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Stepped sine system identification, errors-in-variables and the quotient singular value decomposition

Bart De Moor *

Department of Electrical Engineering
Katholieke Universiteit Leuven
Kardinaal Mercierlaan 94
B-3030 Leuven (Heverlee)
Belgium
tel: 32/16/220931 fax: 32/16/221855
email: demoor@esat.kuleuven.ac.be

Jan Swevers
Hendrik Van Brussel

Department of Mechanical Engineering
Katholieke Universiteit Leuven
Celestijnenlaan 300 B
B-3030 Leuven (Heverlee)
Belgium
tel: 32/16/286611 fax: 32/16/222345
email: swevers@mech.kuleuven.ac.be

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*Research Associate of the Belgian National Fund for Scientific Research

Abstract

We develop a parameter estimation algorithm for the stepped sine identification of the transfer function of a linear system when all measurements are corrupted by noise. It consists of three steps:

- First an overdetermined set of linear equations containing stepped-sine measurements is solved in a Linear Least Squares sense.
- The resulting error covariance matrix is used to construct the noise covariance matrix for a second set of linear equations
- This second set of equations, in which all data are noisy, is solved using the quotient singular value decomposition, which is a generalization of the singular value decomposition.

Simulation results show that this new method remains accurate even at extremely bad signal-to-noise ratios. It is also possible to take into account offset and drift of the measurement sensors. Finally, all algorithmic steps involved lend themselves to recursive updating.

1 Introduction

In many disciplines, such as modal analysis of mechanical systems, frequency domain identification techniques are preferred over time domain approaches. Using measurements of frequency response functions (FRF) that relate inputs to outputs in a certain frequency range, parameters of some a priori postulated models are estimated. A popular approach to scan a certain frequency range is broadbanded excitation such as swept sine, white noise, hammer impact, fast swept sine (chirp) etc ...(see e.g. [12]). These excitation signals and the measured responses are transformed to the frequency domain using the Fast Fourier Transform (FFT). The main drawbacks are limited frequency resolution and dynamic range. Another frequency response method is stepped sine excitation: The frequency response function is evaluated over a number of discrete frequency points within the range of interest. As is well known, the steady state frequency response for linear systems at a particular frequency is determined by the amplitude and phase relations between a sinusoidal input and resulting output. The most important advantages of the stepped sine approach are its high accuracy (typically because the effective signal-to-noise ratio is high) and the possibility to use frequency points that are unequally spaced so that resonance and anti-resonance frequency ranges are measured with a higher resolution.

For mechanical systems, most of these methods identify a modal model, describing the structure in terms of its modal parmeters: natural frequencies, damping ratios and mode shapes. These modal parameters are very useful in structural analysis and design. The frequency domain identification method of this paper is developed with a control application in mind. For such applications, state space models [8]

and input/output models [2] [7] [11] are more appropriate. Besides, the modal parameters can easily be derived from these models.
In this paper, we describe a new input/output model identification technique, which combines ideas of linear least squares estimation, errors-in-variables and numerical linear algebra. Section 2 describes a least squares method to estimate the frequency response function for a pure sinusoidal excitation. It is shown how to eliminate an offset and a trend from the measured response. In section 3, we introduce the quotient singular value decomposition (**QSVD**) as a numerical robust tool to solve errors-in-variables problems. In section 4, we combine the insights of sections 2 and 3 to develop a **QSVD** based parameter estimation algorithm, both for the discrete time and continuous time case. The simulation results in section 5 show that the **QSVD**-based method remains accurate even at extremely bad signal-to-noise ratios, outperforming least squares and SVD based methods.

2 Frequency response function estimation based on stepped sine excitation

Let us consider a linear single input single output system with input signal $u(t)$ and output signal $y(t)$. The steady state response of a linear system to a sinusoidal input is a sinusoid with the same frequency but a different amplitude and phase. Let $u_{\omega_i}(t)$ be such a sinusoidal input at a particular pulsation ω_i:

$$u_{\omega_i}(t) = \alpha_i \sin(\omega_i t + \phi_i)$$

The output $y_{\omega_i}(t)$ is also a sinusoid of the same frequency but with a different amplitude and phase:

$$y_{\omega_i}(t) = \beta_i \sin(\omega_i t + \psi_i)$$

A Fourier analysis of these signals would result in two nonzero components for each signal: a component at ω_i, and a component at $-\omega_i$, which is not relevant for our purpose. The component at ω_i is a complex number which describes the amplitude and phase of the sinusoid. For the input $u_{\omega_i}(t)$ it is represented by $a^u_{\omega_i} + jb^u_{\omega_i}$ while for the output $y_{\omega_i}(t)$ it is $a^y_{\omega_i} + jb^y_{\omega_i}$. The complex value of the frequency response function at that particular pulsation ω_i is the ratio of these two complex numbers. At every particular frequency the complex numbers $a^u_{\omega_i} + jb^u_{\omega_i}$ and $a^y_{\omega_i} + jb^y_{\omega_i}$ can be calculated from the measured time domain input/output data by decomposing the measurements into a sine and a cosine. This decomposition is exact if both signals are pure sinusoidal:

$$u_{\omega_i}(k) = 2a^u_{\omega_i} \cos(\omega_i k T_s) - 2b^u_{\omega_i} \sin(\omega_i k T_s) \quad (1)$$

$$y_{\omega_i}(k) = 2a^y_{\omega_i} \cos(\omega_i k T_s) - 2b^y_{\omega_i} \sin(\omega_i k T_s) \quad (2)$$

where $u_{\omega_i}(k)$ and $y_{\omega_i}(k)$ are the sampled input and output signals and $k = 1, \ldots, r$ is the time index; The number of samples is r and T_s is the sampling period. One

finds easily that

$$\begin{aligned} u_{\omega_i}(k) &= 2a^u_{\omega_i} cos(\omega_i k T_s) - 2b^u_{\omega_i} sin(\omega_i k T_s) \\ &= (a^u_{\omega_i} + j b^u_{\omega_i}) e^{j\omega_i k T_s} + (a^u_{\omega_i} - j b^u_{\omega_i}) e^{-j\omega_i k T_s} \end{aligned}$$

