93_42

Pergamon

0005-1098(95)00016-X

Automatica, Vol. 31, No. 8, pp. 1137–1147, 1995
Elsevier Science Ltd
Printed in Great Britain
0005-1098/95 $9.50 + 0.00

Parameter Identification of Induction Motor Drives*

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The performance of parameter estimation methods can be increased if additional a priori knowledge about the parameters is included in the estimators. This approach is tested on an induction motor.

Key Words—Parameter estimation; electric drives; total least squares estimation.

NOTATION

V_s^q, V_s^d	q-phase and d-phase of stator voltage in the stationary reference frame;
I_s^q, I_s^d	q-phase and d-phase of stator current in the stationary reference frame;
I_s^q, I_r^d	q-phase and d-phase of rotor current in the stationary reference frame;
I_s^a, I_s^b, I_s^c	stator current in a stationary three-phase system;
λ_s^q, λ_s^d	q-phase and d-phase of stator flux in the stationary reference frame;
λ_r^q, λ_r^d	q-phase and d-phase of rotor flux in the stationary reference frame;
ω_r	rotor speed (rad s^{-1});
R_s	stator resistance;
R_r	rotor resistance (referred to stator);
L_s	stator self-inductance;
L_m	stator–rotor mutual inductance;
L_r	rotor self-inductance;
T	electromagnetic torque;
P	number of poles.

Abstract—A parameter identification method is presented that identifies all the parameters of an induction motor simultaneously. The method assumes that the motor can be described by a time-varying linear model. The idea is to filter the current, voltage and speed signals of the motor such that a set of signals results that are related by simple linear equations. The coefficients of these linear equations are the same as the coefficients of the differential equations of the machine. Two estimation methods are investigated: the first tries to identify the parameters with a normal general total least squares method, while the second uses constrained general total least squares. The second method is shown to be more robust than the first with respect to noise sensitivity, which is important since the system must function in an industrial environment. The problem of identifying induction motors is a very difficult one, because interesting phenomena like pole–zero cancellations and non-persistency of excitation may occur.

1. INTRODUCTION

Induction motors are robust and rather cheap motors that hardly need maintenance. Their controllability, however, is quite poor, because there is no simple torque production equation. The resulting characteristic is highly nonlinear. Vector control is the solution to this problem. With powerful microprocessors and digital signal processors (DSP) currently available, this control technique is becoming a good alternative for DC systems. However, some problems still need to be solved, such as reliable on-line parameter identification, which is the subject of this paper. Parameter identification is not only used for vector control but is also important for machine designers and for simulation purposes.

A lot of work has been done on this subject (see e.g. Matsuo and Lipo, 1985; Sugimoto and Tamai, 1987). In this paper we present a new approach, which is based on the identification of continuous systems via linear filtering. The problem then reduces to estimating coefficients of linear equations. The popular recursive least squares (RLS) technique cannot solve this problem, since it puts too many restrictions on the noise signals. A less restrictive method called total least squares (TLS) is used instead. Estimation based on linear filtering is also used for estimating transfer functions of linear systems (Schoukens, 1990). Specific problems arise here, since the goal of our research is not to fit the transfer function correctly but to estimate the physical parameters, which, as we shall show, makes a lot of difference.

* Received 2 November 1993; revised 18 July 1994; received in final form 19 January 1995. This paper was not presented at any IFAC meeting. This paper was initially handled by past Associate Editor Y. Sunahara and recommended for publication in revised form by Editor Torsten Söderström. Corresponding author Dr Christiaan Moons. Tel. +32 16 32 11 11/32 17 09; Fax +32 16 32 19 86; E-mail christiaan.moons@esat.kuleuven.ac.be.

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This paper is organized as follows. The voltage and current equations of induction motors are given in Section 2. The identification method is explained in Section 3. This section also explains the TLS estimation method and its statistical properties. A variant of this approach, which is based on constrained optimization, is explained in Section 4. This variant performs very well under low signal-to-noise ratio conditions. The main contributions of this work are as follows.

