TRENDS IN THE NEED OF MATHEMATICS FOR ENGINEERING AND THE IMPACT ON ENGINEERING EDUCATION.

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0 Summary

It is the aim of this paper to address the needs, opportunities and approaches for mathematics in engineering. This paper is organized in three sections.

The first section deals with trends in the need of mathematics for engineering. It is claimed that engineers need more and different mathematics.

Section 2 deals with the question : what can mathematics offer ?

In section 3 we discuss an approach for the education of mathematics in engineering. It aims at helping the student in constructing a coherent set of concepts and tools on which he or she can build in technical courses.

Since the paper is based on the author's research and teaching experience, it is not obvious that the approach can be carried over to other engineering colleges. Since this experience is considered to be very instrumental, for the approach, it is valuable to briefly summarize it.

The teaching experience includes 1 year at Electrical Engineering Department UCBerkeley and 8 years at K. Univ. Leuven more in areas of applied mathematics initially only for EE students now for all engineering students.

At the K.U.Leuven there is an entrance examination on the knowledge of mathematics (geometry, analytic geometry, algebra, calculation, numerical methods). Some 600 students start each year and the studies last 5 years.

The course experience includes :

- Algebra and analytic geometry (1st year)
- Signal processing and System theory (3rd year)
- Electrical circuit theory (3rd year)
- Nonlinear systems and control (5th year)
- Computer-controlled systems (postgraduate year mechatronics)

The research experience is mainly on the application of mathematics to engineering problems. In system theory numerical reliability of algorithms and the matrix algebra is stressed. Signal processing is done with as special applications biomedical and chip design. In cryptography several algorithmic, number theoretic and statistical problems are studied. Electrical circuit theory is also studied in view of chip design, the design of CAD tools and the analysis of piecewise linear circuits.

1 Trends in the needs of mathematics applied to engineering.

It is the aim of this section to argue that engineers need nowadays *more mathematics* and that they need *different mathematics.*

More mathematics is needed because engineers in research, development and production are being faced with more complex problems. Indeed the design and automation of complex systems (electrical, mechanical, computer, process, organizational...) requires a more systematic, global and formal approach. As illustration one can think of software projects of 100.000 lines of code or chip designs with 10.000 transistors. Different strategies like top-down (fig.1), bottom up, meet-in-the-middle... have been studied for such complex problems. Progress in VLSI, chip technology, microcomputers and software engineering on the other hand make more complex algorithms feasible in many different areas like measurement techniques, information processing, signal processing, automation, telecommunication, data protection, robotics. It suffices to look at the research literature in these domains in order to see that more and more mathematics is being used.

Whereas the emphasis in the education of mathematicians is on building layer by layer a consistent structure using deduction, it is claimed that engineers need a different approach to mathematics and a different kind of mathematics. First of all, there is a need for more discrete mathematics like set theory, number theory, graph theory, Boolean algebra, discrete optimization, iteration, recursion. This need is justified for nearly all disciplines in engineering because it is important for operations research, software engineering, digital electronics, databases, ...Next one should place more emphasis on the concepts and the use of computer tools, than on the mechanics and the hand calculation. The reason for this is that nearly all engineers have in their profession access to a computer (PC or workstation), and software packages. It would be dangerous to claim that academic engineers need less mathematics, because they can have their calculations done by computer. Rather on the contrary in order to invent, design and produce new systems engineers, more advanced and abstract concepts are useful. For example (Fig.2) the structure of a hypercube computer can be understood by reasoning in more than 3 dimensions and by using recursion on the dimension in order to go from an n-cube to an (n+1)-cube. In the same way, while the calculations and the plotting of 3-dimensional objects can be done by computer, the engineer should still have a good geometrical insight for most disciplines in engineering.

2 What can mathematics offer ?

Of course the enormous expansion of mathematics and applied mathematics has many interesting and important contributions towards engineering. First and foremost, it provides formalisms for correct reasoning and calculation such as logic, algebra, calculus, geometry, probability, ...