The right hand side of this last equation is the Fourier transform of $u_{\omega_i}(k)$. From equation (1) one finds immediately that $a^u_{\omega_i} + j b^u_{\omega_i}$ can be calculated from r observations of $u_{\omega_i}(k)$, $k = 1, 2, \ldots, r$, by solving the set of equations:

$$\begin{pmatrix} cos(\omega_i T_s) & -sin(\omega_i T_s) \\ cos(2\omega_i T_s) & -sin(2\omega_i T_s) \\ \cdots & \cdots \\ cos(r\omega_i T_s) & -sin(r\omega_i T_s) \end{pmatrix} \begin{pmatrix} a^u_{\omega_i} \\ b^u_{\omega_i} \end{pmatrix} = \begin{pmatrix} u_{\omega_i}(1) \\ u_{\omega_i}(2) \\ \vdots \\ u_{\omega_i}(r) \end{pmatrix} \tag{3}$$

which will be denoted as

$$A^u_{\omega_i}(z^u_{\omega_i}) = (u) \tag{4}$$

Here $A^u_{\omega_i}$ is an $r \times 2$ matrix. The exact solution is

$$(z^u_{\omega_i}) = (A^{u\,t}_{\omega_i} A^u_{\omega_i})^{-1} A^{y\,t}_{\omega_i} (u) \tag{5}$$

A similar set of equations with output measurements gives the complex numbers $a^y_{\omega_i} + j b^y_{\omega_i}$. In reality however, the steady state response of a system to a sinusoidal input is not purely sinusoidal. In control system applications, such as robotics, the observed steady state response is often a combination of a sinusoid, an offset, a trend and possibly higher harmonics excited by static nonlinearities. The extra terms caused by offsets and linear trends are modelled as:

$$y_{\omega_i}(k) = a^y_{\omega_i} cos(\omega_i k T_s) - b^y_{\omega_i} sin(\omega_i k T_s) + c^y_{\omega_i} + d^y_{\omega_i} k$$

For a set of r observations $y_{\omega_i}(k)$ at a particular frequency ω_i, the linear equations become:

$$\begin{pmatrix} cos(\omega_i T_s) & -sin(\omega_i T_s) & 1 & 0 \\ cos(2\omega_i T_s) & -sin(2\omega_i T_s) & 1 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ cos(r\omega_i T_s) & -sin(r\omega_i T_s) & 1 & r-1 \end{pmatrix} \begin{pmatrix} a^y_{\omega_i} \\ b^y_{\omega_i} \\ c^y_{\omega_i} \\ d^y_{\omega_i} \end{pmatrix} = \begin{pmatrix} y_{\omega_i}(1) \\ y_{\omega_i}(2) \\ \vdots \\ y_{\omega_i}(r) \end{pmatrix} \tag{6}$$

which will be denoted by

$$A^y_{\omega_i}(z^y_{\omega_i}) = (y) \tag{7}$$

Here $A^y_{\omega_i}$ is an $r \times 4$ matrix. The exact solution of (7) is:

$$(z^y_{\omega_i}) = (A^{y\,t}_{\omega_i} A^y_{\omega_i})^{-1} A^{y\,t}_{\omega_i} (y) \tag{8}$$

The factor 2 that normally appears in the Fourier transform of the input and the output signal, is dropped in (3) and (6) since it has no influence on the value of the

transfer function which is the ratio of $a^y_{\omega_i} + jb^y_{\omega_i}$ and $a^u_{\omega_i} + jb^u_{\omega_i}$. Nonlinearities, such as Coulomb friction, often excite higher harmonics. These terms can be modelled as $cos(2\omega_i kT_s)$, $sin(2\omega_i kT_s)$, $cos(3\omega_i kT_s)$, etc... . They will not be taken into account in what follows although it should be kept in mind that such an extension is straightforward.

Another important deviation of ideal circumstances occurs when measurements are corrupted by additive noise, caused by the finite resolution of the measurement equipment. These errors will be represented by $\delta u_{\omega_i}(k)$ and $\delta y_{\omega_i}(k)$. Because of the *central limit theorem* (see e.g. [6]), which acts favourably in most engineering environments, we can assume that these noise sequences are identically and independently normally distributed with mean zero and variances $\sigma^2_{u_{\omega_i}}$ and $\sigma^2_{y_{\omega_i}}$. These measurement noises cause errors on the right hand side of equations (4) and (7). Solving these equations in a least squares sense (which will give the maximum likelihood solution because of the a priori assumption of normality), the obtained solutions will differ from the exact (noisefree) solutions $z^u_{\omega_i}$ and $z^y_{\omega_i}$:

$$\begin{aligned} z^u_{\omega_i} + \delta z^u_{\omega_i} &= (A^{u\,t}_{\omega_i} A^u_{\omega_i})^{-1} A^{u\,t}_{\omega_i}\ (u + \delta u) \\ z^y_{\omega_i} + \delta z^y_{\omega_i} &= (A^{y\,t}_{\omega_i} A^y_{\omega_i})^{-1} A^{y\,t}_{\omega_i}\ (y + \delta y) \end{aligned}$$

where δu and δy are the vectors that contain the error sequences $\delta u_{\omega_i}(k)$ and $\delta y_{\omega_i}(k)$. It follows from the zero mean assumption that

$$\begin{aligned} \mathbf{E}(\delta z^u_{\omega_i}) &= (A^{u\,t}_{\omega_i} A^u_{\omega_i})^{-1} A^{u\,t}_{\omega_i}\ \mathbf{E}(\delta u) = 0 \\ \mathbf{E}(\delta z^y_{\omega_i}) &= (A^{y\,t}_{\omega_i} A^y_{\omega_i})^{-1} A^{y\,t}_{\omega_i}\ \mathbf{E}(\delta y) = 0 \end{aligned}$$

Hence, the least squares estimates of the unknown coefficients are unbiased with error covariance matrices given by

$$\begin{aligned} \mathbf{E}(\delta z^u_{\omega_i} \delta z^{u\,t}_{\omega_i}) &= (A^{u\,t}_{\omega_i} A^u_{\omega_i})^{-1} \sigma^2_{u_{\omega_i}} \\ \mathbf{E}(\delta z^u_{\omega_i} \delta z^{y\,t}_{\omega_i}) &= 0 \\ \mathbf{E}(\delta z^y_{\omega_i} \delta z^{u\,t}_{\omega_i}) &= 0 \\ \mathbf{E}(\delta z^y_{\omega_i} \delta z^{y\,t}_{\omega_i}) &= (A^{y\,t}_{\omega_i} A^y_{\omega_i})^{-1} \sigma^2_{y_{\omega_i}} \end{aligned}$$