- We first demonstrate how a carefully designed filter bank allows us to linearize the problem of estimating the unknown physical parameters of induction motors.
- It turns out that in the resulting set of equations all the data are corrupted by elementwise-correlated noise. Therefore we propose to solve it by a recently introduced technique of total least squares.
- The main cause of the extreme sensitivity of some of the parameter estimates is pointed out.
- It turns out that under low signal-to-noise conditions the parameter sensitivity is still too high. A careful analysis reveals that we get reliable transfer function estimates at high rotor speeds, while the parameter accuracy is very unstable. Therefore we introduce nonlinear constraints into the TLS scheme.

2. INDUCTION MOTOR EQUATIONS

The equations describing the induction motor, are written in a fixed reference frame. The rotor speed (in *electrical* angles) is ω_r. In the linear case we get

$$\begin{aligned} V_s^q &= R_s I_s^q + L_s \dot{I}_s^q + L_m \dot{I}_r^q, \\ V_s^d &= R_s I_s^d + L_s \dot{I}_s^d + L_m \dot{I}_r^d, \\ 0 &= R_r I_r^q + L_m \dot{I}_s^q + L_r \dot{I}_r^q - \omega_r (L_m I_s^d + L_r I_r^d), \\ 0 &= = R_r I_r^q + L_m \dot{I}_s^d + L_r \dot{I}_r^d + \omega_r (L_m I_s^q + L_r I_r^q). \end{aligned} \tag{1}$$

This model does not describe the machine exactly during saturation, which can therefore be interpreted as a disturbance of the model. On the other hand, saturation can also be described as a time-varying inductance L_m as in Zai *et al.* (1992). Since our method can detect time-varying parameters, as will be shown in Section 3.5, it can also deal with saturation effects in an approximate way.

The values I_r^d and I_r^q and their derivatives are not directly measurable. After the elimination of those inmeasurable quantities from (1), the following two equations result:

$$\begin{aligned} &a_2 \frac{d^2 V_s^q}{dt^2} + a_1 \frac{dV_s^q}{dt} + a_0 V_s^q \\ &\quad = b \frac{dI_s^d}{dt} + c_3 \frac{d^3 I_s^q}{dt^3} + c_2 \frac{d^2 I_s^q}{dt^2} + c_1 \frac{dI_s^q}{dt} + c_0 I_s^q, \\ &a_2 \frac{d^2 V_s^d}{dt^2} + a_1 \frac{dV_s^d}{dt} + a_0 V_s^d \\ &\quad = -b \frac{dI_s^q}{dt} + c_3 \frac{d^3 I_s^d}{dt^3} + c_2 \frac{d^2 I_s^d}{dt^2} + c_1 \frac{dI_s^d}{dt} + c_0 I_s^d. \end{aligned} \tag{2}$$

Note that the coefficients of the two equations are identical. We measure I_s^d, I_s^q and V_s^q. The voltage V_s^d is not necessary, because, as can be seen from (2), the second equation is redundant. So it suffices to estimate only the coefficients of one of them. In the sequel we use the first. The coefficients are related to the physical parameters based on the following equations. Let $\sigma = L_s L_r - L_m^2$; then

$$\begin{aligned} a_2 &= L_r^2, \quad a_1 = 2 L_r R_r, \\ a_0 &= \omega_r^2 L_r^2 + R_r^2, \quad b = L_m^2 \omega_r R_r, \\ c_3 &= L_r \sigma, \quad c_2 = R_s L_r^2 + R_r L_r L_s + R_r \sigma, \\ c_1 &= 2 R_s R_r L_r - R_r^2 L_s + \omega_r^2 \sigma L_r, \\ c_0 &= R_r^2 R_s + \omega_r^2 R_s L_r^2. \end{aligned} \tag{3}$$

