix} = \begin{bmatrix} P_1 & P_2 & . & .. & P_i \end{bmatrix} . \begin{bmatrix} D & 0 & 0 & .. & 0 & 0 \\ C.B & D & 0 & .. & . & . \\ C.A.B & C.B & D & .. & . & . \\ . & & & ... & 0 & 0 \\ . & & & ... & . & 0 \\ C.A^{i-2}.B & & & . & .. & . \; D \end{bmatrix} \qquad [17]$$

The following matrix equations result after reordering the block columns of the K matrix into a new long block column.

$$\begin{bmatrix} K_1 \\ K_2 \\ . \\ . \\ . \\ K_i \end{bmatrix} = \begin{bmatrix} P_1 & P_2 & P_3 & . & . & . & P_i \\ P_2 & P_3 & P_4 & . & . & P_i & 0 \\ . & . & . & . & . & 0 & 0 \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ P_i & 0 & 0 & . & . & 0 & 0 \end{bmatrix} . \begin{bmatrix} D \\ C.B \\ C.A.B \\ . \\ . \\ . \\ C.A^{i-2}.B \end{bmatrix} \qquad [18]$$

$$= \begin{bmatrix} P_1 & P_2 & P_3 & . & . & . & P_i \\ P_2 & P_3 & P_4 & . & . & P_i & 0 \\ . & . & . & . & . & 0 & 0 \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ P_i & 0 & 0 & . & . & 0 & 0 \end{bmatrix} . \begin{bmatrix} I_{1x1} & 0_{1xn} \\ 0_{1.(i-1)xn} & \Gamma(i-1) \end{bmatrix} . \begin{bmatrix} D \\ B \end{bmatrix}$$

This leads to the following equation:

$$K^* = \Phi . \begin{bmatrix} D \\ B \end{bmatrix} \qquad \text{with } \dim(K^*) = r.i \times m \quad \dim(\Phi) = r.i \times (1+n) \qquad [19]$$

The matrices B and D are calculated from this matrix equation. This set of equations are overdetermined, to become an unique solution, i.e.:

$$r.i \geq (l+n) \qquad [20]$$

Remark: One should not always chose r equal to l.i-n, because this can lead to an excess of equations.

The calculation of the initial state is very easy and straightforward, because every term in the structured input/output-equation is known except the one with the state vectors.
The equation to calculate these state vectors is:

$$Y_h - H_t.U_h = \Gamma(i).X \qquad [21]$$

The pseudo inverse matrix of $\Gamma(i)$ is equal to the transpose of $\Gamma(i)$. This because $\Gamma(i)$ is an orthonormal matrix.
The solution of this equation is:

$$X = \Gamma(i)^t.[Y_h - H_t.U_h] \qquad [22]$$

This method gives all the state vectors. The first column of X is the initial state x_0.

4. Application of the algorithm on two real systems.

The first application is the identification of the 120 MW power plant of Pont-sur-Sambre (France). This was done by two thesis students of the department of electrical engineering. The data for this identification was provided by Prof. R. P. Guidorzi of the university of Bologna (Italy) [9].
The second application is the identification of the modal parameters of a flexible arm, directly driven by a d.c. motor. This identification was done at the department of mechanical engineering in the context of the ESPRIT project SACODY.

4.1 Identification of the 120 MW power plant of Pont-sur-Sambre (France).

This identification is done using data measured during normal action. The system considered here is a power plant. It has five input and three outputs. The inputs are: the gas flow C_b (first input), the turbine valves opening Q_s (second input), the super heater spray flow Q_d (third input), the gas dampers R_y (fourth input) and the air flow Q_a (fifth input). The outputs are: the steam pressure P_v (first output), the main steam temperature T_s (second output) and the reheat steam temperature T_{rs} (third output) [figure 4].

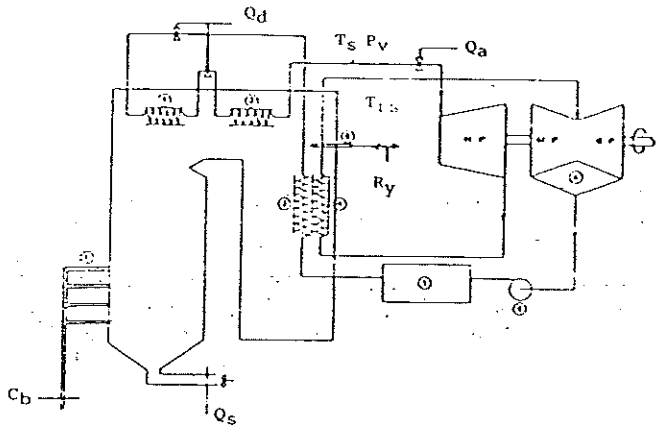


Figure 4:Block diagram of the power plant of Pont-sur-Sambre.

