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Notes on VOLVO PID Control Algorithm (Internal Report)

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VOLVO PID control uses following algorithm[1]:

$$r_n = r_{n-1} + dif_n + p_{gain} * (e_n - e_{n-1}) + i_{gain} * e_n \tag{1}$$

$$dif_n = d_{gain} * (e_n - 2e_{n-1} + e_{n-2}) + \Gamma * dif_{n-1} \tag{2}$$

where r is PID output value and e is input error and

$$\begin{aligned} p_{gain} &= p_{gainf} * 2^{p_{gainp}} \\ i_{gain} &= i_{gainf} * 2^{i_{gainp}+time_{base}} \\ d_{gain} &= d_{gainf} * 2^{d_{gainp}-time_{base}} \end{aligned} \tag{3}$$

For this algorithm, several remarks are made as following.

1 The model of the PID controller

The model of the PID controller with the algorithm as above has the model (continuous time) as:

$$r = P + I + D \tag{4}$$

where

$$P = p_{gain} e \tag{5}$$

$$I = \frac{i_{gain}}{h} \int_0^t e dt \quad or \quad \frac{dI}{dt} = \frac{i_{gain}}{h} e \tag{6}$$

$$D = -\frac{h\Gamma}{1-\Gamma} \frac{dD}{dt} + \frac{d_{gain} h}{1-\Gamma} \frac{de}{dt} \tag{7}$$

h is the sampling period. By discretization of Eq. (5),(6) and (7), we can prove this. The derivatives are assumed to be approximated by backward differences. Then, in the digital form, Eq. (5) becomes

$$P_n = p_{gain}e_n = p_{gain}e_{n-1} + p_{gain}(e_n - e_{n-1}) \tag{8}$$

and Ed. (6) becomes

$$\frac{I_n - I_{n-1}}{h} = \frac{i_{gain}}{h}e_n \tag{9}$$

which can be written as

$$I_n = I_{n-1} + i_{gain}e_n \tag{10}$$

By discretization, Eq. (7), the derivative term, turns to be

$$D_n = -\frac{h\Gamma}{1-\Gamma}(\frac{D_n - D_{n-1}}{h}) + \frac{d_{gain}h}{1-\Gamma}(\frac{e_n - e_{n-1}}{h}) \tag{11}$$

Let

$$D_n - D_{n-1} = dif_n \tag{12}$$

Then Eq. (11) becomes

$$D_n = -\frac{\Gamma}{1-\Gamma}dif_n + \frac{d_{gain}}{1-\Gamma}(e_n - e_{n-1}) \tag{13}$$

From (13), following equation can be obtained:

$$dif_n = D_n - D_{n-1} = -\frac{\Gamma}{1-\Gamma}(dif_n - dif_{n-1}) + \frac{d_{gain}}{1-\Gamma}(e_n - 2e_{n-1} + e_{n-2}) \tag{14}$$

By solving Eq. (14), we can have:

$$dif_n = d_{gain}(e_n - 2e_{n-1} + e_{n-2}) + \Gamma dif_{n-1} \tag{15}$$

Finally, by adding Eq. (8), (10) and (12), we have the PID output as:

$$\begin{aligned} r_n &= P_n + I_n + D_n \\ &= P_{n-1} + I_{n-1} + D_{n-1} + p_{gain}(e_n - e_{n-1}) + i_{gain}e_n + dif_n \\ &= r_{n-1} + p_{gain}(e_n - e_{n-1}) + i_{gain}e_n + dif_n \end{aligned} \tag{16}$$

Eq. (16) and (15) are exactly same as Eq. (1) and (2) which are used in the algorithm.