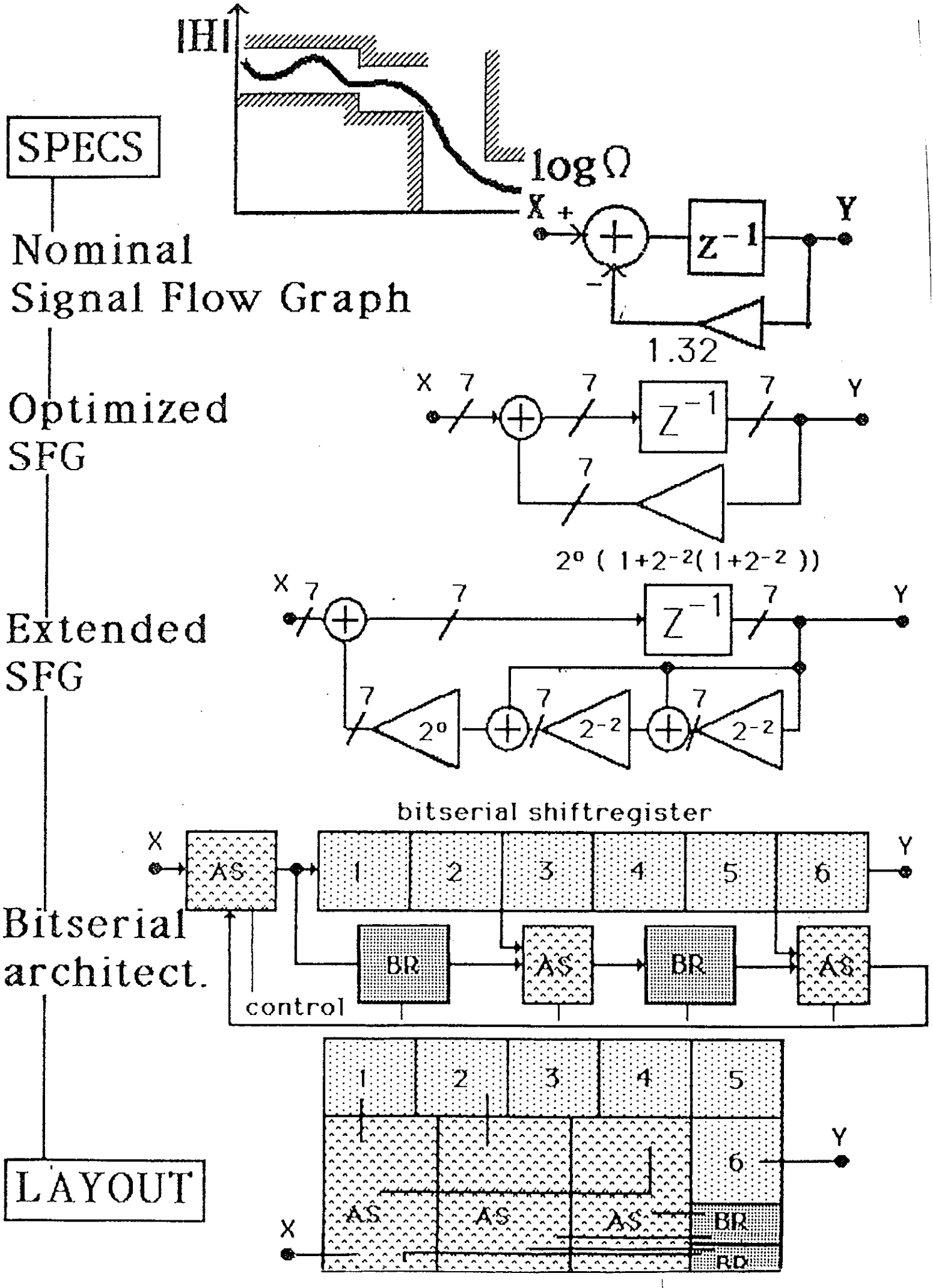


Figure 1: Top down design process of a digital filter chip.