We are only interested in the correlations between $\delta a^u_{\omega_i}$, $\delta b^u_{\omega_i}$, $\delta a^y_{\omega_i}$, and $\delta b^y_{\omega_i}$ (and not in those involving $\delta c^y_{\omega_i}$ and $\delta d^y_{\omega_i}$). These correlations depend only on $(A^{u\,t}_{\omega_i} A^u_{\omega_i})^{-1}$ and on the leading principal 2×2 submatrix of $A^{y\,t}_{\omega_i}$. These two matrices are identical and will be denoted by Σ_{ω_i}. Its (k,l)-th element is denoted by $\Sigma^{(k,l)}_{\omega_i}$. For instance the correlation between $\delta a^u_{\omega_i}$ and $\delta b^u_{\omega_i}$ is $\mathbf{E}(\delta a^u_{\omega_i} \delta b^u_{\omega_i}) = \sigma^2_{u_{\omega_i}} \Sigma^{(1,2)}_{\omega_i}$. In the next section, we show how the knowledge of these noise covariance matrices can be exploited in an errors-in-variables scheme to estimate the parameters of a transfer function.

3 Errors-in-variables and the quotient singular value decomposition

In this section we first present a most elementary derivation of the idea of errors-in-variables. Its main feature is that it always leads to a (generalized) eigenvalue problem, which can be solved by the quotient singular value decomposition.

3.1 Errors-in-variables

Let A be an $m \times n$ $(m > n)$ matrix of 'exact' data, which is rank deficient:

$$rank(A) = n - 1 < n \tag{9}$$

Let the vector x describe the linear relation between the columns of A:

$$Ax = 0 \tag{10}$$

Let N be the perturbation matrix (the matrix of errors, N stands for 'noise') and assume that

$$\mathbf{E}(N) = 0 \tag{11}$$

which of course implies that $\mathbf{E}(A^tN) = 0$ because A is a deterministic matrix. Let C be the $m \times n$ matrix which is the sum of A and N:

$$C = A + N$$

Then it follows that

$$\mathbf{E}(C^tC) = A^tA + \mathbf{E}(A^tN) + \mathbf{E}(N^tA) + \mathbf{E}(N^tN) = A^tA + \mathbf{E}(N^tN) \tag{12}$$

Define the matrices

$$\begin{aligned} \Sigma_C &= \mathbf{E}(C^tC) \\ \Sigma_N &= \mathbf{E}(N^tN) \end{aligned}$$

and assume that we know Σ_N up to an unknown constant of proportionality λ. If we postmultiply (12) with x, we find the generalized eigenvalue problem

$$\Sigma_C x = \Sigma_N x \lambda \tag{13}$$

When $\Sigma_N = I$, (13) reduces to an ordinary eigenvalue problem. The conclusion is the following:

- Under the standing statistical assumptions (9)-(10)-(11)-(12), the identification of a linear relation always leads to a generalized eigenvalue problem.

- The linear relation x is the eigenvector corresponding to the smallest eigenvalue λ (because $\Sigma_C - \Sigma_N\lambda$ must be nonnegative definite since it is a model for the 'covariance' matrix of the noisefree data).

- A consistent estimate of the noise variance can be obtained from the smallest eigenvalue because $\mathbf{E}\|N\|_F^2 = trace(\lambda\Sigma_N)$. In particular, when $\Sigma_N = I$ and each element of N has a variance σ^2, we find $\lambda \approx m\sigma^2$.

Observe that we have derived the solution without minimizing any norm, as is the case in the derivation of total linear least squares [5] and its generalizations [13]. In essence, these techniques also lead to an eigenvalue problem, but their formal derivation only 'works' for unstructured matrices. In our application however, which will be developed in section 4, the data matrices have a special structure. Despite their structure, the solution as a generalized eigenvalue problem will simply follow from the standing assumptions (9)-(10)-(11)-(12)!

In practice the expected values of the matrices are replaced by their sample estimates. A discussion at length of the corresponding finite sample properties of both the covariance estimates and the generalized eigenvalue problems as well as the consistency properties as $m \to \infty$, can be found in [1].

3.2 The quotient singular value decomposition

The relation between the singular value decomposition and the symmetric eigenvalue problem is well known (see e.g. [5]). Let the SVD of an $m \times n$ matrix A be $A = USV^t$. Then

$$\begin{aligned} AA^tU &= U(SS^t) \\ A^tAV &= V(S^tS) \end{aligned}$$

Hence, the left singular vectors of A are the eigenvectors of AA^t and the right singular vectors are the eigenvectors of A^tA. The corresponding eigenvalues are the squares of the singular values. It is well known that the explicit and direct decomposition of the SVD is to be preferred over its computation as a symmetric eigenvalue problem. Indeed, the 'squaring' of A in A^tA and AA^t causes a loss of half of the numerical relative accuracy and hence is to be avoided.
As an example, consider the matrix

$$A = \begin{pmatrix} -2.8226e+01 & -1.6545e+00 & -3.8510e+01 & 2.0614e+01 \\ 3.7042e+01 & 2.9372e+00 & 5.0627e+01 & -2.7723e+01 \\ 1.6885e+01 & 1.9344e+00 & 2.3147e+01 & -1.3158e+01 \\ 2.5953e+01 & 1.7302e+00 & 3.5433e+01 & -1.9137e+01 \\ 1.2205e+01 & 8.4267e-01 & 1.6666e+01 & -9.0248e+00 \\ -2.3699e+01 & -2.0953e+00 & -3.2416e+01 & 1.7926e+01 \\ -2.2129e+01 & -2.0470e+00 & -3.0279e+01 & 1.6817e+01 \\ -1.6855e+01 & -1.9139e+00 & -2.3104e+01 & 1.3119e+01 \\ -3.4587e+01 & -2.6487e+00 & -4.7262e+01 & 2.5804e+01 \\ -3.7946e+00 & -2.5719e-01 & -5.1812e+00 & 2.8017e+00 \end{pmatrix}$$