The coefficients of the differential equations are time-varying. If we measure the rotor speed ω_r, we transform the equations such that the coefficients become constant:

$$\begin{aligned} &d_2 \left(\frac{d^2 V_s^q}{dt^2} + \omega_r^2 V_s^q \right) + d_1 \frac{dV_s^q}{dt} + d_0 V_s^q \\ &\quad = e \omega_r \frac{dI_s^d}{dt} + f_4 \left(\frac{d^3 I_s^q}{dt^3} + \omega_r^2 \frac{dI_s^q}{dt} \right) \\ &\qquad + f_3 \frac{d^2 I_s^q}{dt^2} + f_2 \frac{dI_s^q}{dt} + f_1 \left(\frac{d^2 I_s^q}{dt^2} + \omega_r^2 I_s^q \right) + f_0 I_s^q, \end{aligned} \tag{4}$$

with

$$\begin{aligned} \sigma &= L_s L_r - L_m^2, \quad d_2 = L_r^2, \\ d_1 &= 2 L_r R_r, \quad d_0 = R_r^2, \\ e &= L_m^2 R_r, \quad f_4 = L_r \sigma, \\ f_3 &= 2 R_r L_r L_s, \quad f_2 = 2 R_s R_r L_r + R_r^2 L_r, \\ f_1 &= L_r^2 R_s, \quad f_0 = R_r^2 R_s. \end{aligned} \tag{5}$$

The identification method explained by Capolino and Boussat (1990) is similar to ours, although it was developed independently. The major difference is the resulting differential equation. We use a higher-order differential equation compared with that of Capolino and Boussat.

The advantage of our approach is that all the physical parameters (R_r, R_s, L_r, L_s, L_m) can be estimated independently. In their paper it is only possible to find linear combinations of the parameters (e.g. it is possible to estimate the rotor time constant, but not R_r and L_r separately). Most vector controllers use the rotor time constant and not R_r and L_r separately. The L_m parameter, which gives the saturation condition of the machine, can only be estimated with our method. It is obvious that all parameters must be known separately for design and simulation purposes. The price we have to pay to estimate all the parameters is the complexity of the identification algorithm.

Observe that there are eight coefficients and five unknown physical parameters. This means that there are nonlinear relations between these coefficients. This redundancy will be exploited later. All the voltages and currents in (2) can be measured. It is our goal to identify the coefficients of this equation.

3. IDENTIFICATION SCHEME

In this section we describe the basic idea of the identification method. Section 3.1 explains the principle of the method. Restrictions on the filters are posed, since we are dealing with a time varying system. The total least squares method, which will be explained, is used to estimate the parameters. Section 3.2 explains the use of this method for the parameter identification of the induction motor. Some statistical properties of the method are given in Section 3.3. The implementation of the algorithm is explained in Section 3.4. The results obtained for an induction motor model are given in Section 3.5.

3.1. *Principle*

First suppose that all the coefficients are constant and the rotor speed is constant. We describe the system equations in the Laplace domain. Apply a linear operator (system) $G(s)$ to both sides of the differential equation (4). If we neglect the initial conditions, the following equation results:

$$\begin{aligned} &G(s)[d_2(s^2V_s^q + \omega_r^2V_s^q) + d_1sV_s^q + d_0V_s^q] \\ &\quad = G(s)[e\omega_r sI_s^d + f_4(s^3I_s^q + \omega_r^2 sI^q) \\ &\qquad + f_3s^2I_s^q + f_2sI_s^q + f_1(s^2I_s^q + \omega_r^2I_s^q) + f_0I_s^q]. \end{aligned}$$

After measuring V_s^q, I_s^q and I_s^d, generate signals of the form $G(s)[s^2V_s^q + \omega_r^2V_s^q]$, $G(s)[sV_s^q]$, $G(s)[V_s^q]$ and $G(s)[\omega_r sI_s^d]$, etc. from the inputs and outputs. The currents are treated as inputs and the voltages as outputs. We get a linear relation between the generated signals that has the same coefficients as the differential equation (4):

$$\begin{aligned} &d_2[G(s)(s^2V_s^q + \omega_r^2V_s^q)] \\ &\qquad + d_1G(s)[sV_s^q] + d_0G(s)[V_s^q] \\ &\quad = eG(s)[\omega_r sI_s^d] + f_4G(s)[s^3I_s^q + \omega_r^2 sI^q] \\ &\qquad + f_3G(s)[s^2I_s^q] + f_2G(s)[sI_s^q] \\ &\qquad + f_1G(s)[s^2I_s^q + \omega_r^2I_s^q] + f_0G(s)[I_s^q]. \end{aligned} \tag{6}$$