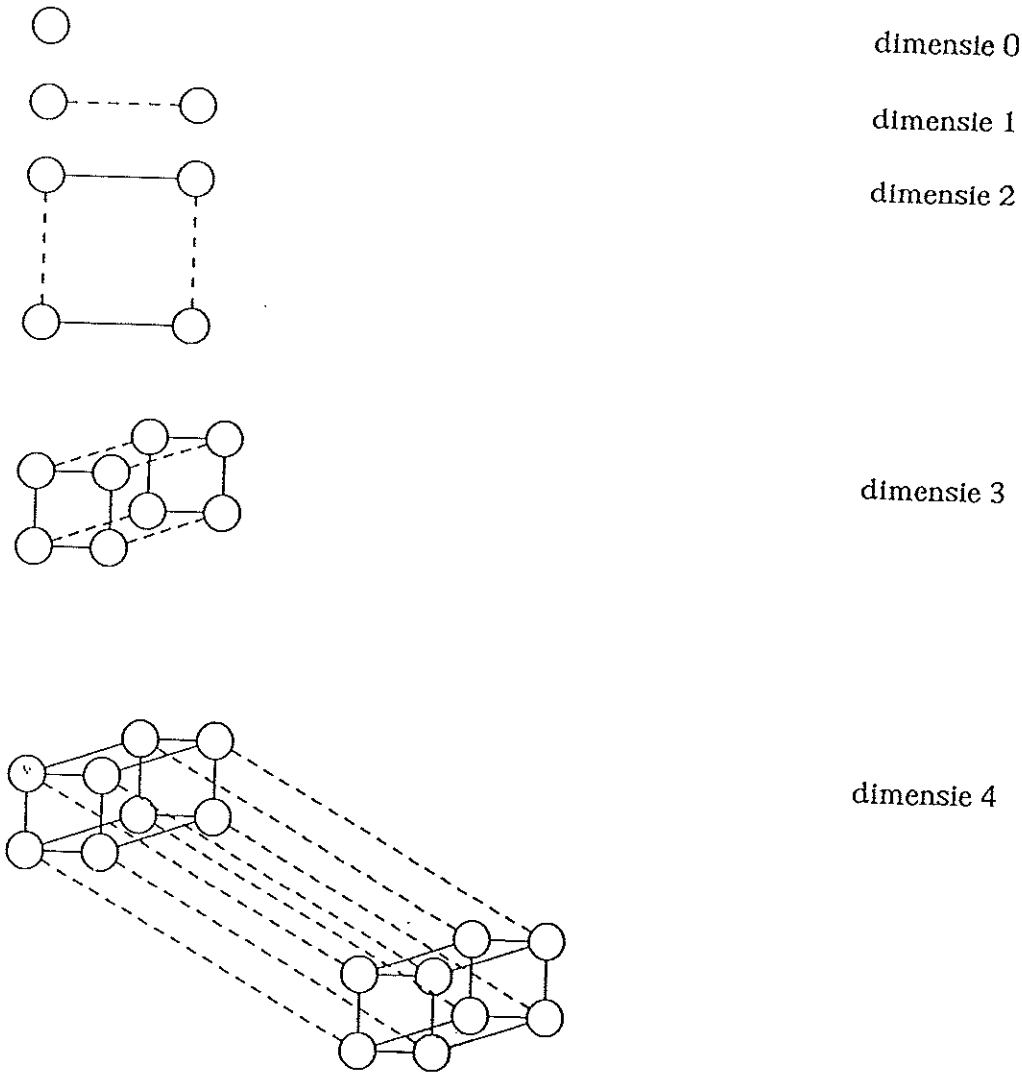


Figure 2: Recursive process to generate hypercubes of increasing dimension.