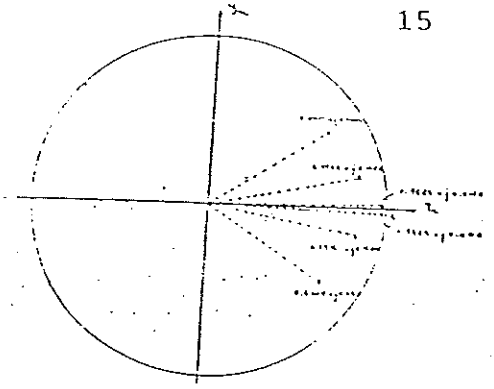
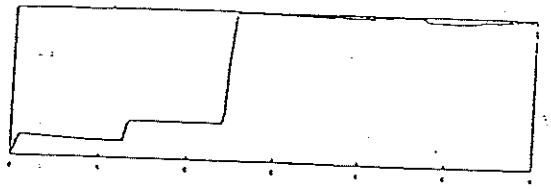
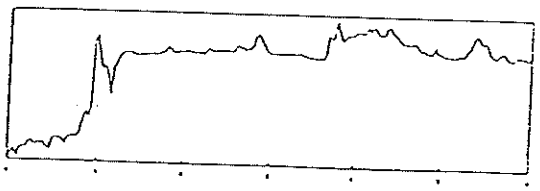


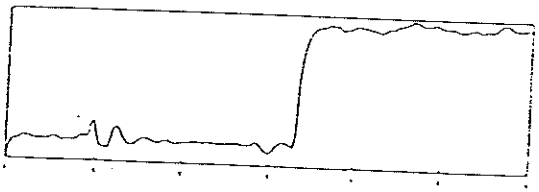
Figure 5:The z-plane representation of the poles of the identified power plant.



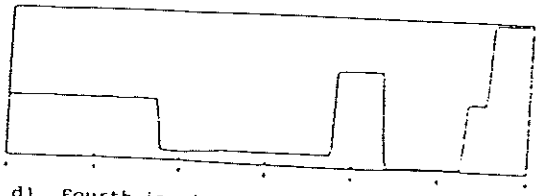
a) First input C_b (measured data).



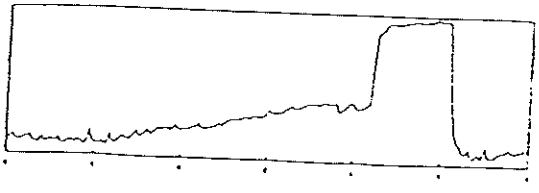
b) Second input Q_s (measured data).



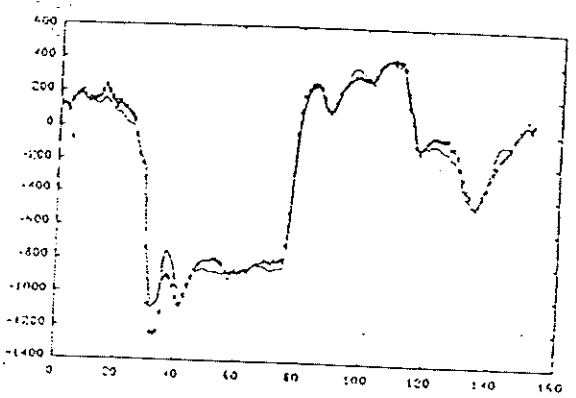
c) Third input Q_d (measured data).



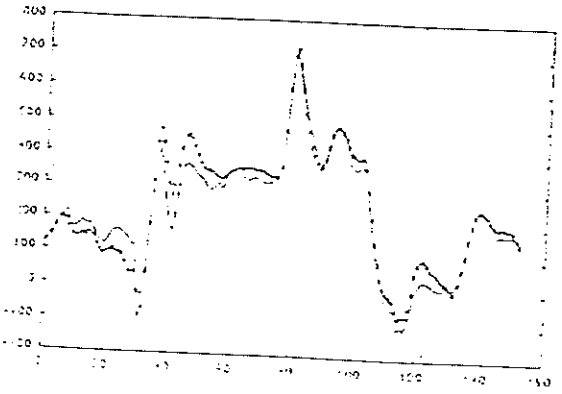
d) Fourth input R_y (measured data).



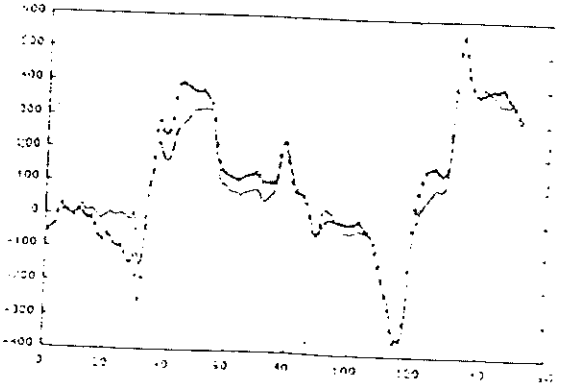
e) Fifth input Q_a (measured data).



f) First output P_v (measured and simulated (crosses) data).



g) Second output T_s (measured and simulated (crosses) data).



h) Third output T_{rs} (measured and simulated (crosses) data).

Figure 6:The measured inputs and outputs of the power plant.Figures f and g also show (with crosses) the simulated outputs using the calculated model and the inputs.