Observe that at first sight, there is nothing wrong with this matrix; in particular, the data seem to be balanced in their orders of magnitude.
The singular values of A [1] are 1.4142e+02, 1.4142e+00, 1.0000e-07 and 9.9999e-10. The numerical rank (defined as the index of the smallest singular value σ_r satisfying $\sigma_r > \sigma_1 \epsilon_m$) is clearly 4. Obviously, the right singular vector corresponding to the smallest singular value is well determined. The computed eigenvalues of $fl(A^tA)$ [2] however are 2.0000e+04, 2.0000e+00, 2.6495e-12 and -9.8821e-13 . The numerical rank is 2. The elements of the eigenvector corresponding to the smallest eigenvalue do not have even a vague ressemblance to the corresponding singular vector of A. As a matter of fact, this eigenvector is not uniquely defined within the limited numerical precision. We want to emphasize that this phenomenon is **not** a consequence of the numerical algorithms used to compute the SVD and eigenvalue decompositions. The only cause of this catastrophic loss of accuracy is the explicit squaring in A^tA. This is clarified if we reveal how we have generated our example. The matrix A is unitarily equivalent to the matrix

$$B = \begin{pmatrix} 1.0000e+00 & 1.0000e+00 & 0 & 0 \\ 1.0000e-09 & 0 & 0 & 0 \\ 0 & 1.0000e-09 & 0 & 0 \\ 0 & 0 & 1.0000e+02 & 1.0000e+02 \\ 0 & 0 & 1.0000e-07 & 0 \\ 0 & 0 & 0 & 1.0000e-07 \end{pmatrix}$$

The singular vector corresponding to the smallest singular value is $[\, -1/\sqrt{2}, 1/\sqrt{2}, 0, 0\,]$ and is well determined. The finite precision computation with $\epsilon_m \approx 1.00e-016$ of B^tB gives

$$fl(B^tB) = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 10000 & 10000 \\ 0 & 0 & 10000 & 10000 \end{pmatrix}$$

which has eigenvalues 20000,2,0 and 0. Obviously, there is no *unique* eigenvector corresponding to the smallest eigenvalue (only the corresponding two-dimensional subspace is well determined).
In the context of identification with noisy data, this example illustrates the importance of using the SVD instead of the corresponding symmetric eigenvalue problem. Imagine that the data in A are noisy measurements and that the covariance matrix of the noise is an unknown multiple of the identity matrix. It follows from section 3.1., that the right singular vector of A corresponding to the smallest singular vector is an *unbiased* estimator of the 'exact' (noisefree) linear relation. While we can find a good estimate of this linear relation, using the matrix A, this is clearly impossible if we first compute A^tA.

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[1] Computed with MATLAB, numerical precision $\epsilon_m \approx 1e-016$

[2] fl(.) denotes the floating point result of a computation.

In a similar way one has to avoid the explicit formation of the matrices A^tA and B^tB in the symmetric generalized eigenvalue problem. This can be avoided by using the **QSVD**:

Lemma 1 The quotient singular value decomposition [3]
Every pair of matrices $A \in \Re^{m\times n}$ and $B \in \Re^{p\times n}$ can be factorized as

$$\begin{aligned} A &= U_a S_a X^{-1} \\ B &= U_b S_b X^{-1} \end{aligned}$$

where U_a and U_b are orthonormal. The matrices S_a and S_b have the following structure:

$$S_a = \begin{array}{c} \\ r_{ab}-r_b \\ r_a+r_b-r_{ab} \\ m-r_a \end{array} \begin{array}{c} \begin{array}{cccc} r_{ab}-r_b & r_a+r_b-r_{ab} & r_{ab}-r_a & n-r_{ab} \end{array} \\ \begin{pmatrix} I & 0 & 0 & 0 \\ 0 & D_a & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

$$S_b = \begin{array}{c} \\ p-r_b \\ r_a+r_b-r_{ab} \\ r_{ab}-r_a \end{array} \begin{array}{c} \begin{array}{cccc} r_{ab}-r_b & r_a+r_b-r_{ab} & r_{ab}-r_a & n-r_{ab} \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & D_b & 0 & 0 \\ 0 & 0 & I & 0 \end{pmatrix} \end{array}$$

where $r_{ab} = rank\begin{pmatrix} A \\ B \end{pmatrix}$ and $D_a^2 + D_b^2 = I_{r_a+r_b-r_{ab}}$. The matrix $X \in \Re^{n\times n}$ is a nonsingular matrix and is a common factor in the decompositions of A and B.

The relation of the **QSVD** with the generalized eigenvalue problem is an immediate consequence of the orthonormality of U_a and U_b. It is easy to show that

$$A^tAXS_b^tS_b = B^tBXS_a^tS_a$$

and when B^tB is nonsingular:

$$A^tAX = B^tBX(S_a^tS_a(S_b^tS_b)^{-1})$$

Hence, the columns of X are the eigenvectors of the generalized eigenvalue problem while the diagonal elements of $(S_a^tS_a(S_b^tS_b)^{-1})$ are the corresponding generalized eigenvalues. Observe however that we have avoided completely the squaring of A and B.
Algorithms for the computation of the **QSVD** are described in [5] and the references therein. Observe that a QR-decomposition for both A and B might greatly reduce the computational complexity. This follows from

$$\begin{aligned} A &= Q_aR_a &= U_aS_aX^{-1} \\ B &= Q_bR_b &= U_bS_bX^{-1} \end{aligned}$$

[3] This factorization for two matrices is called the generalized SVD in [5]. However recently several other generalizations have been discovered [4] and the name **QSVD** was proposed in [3].

This implies that

$$\begin{aligned} R_a &= (Q_a^t U_a) S_a X^{-1} \\ R_b &= (Q_b^t U_b) S_b X^{-1} \end{aligned}$$

is a **QSVD** of the triangular factors of the QR-decomposition of A and B with other orthonormal factors but with the same quasi-diagonal factors S_a and S_b and nonsingular factor X as the **QSVD** of (A, B). When a new row is appended to both A and B, it is relatively easy to update the QR-decomposition of each using $n-1$ Givens rotations (see e.g. [5]). Observe that in most applications we don't need the orthogonal factors explicitly. Only the triangular factors need to be updated (For more details about updating the **QSVD** and related questions, we refer to [9]).

4 Transfer function parameter estimation

In this section, we combine the ideas of section 2 and 3 to develop an identification technique that estimates the parameters of a transfer function, starting from noise corrupted stepped sine measurements. In particular, it will be demonstrated how we can use the error covariance matrices of the least squares estimation step of section 2 to construct 'square roots' M of the error covariance matrix Σ_N of the errors-in-variables method described in section 3. A **QSVD** of the matrix pair (C, M) where C is a certain structured matrix containing the results of the least squares step, will then deliver the parameters of the transfer function. Both discrete time and continuous time input/output models are considered.