2 The differences with the standard PID control

The standard PID control has the model:

$$r = P + I + D \tag{17}$$

$$P = K_{ps}e \tag{18}$$

$$I = K_{is}\int_0^t e dt \tag{19}$$

$$D = K_{ds}\frac{de}{dt} \tag{20}$$

If we let

$$\begin{aligned} K_{ps} &= p_{gain} \\ K_{is} &= \frac{i_{gain}}{h} \\ K_{ds} &= d_{gain}h \end{aligned} \tag{21}$$

and compare Eq. (17),(18), (19) and (20) with (4),(5),(6) and (7), it can be found that the difference is only in the derivative term D. This modified PID controller is original designed for the purpose that the derivative action (D) only operates on low frequence components(depending on Γ) and at high frequencies the derivative gain is limited to d_{gain}/Γ. This can be seen from the Laplace transformation of D:

$$D(s) = \frac{d_{gain}hs}{1 - \Gamma + h\Gamma s}e(s) \tag{22}$$

When $s \to \infty$,$D(s) \to d_{gain}/\Gamma$. When Γ goes to zero, however, the difference will diminish and the modified PID controller will become a standard one. Let $\Gamma = 2^{-16}$ and $h = 0.016$ as in [1], for example, then Eq. (22) becomes as:

$$D(s) = \frac{d_{gain}hs}{0.99998 + 0.000000244s}e(s) \tag{23}$$

The simulation shows that the secondary pressure may not change at frequencies larger then 1 Khz. [The critical frequency which is limited by sampling law is even lower (when h=0.016, it is 31.25hz)]. In this case, Eq. (23) can be approximated as:

$$D(s) = d_{gain}hse(s) \tag{24}$$

or in time domain

$$D = d_{gain}h\frac{de}{dt} \tag{25}$$

which is a derivative term of a standard PID controller. The result is that when the modified PID controller is used for secondary pressure control with $\Gamma = 2^{-16}$, it will actually act as a standard PID controller.

3 Parameters of the PID controller

From Eq. (3) and (21), we can have the parameters as following:

$$\begin{aligned} K_{ps} &= p_{gainf} * 2^{p_{gainp}} \\ K_{is} &= i_{gainf} * 2^{i_{gainp}+time_{base}}/h \\ K_{ds} &= d_{gainf} * 2^{d_{gainp}-time_{base}} * h \end{aligned} \tag{26}$$

From these equations, we can see that only when

$$h = 2^{time_{base}} \tag{27}$$

the parameters K_{is} and K_{ds} are:

$$K_{is} = i_{gainf} * 2^{i_{gainp}}; \quad K_{ds} = d_{gainf} * 2^{d_{gainp}} \tag{28}$$

Let

$$\begin{gathered} p_{gainf} = 5, \quad p_{gainp} = 8, \quad i_{gainf} = 8, \quad i_{gainp} = 3, \\ d_{gainf} = 5, \quad d_{gainp} = 9, \quad time_{base} = 4 \end{gathered}$$

as given in [1], notice that $h = 0.016$, which is not equal to $2^4 = 16$! We have the parameters based on the standard model:

$$\begin{aligned} K_{ps} &= 5 * 2^8 = 1280 \\ K_{is} &= 8 * 2^{3+4}/0.016 = 64000 \\ k_{ds} &= 5 * 2^{9-4} * 0.016 = 2.56 \end{aligned} \tag{29}$$

4 Data Transformation

The input (ϵ) and the output (r) of the PID controller discussed above is in the digital form. If we consider the input is the measured pressure value (ϵ_p) with unit 'bar' and the output is the percentage of the duty cycle (r_p). The PID parameters should be modified. The next two equations give the relations between ϵ and ϵ_p, r and r_p:

$$\epsilon = 100\epsilon_p \tag{30}$$

$$r = \frac{255}{100} * 2^{16} r_p \tag{31}$$

Replacing ϵ and r in Eq. (4), (5), (6) and (7), by Eq. (30) and (31), we can have PID parameters:

$$K_p = \frac{100 * 100}{255} * 2^{-16} K_{ps}$$

$$K_i = \frac{100 * 100}{255} * 2^{-16} K_{is} \tag{32}$$
$$K_d = \frac{100 * 100}{255} 2^{-16} K_{ds}$$

For the given condition,

$$K_p = \frac{100 * 100}{255} * 1280 = 50196 * 2^{-16} = 0.766$$
$$K_i = \frac{100 * 100}{255} * 64000 * 2^{-16} = 2509804 * 2^{-16} = 38.3 \tag{33}$$
$$K_d = \frac{100 * 100}{255} * 2.56 * 2^{-16} = 100 * 2^{-16} = 0.00152$$

References

[1] "PID-controller", letter from VOLVO, Jan., 1992