The identification method calculates a state space model of the system. The model is not presented here because it is not very explanatory. Only the poles of the system and the resulting simulation of the sixth order model is shown in the figures 5 and 6. These simulations and the comparison with the measured outputs is the most direct and objective method to validate the calculated model. Comparing simulation and measurements with data used during the identification itself, will always give good or "optimal" results, because every identification method calculates a sort of "optimal" model for the measured data. More objective, but only appropriate with time invariant systems, is comparing simulation and measurements with different data from the one used for the identification. The simulations show that the model and the reality match very well.
It's important to notice here that the data were measured during normal action, without interrupting the system, without applying special signals to it.

4.2. Identification of a flexible beam directly driven by a d.c. motor.

The system considered here is shown in figure 7.

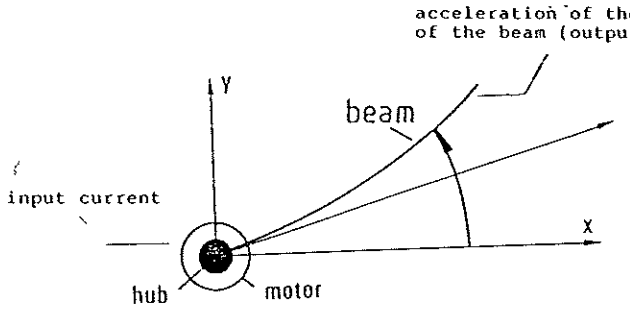


Figure 7: Schematic representation of the flexible beam and the d.c.-motor.

The input to the system is the current in the d.c. motor. The output is the acceleration at the end of the beam. This is measured with an accelerometer placed at the tip of the beam.The motion takes place in an horizontal plane, such that the system can be considered as time invariant. It's assumed that every point of the beam makes a circular displacement in the plane perpendicular to the axis of the motor.
This structure has many resonance frequencies. Only the first two eigenfrequences are considered, to restrict the calculation time and the matrix dimensions. This is done by filtering the data with a low pass filter with a cut off frequency at 80 Hz. This results in a system with two pairs of complex conjugated poles.

The influence of the dimensions of the input/output block Hankelmatrices on the order estimation criterion is considered in figure 8. This criterion looks at the singular values of the matrix $Y_h.U_h^{\perp}.U_h^{\perp t}$. As explained in paragraph 3.4, the gap in the singular value spectrum is the boundary between the high energy modes, which are assigned to the system, and the low energy modes, which are assigned to the noise on the data. In this figure the number of block rows (i) are increased as well as the number of columns (j), such that j/i is a constant equal to 4. The figure shows that as i increases the gap in the singular value spectrum increases. This means that the criterion to estimate the order becomes more and more clear if more data is taken in consideration. From this figure it's very clear that the system is a fourth order system. The other non zero singular values are assumed to result from the noise on the data. As four singular values lie above the gap the system order is four.
Remark that as i increases, the number of singular values increases also. This is due to the growth of the dimensions of the block Hankelmatrices.
With increasing number of columns of the input\output Hankelmatrices, 80 different identifications were done. Each of these identifications result in a discrete time state space model. These models are not shown in this paper, because they are not very explanatory. More explanatory are the eigenfrequencies and the damping ratios of the system. These are calculated from the real and imaginary parts of the eigenvalues of the system matrix A, after transforming the discrete time state space model to a continues time state space model.
Figure 9 shows the eigenfrequencies of the system as a function of the number of columns of the input/output matrices. The number of the block rows (i) is equal to 7.
Figure 10 shows the damping ratio as a function of j.
In both figures the curves reach a constant value if j is large enough. This because increasing the column dimension of the block Hankelmatrices, results in taking more sampled data into account,and so more periods of the eigenfrequencies. This leads to a more precise determination of the frequencies and damping ratios.

Figure 11 and figure 12 show the first eigenvalue, respectively the first damping ratio of the system with 3 different datasets and 2 different methods to calculate. The first method of calculation is explained in this text. The second method is a new method developed recently. It's a total linear least squares version of the algorithm. The algorithm discussed in this text is a linear least squares algorithm. Figure 11 shows that for j>80 the curves lie within a domain of 0.2 Hz. Figure 12 shows for j>80 that all the curves lie within a range of 0.005 of the damping ratio.

Remark that only 100 data points where used at a sampling ratio of 125 Hz. This means that all data were taken in an interval of approximately 1 second.

These results are compared with the results derived with conventional frequency domain methods. Both methods give the same eigenfrequency and damping ratio.The transfer function calculated with this frequency domain method is shown in figure 13.

5. CONCLUSIONS.

In this paper a classification of the different time domain identification methods for linear systems is given. The advantages and drawbacks of the different groups of identification methods led to the development of a new direct identification method. The approach of the identification method is discussed and illustrated with two examples.
The algorithm is further investigated. This led to a total least squares approach, which is an improvement of the linear least squares approach, discussed in this text, if both the input and output measurements are perturbated by noise.
The structured Hankelmatrices make it possible to change the algorithm to an adaptive strateg, which are useful for on-line identification of time variant systems. The algorithm uses QR factorization in an adaptive way, and avoids singular value decomposition as much as possible, because this factorization takes much calculation time.

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