A continuous time input/output model relates the Laplace transform of the system input $U(s)$ to the Laplace transform of the system output $Y(s)$.

$$\frac{Y(s)}{U(s)} = H_c(s) \tag{14}$$

$H_c(s)$ is the Laplace transform of the impulse response of the system, and can be modelled as a ratio of two polynomials in s

$$H_c(s) = \frac{\beta_0 + \beta_1 s + \ldots + \beta_q s^q}{\alpha_0 + \alpha_1 s + \ldots + \alpha_p s^p} \tag{15}$$

where q is the order of the numerator and p is the order of the denominator.
A discrete time input/output model relates the z-transform of the system input $U(z^{-1})$ to the z-transform of the system output $Y(z^{-1})$.

$$\frac{Y(z^{-1})}{U(z^{-1})} = H_d(z^{-1}) \tag{16}$$

$H_d(z^{-1})$ is the z-transform of the unit step response of the system, and can be modelled as a ratio of two polynomials in z^{-1}

$$H_d(z^{-1}) = \frac{\acute{\beta}_0 + \acute{\beta}_1 z^{-1} + \ldots + \acute{\beta}_q z^{-q}}{\acute{\alpha}_0 + \acute{\alpha}_1 z^{-1} + \ldots + \acute{\alpha}_p z^{-p}} \tag{17}$$

where q is the order of the numerator and p is the order of the denominator.
The orders of these continuous time and discrete time domain models are often estimated by inspection of the magnitude and phase of the measured frequency response function and we assume that they are known.
To identify the continuous time domain model, the transfer function $H_c(s)$ when evaluated at $s = j\omega_i$, is equated with the value of the frequency response function at that particular pulsation ω_i:

$$H_c(j\omega_i) = \frac{a^y_{\omega_i} + jb^y_{\omega_i}}{a^u_{\omega_i} + jb^u_{\omega_i}} \tag{18}$$

Combining (15) and (18) results in the following complex linear relation:

$$\begin{aligned} 0 &= (\alpha_0 + \alpha_1(j\omega_i) + \ldots + \alpha_p(j\omega_i)^p)(a^y_{\omega_i} + jb^y_{\omega_i}) \\ &- (\beta_0 + \beta_1(j\omega_i) + \ldots + \beta_q(j\omega_i)^q)(a^u_{\omega_i} + jb^u_{\omega_i}) \end{aligned} \tag{19}$$

Both the real part and the imaginary part of this equation have to be zero. This results in the following two linear equations:

$$\begin{pmatrix} a^y_{\omega_i} & -b^y_{\omega_i}\omega_i & -a^y_{\omega_i}\omega_i^2 & \ldots & -a^u_{\omega_i} & b^u_{\omega_i}\omega_i & a^u_{\omega_i}\omega_i^2 & \ldots \\ b^y_{\omega_i} & a^y_{\omega_i}\omega_i & -b^y_{\omega_i}\omega_i^2 & \ldots & -b^u_{\omega_i} & -a^u_{\omega_i}\omega_i & b^u_{\omega_i}\omega_i^2 & \ldots \end{pmatrix} \begin{pmatrix} \alpha_0 \\ \alpha_1 \\ \vdots \\ \alpha_p \\ \beta_0 \\ \beta_1 \\ \vdots \\ \beta_q \end{pmatrix} = 0 \tag{20}$$

In order to identify the discrete time domain model, the transfer function $H_d(z^{-1})$ when evaluated at $z = e^{j\omega_i T_s}$, is equated with the value of the frequency response function at that particular pulsation ω_i:

$$H_d(e^{-j\omega_i T_s}) = \frac{a^y_{\omega_i} + jb^y_{\omega_i}}{a^u_{\omega_i} + jb^u_{\omega_i}} \tag{21}$$

T_s is the sampling period for which the discrete model has to be identified. Combining (17) and (21) results in the following complex linear relation:

$$\begin{aligned} 0 &= (\acute{\alpha}_0 + \acute{\alpha}_1(e^{-j\omega_i T_s}) + \ldots + \acute{\alpha}_p(e^{-j\omega_i pT_s})(a^y_{\omega_i} + jb^y_{\omega_i}) \\ &- (\acute{\beta}_0 + \acute{\beta}_1(e^{-j\omega_i T_s}) + \ldots + \acute{\beta}_q(e^{-j\omega_i qT_s})(a^u_{\omega_i} + jb^u_{\omega_i}) \end{aligned} \tag{22}$$

Both the real part and the imaginary part of this equation have to be zero. This results in the following two linear equations:

$$\begin{pmatrix} a^y_{\omega_i} & \Re((a^y_{\omega_i}+jb^y_{\omega_i})e^{-j\omega_i T_s}) & \dots & -a^u_{\omega_i} & \Re(-(a^u_{\omega_i}+jb^u_{\omega_i})e^{-j\omega_i T_s}) & \dots \\ b^y_{\omega_i} & \Im((a^y_{\omega_i}+jb^y_{\omega_i})e^{-j\omega_i T_s}) & \dots & -b^u_{\omega_i} & \Im(-(a^u_{\omega_i}+jb^u_{\omega_i})e^{-j\omega_i T_s}) & \dots \end{pmatrix} \begin{pmatrix} \acute{\alpha}_0 \\ \acute{\alpha}_1 \\ \vdots \\ \acute{\alpha}_p \\ \acute{\beta}_0 \\ \acute{\beta}_1 \\ \vdots \\ \acute{\beta}_q \end{pmatrix} = 0 \quad (23)$$

$\Re(.)$ denotes the real part of a complex number and $\Im(.)$ denotes the imaginary part of a complex number.
If the frequency response function is measured at m different frequencies ω_i, $i = 1, 2, \dots, m$, (20) and (23) result in a set of linear equations of the following form

$$Aa = 0 \quad (24)$$

where A is a $(2m) \times (p+q+2)$ matrix and

$$a = (\ \alpha_0\ \ \alpha_1\ \ \dots\ \ \alpha_p\ \ \beta_0\ \ \beta_1\ \ \dots\ \ \beta_q\ )'$$

in the continuous time case and,

$$a = (\ \acute{\alpha}_0\ \ \acute{\alpha}_1\ \ \dots\ \ \acute{\alpha}_p\ \ \acute{\beta}_0\ \ \acute{\beta}_1\ \ \dots\ \ \acute{\beta}_q\ )'$$

in the discrete case.
In reality however, the matrix A of (24), which contains the exact data, is not available. In particular, the data are corrupted by additive noise and we only observe the matrix

$$C = A + N$$

where N is the noise matrix which consists of two submatrices N_y and N_u:

$$N = (\ N_y\ \ N_u\ )$$

N_u contains the errors on $a^u_{\omega_i}$ and $b^u_{\omega_i}$ while N_y contains the errors on $a^y_{\omega_i}$ and $b^y_{\omega_i}$. In the continuous time domain, both matrices have the following 'graded' structure:

$$N_y = \begin{pmatrix} \delta a^y_{\omega_1} & -\omega_1 \delta b^y_{\omega_1} & -(\omega_1)^2 \delta a^y_{\omega_1} & (\omega_1)^3 \delta b^y_{\omega_1} & \dots \\ \delta b^y_{\omega_1} & \omega_1 \delta a^y_{\omega_1} & -(\omega_1)^2 \delta b^y_{\omega_1} & -(\omega_1)^3 \delta a^y_{\omega_1} & \dots \\ \delta a^y_{\omega_2} & -\omega_2 \delta b^y_{\omega_2} & -(\omega_2)^2 \delta a^y_{\omega_2} & (\omega_2)^3 \delta b^y_{\omega_2} & \dots \\ \delta b^y_{\omega_2} & \omega_2 \delta a^y_{\omega_2} & -(\omega_2)^2 \delta b^y_{\omega_2} & -(\omega_2)^3 \delta a^y_{\omega_2} & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \delta a^y_{\omega_m} & -\omega_m \delta b^y_{\omega_m} & -(\omega_m)^2 \delta a^y_{\omega_m} & (\omega_m)^3 \delta b^y_{\omega_m} & \dots \\ \delta b^y_{\omega_m} & \omega_m \delta a^y_{\omega_m} & -(\omega_m)^2 \delta b^y_{\omega_m} & -(\omega_m)^3 \delta a^y_{\omega_m} & \dots \end{pmatrix}$$

and

$$N_u = \begin{pmatrix} -\delta a^u_{\omega_1} & \omega_1 \delta b^u_{\omega_1} & (\omega_1)^2 \delta a^u_{\omega_1} & -(\omega_1)^3 \delta b^u_{\omega_1} & \dots \\ -\delta b^u_{\omega_1} & -\omega_1 \delta a^u_{\omega_1} & (\omega_1)^2 \delta b^u_{\omega_1} & (\omega_1)^3 \delta a^u_{\omega_1} & \dots \\ -\delta a^u_{\omega_2} & \omega_2 \delta b^u_{\omega_2} & (\omega_1)^2 \delta a^u_{\omega_2} & -(\omega_2)^3 \delta b^u_{\omega_2} & \dots \\ -\delta b^u_{\omega_2} & -\omega_2 \delta a^u_{\omega_2} & (\omega_2)^2 \delta b^u_{\omega_2} & (\omega_1)^2 \delta a^u_{\omega_2} & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -\delta a^u_{\omega_m} & \omega_m \delta b^u_{\omega_m} & (\omega_m)^2 \delta a^u_{\omega_m} & -(\omega_m)^3 \delta b^u_{\omega_m} & \dots \\ -\delta b^u_{\omega_m} & -\omega_m \delta a^u_{\omega_m} & (\omega_m)^2 \delta b^u_{\omega_m} & (\omega_m)^3 \delta a^u_{\omega_m} & \dots \end{pmatrix}$$

This follows directly from (20).

The structure of submatrices N_y and N_u in the discrete time domain scheme is:

$$N_y = \begin{pmatrix} \delta a^y_{\omega_1} & \Re((\delta a^y_{\omega_1} + j\delta b^y_{\omega_1}) e^{-j\omega_1 T_s}) & \Re((\delta a^y_{\omega_1} + j\delta b^y_{\omega_1}) e^{-2j\omega_1 T_s}) & \dots \\ \delta b^y_{\omega_1} & \Im((\delta a^y_{\omega_1} + j\delta b^y_{\omega_1}) e^{-j\omega_1 T_s}) & \Im((\delta a^y_{\omega_1} + j\delta b^y_{\omega_1}) e^{-2j\omega_1 T_s}) & \dots \\ \delta a^y_{\omega_2} & \Re((\delta a^y_{\omega_2} + j\delta b^y_{\omega_2}) e^{-j\omega_2 T_s}) & \Re((\delta a^y_{\omega_2} + j\delta b^y_{\omega_2}) e^{-2j\omega_2 T_s}) & \dots \\ \delta b^y_{\omega_2} & \Im((\delta a^y_{\omega_2} + j\delta b^y_{\omega_2}) e^{-j\omega_2 T_s}) & \Im((\delta a^y_{\omega_2} + j\delta b^y_{\omega_2}) e^{-2j\omega_2 T_s}) & \dots \\ \vdots & \vdots & \vdots & \vdots \\ \delta a^y_{\omega_m} & \Re((\delta a^y_{\omega_m} + j\delta b^y_{\omega_m}) e^{-j\omega_m T_s}) & \Re((\delta a^y_{\omega_m} + j\delta b^y_{\omega_m}) e^{-2j\omega_m T_s}) & \dots \\ \delta b^y_{\omega_m} & \Im((\delta a^y_{\omega_m} + j\delta b^y_{\omega_m}) e^{-j\omega_m T_s}) & \Im((\delta a^y_{\omega_m} + j\delta b^y_{\omega_m}) e^{-2j\omega_m T_s}) & \dots \end{pmatrix} \tag{25}$$

and

$$N_u = \begin{pmatrix} -\delta a^y_{\omega_1} & \Re(-(\delta a^y_{\omega_1} + j\delta b^y_{\omega_1}) e^{-j\omega_1 T_s}) & \Re(-(\delta a^y_{\omega_1} + j\delta b^y_{\omega_1}) e^{-2j\omega_1 T_s}) & \dots \\ -\delta b^y_{\omega_1} & \Im(-(\delta a^y_{\omega_1} + j\delta b^y_{\omega_1}) e^{-j\omega_1 T_s}) & \Im(-(\delta a^y_{\omega_1} + j\delta b^y_{\omega_1}) e^{-2j\omega_1 T_s}) & \dots \\ -\delta a^y_{\omega_2} & \Re(-(\delta a^y_{\omega_2} + j\delta b^y_{\omega_2}) e^{-j\omega_2 T_s}) & \Re(-(\delta a^y_{\omega_2} + j\delta b^y_{\omega_2}) e^{-2j\omega_2 T_s}) & \dots \\ -\delta b^y_{\omega_2} & \Im(-(\delta a^y_{\omega_2} + j\delta b^y_{\omega_2}) e^{-j\omega_2 T_s}) & \Im(-(\delta a^y_{\omega_2} + j\delta b^y_{\omega_2}) e^{-2j\omega_2 T_s}) & \dots \\ \vdots & \vdots & \vdots & \vdots \\ -\delta a^y_{\omega_m} & \Re(-(\delta a^y_{\omega_m} + j\delta b^y_{\omega_m}) e^{-j\omega_m T_s}) & \Re(-(\delta a^y_{\omega_m} + j\delta b^y_{\omega_m}) e^{-2j\omega_m T_s}) & \dots \\ -\delta b^y_{\omega_m} & \Im(-(\delta a^y_{\omega_m} + j\delta b^y_{\omega_m}) e^{-j\omega_m T_s}) & \Im(-(\delta a^y_{\omega_m} + j\delta b^y_{\omega_m}) e^{-2j\omega_m T_s}) & \dots \end{pmatrix} \tag{26}$$