The most obvious idea is to choose the order of the denominator of $G(s)$ as large as the highest order of differentiation that appears in the differential equation. This prevents the filters from amplifying the high-frequency noise. However, the fact that the real system is time-varying, and not constant as assumed above, causes a problem. In the time domain

$$G[a\omega_r(t)y(t)] \neq a\omega_r(t-\tau)G[y(t)], \tag{7}$$

where $\tau > 0$ is a delay (see below for an explanation of τ).

One cannot apply the law of convolution when $G(s)$ is applied to a time-varying differential equation. Van hamme (1992) has proved that this can only be tolerated if $G(s)$ has a flat amplitude characteristic and a linear phase characteristic. So we require

$$|G(j\omega)| = 1, \tag{8}$$

$$\angle(G(j\omega)) = k\omega \quad \text{(for some positive constant } k\text{)} \tag{9}$$

in a large frequency region. This means that the signals that will be generated ($G(s)[V_s^q]$, $G(s)[sV_s^q]$ and $G(s)[s^2V_s^q]$) are derivatives of V_s^q. So we need pure differentiators. Differentiators are not very popular in signal processing, because they amplify the noise in the signal. But high-frequency components in the signal are also greatly amplified. So the frequency band from DC to the Nyquist frequency has to be filled with the signal, which means that we need a suitable excitation of the motor. Oversampling has to be avoided because it degrades the signal-to-noise ratio, since differentiators will, in some frequency bands, only amplify the noise. The global identification scheme is shown in Fig. 1.

All the preprocessing filters, like the anti-aliasing filters and the correcting filters, form the linear operator $G(j\omega)$. All the filtering will be done with digital filters, because these can be made identical, while this is impossible with (hardware) analog filters. The anti-aliasing filters must be analog. In our case they are sixth-order