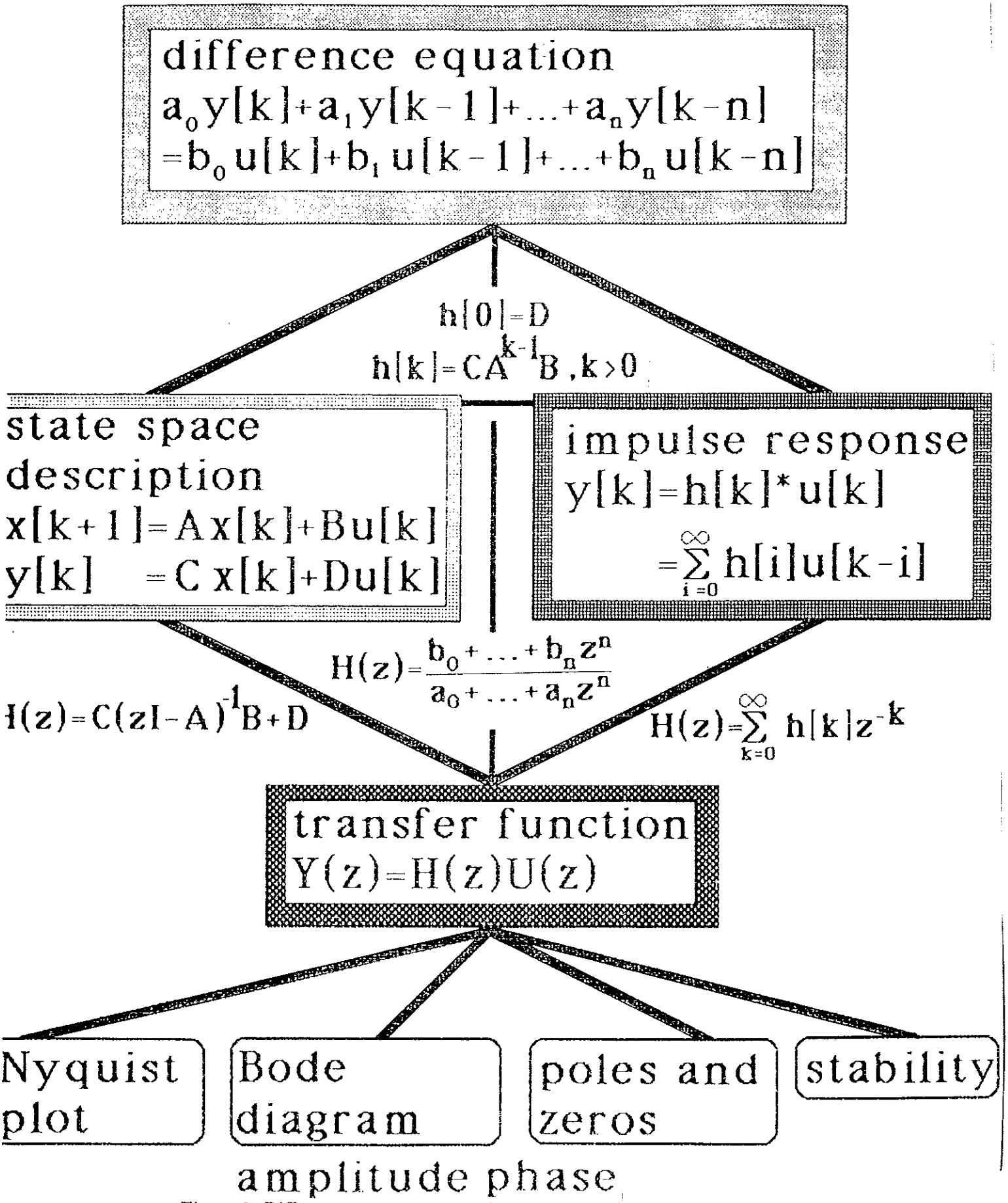


Figure 3: Different representations of a discrete time dynamical system.

Next mathematics is very remarkable and productive for engineers in making available a wealth of unifying concepts, e.g. convolution, chain fractions expansion, sensitivities, orthogonality, singular value decomposition, least squares, modulo calculus, recursion, fast Fourier transform, simulated annealing, stability, convergence, adaptivity, complexity theory, dynamic systems, average, standard deviation, yield, graph theory. In order to illustrate the first example, one can observe that convolution is used in the study of the sum of two random variables, the multiplication of polynomials, the multiplication of integers with a digital circuit, the relationship between input and output of a dynamical system ... Chain fractions expansion of rational numbers appear to be useful in gear boxes and electronic counters, whereas chain fractions expansions of ratios of polynomials are used in numerical analysis and filter design.