The covariance matrix Σ_N of the noise can be estimated because of the knowledge of the correlations between the noise variables $\delta a^u_{\omega_i}, \delta b^u_{\omega_i}, \delta a^y_{\omega_i}, \delta b^y_{\omega_i}$ from the least squares estimation in section 2.

$$\Sigma_N = \mathbf{E}(N^t N) = \begin{pmatrix} \mathbf{E}(N_y^t N_y) & 0 \\ 0 & \mathbf{E}(N_u^t N_u) \end{pmatrix}$$

The expression for $\mathbf{E}(N_y^t N_y)$ in the continuous time domain scheme is:

$$\mathbf{E}(\mathrm{N}_y^t \mathrm{N}_y) = \begin{pmatrix} \sum_{i=1}^m \sigma^2_{y_{\omega_i}} \Lambda_{\omega_i} & 0 & -\sum_{i=1}^m (\omega_i)^2 \sigma^2_{y_{\omega_i}} \Lambda_{\omega_i} & \dots \\ \cdot & \sum_{i=1}^m (\omega_i)^2 \sigma^2_{y_{\omega_i}} \Lambda_{\omega_i} & 0 & \dots \\ \cdot & \cdot & \sum_{i=1}^m (\omega_i)^4 \sigma^2_{y_{\omega_i}} \Lambda_{\omega_i} & \dots \\ \cdot & \cdot & \cdot & \dots \\ \vdots & \vdots & \vdots & \vdots \end{pmatrix} \tag{27}$$

with Λ_{ω_i} equal to $\Sigma^{(1,1)}_{\omega_i} + \Sigma^{(2,2)}_{\omega_i}$. The expression for $\mathbf{E}(N_y^t N_y)$ in the discrete time domain scheme is:

$$\mathbf{E}(\mathrm{N}_y^t \mathrm{N}_y) = \begin{pmatrix} \sum_{i=1}^m \sigma^2_{y_{\omega_i}} \Lambda_{\omega_i} & \sum_{i=1}^m \Re(e^{-j\omega_i T_s}) \sigma^2_{y_{\omega_i}} \Lambda_{\omega_i} & \sum_{i=1}^m \Re(e^{-2j\omega_i T_s}) \sigma^2_{y_{\omega_i}} \Lambda_{\omega_i} & \dots \\ \cdot & \sum_{i=1}^m \sigma^2_{y_{\omega_i}} \Lambda_{\omega_i} & \sum_{i=1}^m \Re(e^{-j\omega_i T_s}) \sigma^2_{y_{\omega_i}} \Lambda_{\omega_i} & \dots \\ \cdot & \cdot & \sum_{i=1}^m \sigma^2_{y_{\omega_i}} \Lambda_{\omega_i} & \dots \\ \cdot & \cdot & \cdot & \dots \\ \vdots & \vdots & \vdots & \vdots \end{pmatrix}$$

The expressions for the corresponding covariance matrix $\mathbf{E}(N_u^t N_u)$ are exactly the same, except that $\sigma^2_{y_{\omega_i}}$ is replaced by $\sigma^2_{u_{\omega_i}}$.
It is straightforward to find a 'square root' M_y of the matrix $\mathbf{E}(N_y^t N_y)$ in (27), i.e. a matrix M_y of size $(2m) \times (p+q+2)$ such that $\mathbf{E}(N_y^t N_y) = M_y^t M_y$. Indeed, define

$$\begin{aligned} \epsilon_i &= \sqrt{\Sigma_{\omega_i}^{(1,1)}} \\ \eta_i &= \sqrt{\Sigma_{\omega_i}^{(2,2)}} \end{aligned}$$

Then it is easily verified that

$$M_y = \begin{pmatrix} \epsilon_1 & -\omega_1\eta_1 & -(\omega_1)^2\epsilon_1 & (\omega_1)^3\eta_1 & \dots \\ \eta_1 & \omega_1\epsilon_1 & -(\omega_1)^2\eta_1 & -(\omega_1)^3\epsilon_1 & \dots \\ \epsilon_2 & -\omega_2\eta_2 & -(\omega_2)^2\epsilon_2 & (\omega_2)^3\eta_2 & \dots \\ \eta_2 & \omega_2\epsilon_2 & -(\omega_2)^2\eta_2 & -(\omega_2)^3\epsilon_2 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \epsilon_m & -\omega_m\eta_m & -(\omega_m)^2\epsilon_m & (\omega_m)^3\eta_m & \dots \\ \eta_m & \omega_m\epsilon_m & -(\omega_m)^2\eta_m & -(\omega_m)^3\epsilon_m & \dots \end{pmatrix}$$

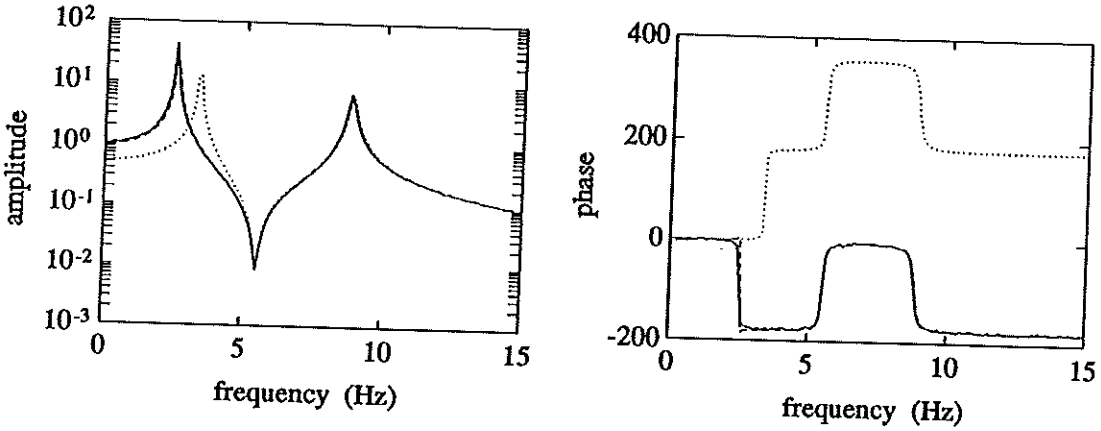
Square roots of the matrices $\mathbf{E}(N_y^t N_y)$ for the discrete time case and for the matrices $\mathbf{E}(N_u^t N_u)$ can be obtained in a similar way.