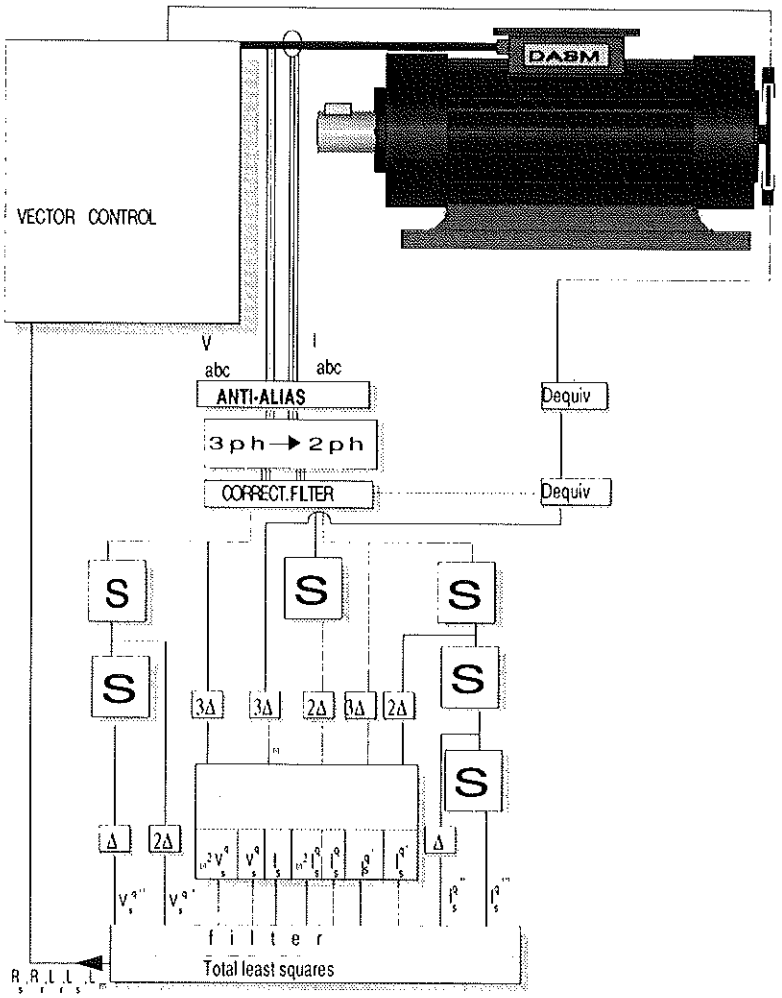

Fig. 1. Global identification scheme. The large block in the middle at the bottom of the scheme contains all the nonlinear combinations of the measured signals. The blocks labeled 'Dequiv' are delays that shift the rotor speed signal in time by the same amount as the anti-aliasing filter and the correcting filters.

Butterworth switched capacitor filters, which have a flat amplitude characteristic (needed for requirement (8)). The phase characteristic is not linear (a requirement expressed by (9)). This means that a digital filter is needed that compensates for this phase error and also for mutual differences from one anti-aliasing filter to another. Only IIR filters can be used to correct these phase errors. So we need FIR or IIR filters that behave like differentiators and IIR filters for the phase corrections. Fitting an IIR (FIR) filter of known order with a given frequency characteristic is thoroughly explained in the literature (Pärks and Burrus, 1987). The inequality (7) then becomes

$$G[a\omega_r(t)y(t)] = a\omega_r(t-\tau)G[y(t)], \qquad (10)$$

with τ the integer delay of G.

The approximation for a differentiator poses an extra problem. A phase characteristic of 90° (a property of a differentiator) is impossible to achieve, since this would mean that the phase is independent of the frequency. For linear transfer functions this is not possible. It is not strictly necessary to have a phase characteristic of 90°. A pure delay (linear phase characteristic) is also allowed, because the signals can be reshifted in time after the filter action. The characteristics of the differentiators are shown in Fig. 2. The combined characteristic of the analog anti-aliasing filter and the correcting IIR filter is shown in Fig. 3.

3.2. *Parameter estimation*

After digital filtering, we would in principle obtain signals that satisfy

$$H_e\Theta = 0, \qquad (11)$$

where

$$\Theta_e = [d_2 \quad d_1 \quad d_0 \quad e \quad f_4 \quad f_3 \quad f_2 \quad f_1 \quad f_0]^t. \qquad (12)$$

The matrix H_e is rank-deficient, as follows from

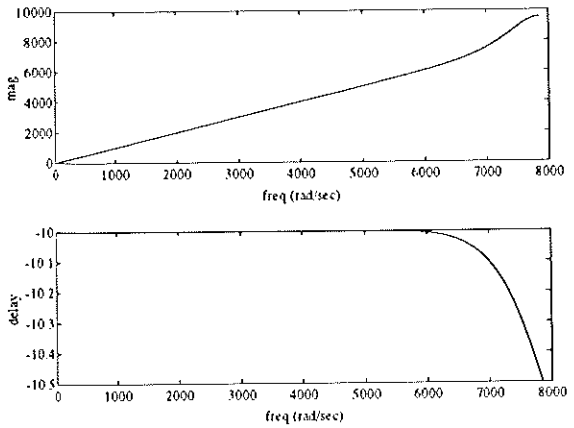

Fig. 2. Filter characteristics of the differentiators. Top: magnitude characteristic of the differentiator. Bottom: delay characteristic.