A third important contribution of mathematics is a box of tools and insights. There are a wealth of properties which are useful in order to calculate and to convert from one representation to another. Quite often one can see that many objects can be described in a geometric form (plots, block diagram, flow charts, ...) as well as in a linguistic form (data, formulas, programs, ...). Fig. 3 gives as an illustration a number of plots which are useful in order to study the behaviour of a discrete time dynamical system represented by a block diagram (generated with the MATLAB program and used in a third year course on systems). All these representations can also be described by algebraic expressions. Most often an engineer is more interested in a graphical or geometric representation. Students can train this understanding of the different representations and the relationships by varying some parameters and observing the differences between the plots as generated by a PC. It is not only in engineering but also in mathematics that one can observe an evolution towards the use of personal computers and of graphical representations. The publications of the Visual Mathematics Libraary [24] are quite interesting in this respect.

A fourth contribution is that mathematics provides links between reality and computer. Quite often engineering students and engineers feel so confident about the simulation environment (workstation or PC and a user friendly softwarepackage), that they accept whatever results are obtained by the simulation. However computers need a mathematical model of reality for simulation and optimization and depending on the kind of model one can observe certain phenomena. Hence engineers should be quite critical with respect to the results generated by the computer. In fact the only way a computer can deal with a technical system is by having a mathematical model of the system. Here again a solid mathematical understanding of the models, the phenomena as well as the numerical methods is important in order to make the correct decisions. Often one can develop for the same technical system or product a hierarchy of models (Fig. 4), where one should use the appropriate model if one is interested in some behaviour. E.g. if one is interested in the distortion behaviour of an amplifier, one should use a nonlinear model of the electronic circuit.

3 Approach for the education of mathematics in engineering.

From the previous observations one should place more emphasis on the integration of concepts than on deductive abstract reasoning, mechanics of the algebra and hand calculation. The reasons for this are that an engineer will not perform many calculations by hand but rather by a computer and that he will not have the time to reproduce the whole deductive structure of mathematics in order to be able to apply it. Hence the aim of the education in math should be