Summarizing, we have the following algorithm to estimate the parameters of a transfer function:

- Obtain least squares estimates of the values of the transfer function from stepped sign measurements, together with estimates of the error covariances (section 2).
- Construct the matrices C and the square roots M of the error covariance matrices (section 4).
- Compute the **QSVD** of the matrix pair (C, M) as outlined in section 3.

5 Simulation Results

In this section, we illustrate the accuracy of our **QSVD** based identification method. Both the continuous time domain and the discrete time domain schemes are tested. The test case is a fourth order continuous time model and its discrete time equivalent (for a sampling period of 10 $msec$). The model describes a system with resonance frequencies at $2.5Hz$ and $8.8Hz$, and an anti-resonance frequency at $5.5Hz$. The damping ratio of these modes is 0.01. The model was treated as a black box and submitted to a stepped sine excitation with a frequency step of $0.1Hz$ in a frequency range going from $0.2Hz$ to $15Hz$. Additive measurement noise is simulated by adding normally distributed random sequences to the input sinusoids and corresponding model outputs. An offset and a linear trend were added to the outputs.



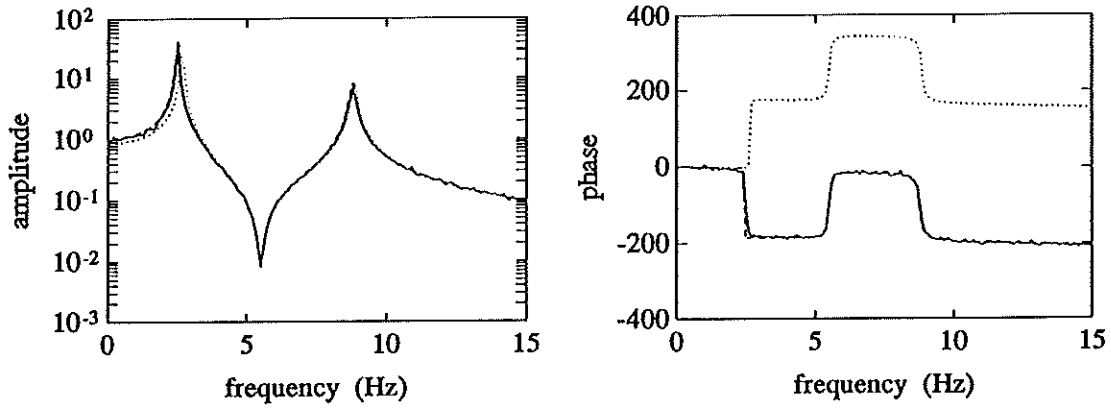
solid line	:	frequency response function obtained from a stepped sine analysis of the model
dashed line	:	**QSVD** identification result
dotted line	:	**SVD** result (=**QSVD** with identity covariance weighting)

Figure 1: Identification result of the continuous time domain model

Figure 1 shows the amplitude and phase diagrams of the identified continuous time domain models. The standard deviation of the noise sequences on the input and output sinusoids is 20% of the amplitude of the respective signals. The solid line shows the frequency response function that resulted from the corrupted time domain input/output data by solving equations (3) and (6). The dashed line shows the transfer function obtained with the **QSVD** algorithm outlined in section 4. The dotted line shows the result when the covariance weighting is not taken into account (i.e. when it is taken to be equal to the identity matrix, in which case the **QSVD** reduces to an **SVD** of the data matrix C). This last result shows the importance of the appropriate covariance weigting. For this example, if the appropriate covariances are not taken into account, the estimation of the first mode is bad, resulting in an erroneous eigenfrequency and even a negative damping. The **QSVD** method finds a model that is much closer to the 'exact' model.

Figure 2 shows the amplitude and phase diagrams of the identified discrete time domain models. The standard deviation of the noise sequences on the input and output sinusoids is 40% of the amplitude of the respective signal. The solid line shows the frequency response estimates that resulted from the corrupted time domain input/output data by solving equations (3) and (6). The dashed line shows **QSVD** results. The dotted line shows the solution when the noise covariance is set equal to the identity.

One might observe that that the identification results for the discrete time domain model are more accurate than the continuous time results. In the continuous time case, the elements of matrix C range over several orders of magnitude as they are proportional to powers of the pulsations. This leads to a bad condition number of



solid line	:	frequency response function obtained from a stepped sine analysis of the model
dashed line	:	**QSVD** based identification result
dotted line	:	**SVD** result (=**QSVD** with identity covariance weighting)

Figure 2: Identification result of the discrete time domain model

the matrix which affects the sensitivity of the singular vector corresponding to the smallest (generalized) singular value. Unfortunately, there is no appropriate scaling to remedy this ill-conditioning. In the discrete time case, the elements of the data matrix C are all of the same order of magnitude leading to a much better balanced matrix and a smaller sensitivity of the solution vector.

6 Conclusions

We have presented a new identification method to estimate the parameters of the transfer function of a single input single output linear system.

- Starting from noisy stepped sine input-output measurements, the frequency response is estimated using linear least squares. This also gives us error covariance matrices. Offsets and trends can be taken into account as well as nonlinearities that generate higher harmonics.

- The results of the least squares step are then used to estimate the parameters of the transfer function. Taking into account the structure of the data matrix and the noise covariances, this leads to a generalized eigenvalue problem.

- Finally, we have shown how to solve the 'square root' version of the generalized eigenvalue problem in a numerically robust way, using the quotient singular value decomposition for two matrices. We have also emphasized the feasibility of recursively updating all calculations.

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