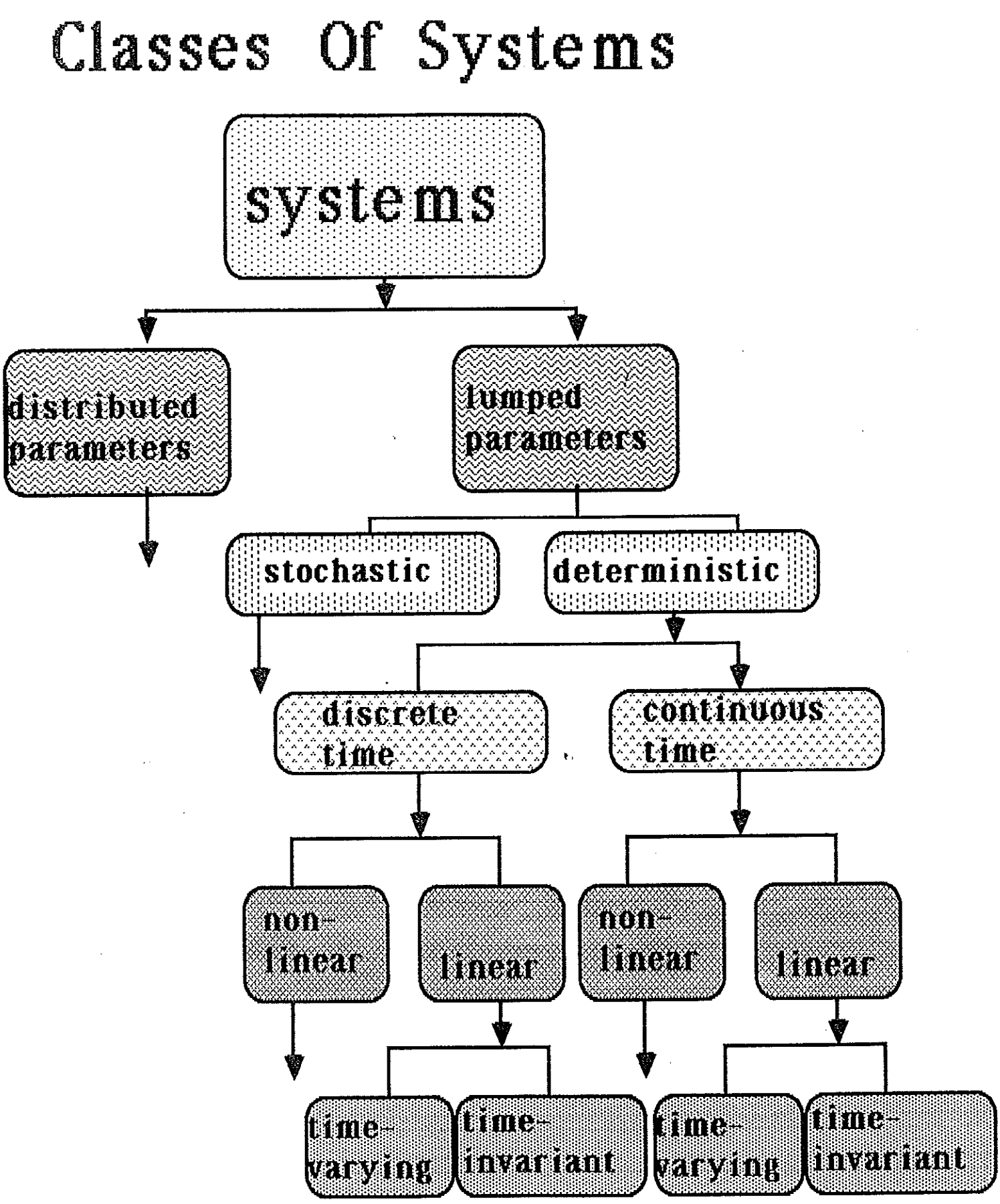


Figure 4: Hierarchy of models for systems

to help the student in constructing a coherent set of concepts and tools, on which he can build in technical courses. The quality of the tools determines the quality of the craftsman ! An important caveat is even more valid for engineers than for mathematicians. If the approach is too abstract , the integration will never happen. Hence I advocate a method whereby the mathematical techniques are illustrated with engineering case studies.

Mathematical topics	Engineering case studies
Boolean algebra	digital circuits
discrete mathematics	VLSI
graph theory	algorithm design
number theory	cryptography
geometry	shell structures, image processing, CAD
group theory	error correcting codes, symmetry
relations, set theory	relational databases
matrices	system theory, signal processing
...	...

This does not imply that the methods should not be rigorous or mathematically correct. But this implies that whenever there is a choice between a long deductive path and a path which can be illustrated with an engineering case study, one should choose the latter. In this way the student is more quickly motivated by the applicability rather than having to carry out a long string of abstract reasonings.

Another important element of the approach is to use all available tools such as audiovisual tools, computer tools (spreadsheet, symbolic manipulation e.g. MACSYMA), simulation tools, optimization tools. As mentioned before there is always some danger that the student might confuse reality and simulation results, but this argument should not be used in order to ban computer tools. In this respect one can observe that the distinction between software and hardware becomes more and more artificial. Software is used to design hardware and hardware can help to accelerate the execution of software. Here again it is important that engineers have an integrated and broad view.

4 Conclusion

Within the scope of this paper it was not possible to give all illustrations and arguments for the approach presented, but the references [1,7,15,16,18,24,26,27] contain more details. At the same time it is important to carry to the engineering students the broader scope of mathematics and its impact on society, history and art as described in some references [2,3,4,5,20].

In this paper it is stressed that mathematics has an essential and important contribution in the engineering education, but that it should be taught in a different way to engineers than to mathematicians. There should be less emphasis on the deduction and observations on applicability should enter very soon. One should stress the construction of a coherent set of tools and concepts.

References

[1] G. Birkhoff en Th. Bartee, "Modern Applied Algebra", Mc Graw Hill, 1970.

[2] J. Bronowoski, "The Ascent of Man", Little Brown and Co., Boston, 1973, ISBN 0-316-10930-4 Ch. 5 The Music of the Spheres.

[3] D.G. Hofstädter, "Metamagical themes", Viking Perguin Brooks Ltd., 1985.

[4] D.G. Hofstädter, "Gödel, Escher, Bach. An external golden braid". Vintage Brooks, Random House, 1980.

[5] B. Mandelbrot, "The fractal geometry of nature". Freeman, New York, 1983. ISBN 0-7167-1186-9.

[6] B. Ernst, "Das Zauberspiel des M.C. Escher", Taco Berlin 1986, ISBN 3-89268-005-1.

[7] M. Fogiel, "The linear Algebra Problem Solver", Research and Education Association. New York, 1980, ISBN 0-87891-518-4.

[8] F. Ayres, "Matrices", Schaum's Outline Series, Mc Graw Hill Book Co., 1974, ISBN 07-084379-1.

[9] F.R. Gantmacher, "Théorie des Matrices" : tome 1.2, Dunod, Paris, 1966 & "Theory of Matrices".

[10] A. Gibbons, "Algorithmic Graph Theory", Cambridge University Press, 1985, 0-521-2888-9.

[11] C. Berge, "Graphes et Hypergraphes", Dunod, Paris, 1973, ISBN 2-04-009755-1.

[12] M. Davio, J.P. Deschamps en A. Thayse, "Digital Systems with Algorithm Implementation", John Whiley and Sons, 1983, ISBN 0-471-10414-0.

[13] G. Birkhoff en S. Mac Lane, "A Survey of Modern Algebra", The Macmillan Company, New York, 1985.

[14] Z. Grossman en W. Magnus, "Groups and their Graphs", New Mathematical Library. The Mathematical Association of America, 1964.

[15] G. Rorres en H. Anton, "Applications of Linear Algebra", John Whiley 1984 (3rd edition), ISBN 0-471-86800-0.

[16] M.R. Schroeder, "Number Theory in Science and Communication with Applications in Cryptography, Physics, Digital Information, Computing and Self-Similarity", Springer Verlag, Berlin, ISBN 0-387-15800-6.

[17] T. Cuthbert, "Optimization using Personal Computers with Application to Electrical Networks", John Whiley and Sons, New York, 1987, ISBN 0-471-81863-1.

[18] P. Gasson, "Geometry of Spatial Forms : Analysis, Synthesis, Concept, Formulation and Space Vision for CAD", John Whiley and Sons, 1983, ISBN 0-85312-652-6.

[19] H. Cohn, "Advanced Number Theory", Dover Publ., New York, 1962, ISBN 0-486-64023-X.

[20] T. Dantzig, "Number, The Language of Science", Macmillan Publ. Co., New York, 1954.

[21] M. Hirsch, S. Smale, "Differential Equations, Dynamical Systems and Linear Algebra", Academic Press, New York 1974, ISBN 0-12-349550-4.

[22] R. Stoll, "Linear Algebra and Matrix theory", Dover Publ., New York, 1952.

[23] R.E. Blahut, "Theory and Practice of Error Control Codes", Addison-Wesley, Reading, 1983, ISBN 0-201-10102-5.

[24] R. Abraham and C. Shaw, "Dynamics - the geometry of behaviour, part 1,2,3" The visual mathematics library, Aerial Press, Santa Cruz, CA, 1985.

[25] F.P. Brooks, "No silver bullet, Essence and accidents of software engineering", IEEE Computer pp. 10-19, April 1987.

[26] J. Radatz, "A standard dictionary of computer terminology : Project 610", IEEE Computer, pp. 72-77, Febr. 1988.

[27] R.J. McEliece, "Finite fields for computer scientists and engineers", Kluwer Academic Publishers, Dordrecht, 1987.

[28] J. Vandewalle and A. Bultheel, "Algebra and analytic geometry for engineers", Notes for the course in the first year engineering at the K.U.Leuven, (in Dutch), Vlaamse Technische Kring, Heverlee 1987.