(11). Here H_e is generated by writing down (6) for every time step and putting all these vectors in one matrix. We assume that H_e has m rows, where $m > g$ (g is the number of unknowns in (12)). In practice, the exact data is corrupted with noise, which is assumed to be additive. The measured data matrix H_m then satisfies

$$H_m = H_e + H_n, \tag{13}$$

where H_e is the exact data matrix and H_n contains the additive noise signals. We assume that all the noise is generated in the measurement channel. This implies that the identification technique to be used must be capable of taking into account that all the data in the observation matrix are corrupted with noise. This can be done by using an estimation technique known as the total linear least squares (TLS) method. The mathematical background of this method is explained below. Without loss of generality, we can fix the coefficient d_2 (see (3)) at 1. Only H_m is available, because this matrix is generated from measurements. The problem now is to find,

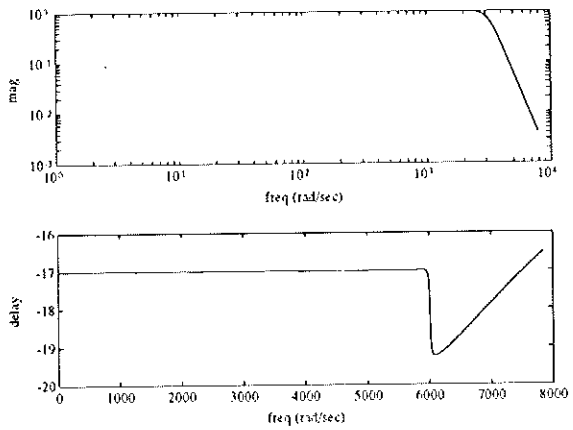

Fig. 3. Correcting filter and anti-aliasing filter. Top: magnitude characteristic. Bottom: delay characteristic.

or at least to approximate, the matrix H_e. The idea of TLS is to look for a matrix H_{apr} that, in a certain sense, is close to H_m and is rank-deficient. The linear relation, which follows from the rank deficiency, is characterized by the coefficients we are looking for. We have to define a measure of the distance between the two matrices H_m and H_{apr}, for which we take the Frobenius norm of the difference of the two matrices. So, H_{apr} as close as possible to H_m means that we must minimize the Frobenius norm of the difference, with the constraint that H_{apr} is rank-deficient. It turns out that H_{apr} can be deduced from H_m by putting the smallest singular value of the data matrix H_m equal to zero, as derived for example in Golub and Van Loan (1989).

3.3. *Statistical properties*

TLS is proved to be unbiased and statistically efficient if the noise on the data is elementwise uncorrelated and normally distributed with equal variance (see Van Huffel and Vandewalle, 1991). If the noise has a Gaussian distribution, the TLS method is the maximum likelihood estimator. In our case, because of differentiation of the data, the noise becomes elementwise-correlated. So the requirements necessary to use TLS are not fulfilled. In this case we have to use the maximum likelihood estimator (ML) (Scharf, 1992). However, ML needs much more computer power than the TLS method. In particular, in real-time applications, with higher sampling rates (kHz), the ML method cannot be used. Some extensions to the TLS method exist that allow some correlations between the noise. One of these extensions, called generalized total least squares (GTLS) (Van Huffel and Vandewalle, 1991) will be used in this paper. Both TLS and GTLS methods result in unbiased estimates if the data is generated by a system that is in the model class. GTLS, however, gives a smaller error covariance matrix than the normal TLS method. The reason is that GTLS exploits the noise structure more accurately than TLS does. The underlying idea of GTLS is that a 'prewhitening' is applied to the signals before the ordinary TLS method is used. In this way a data matrix H_m is generated with a noise matrix H_n where the requirement (14) is satisfied. For ordinary TLS the covariance matrix of the noise of the data matrix is assumed to be

$$\begin{aligned} &\mathrm{COV}_{\mathrm{noise}}(i(g-1)+j, k(g-1)+l) \\ &\quad = E(H_n^{\mathrm{t}}(i,j)H_n(k,l)) = \sigma^2\delta_{ijkl} \quad \forall i,j,k,l \end{aligned} \tag{14}$$

where $E(.)$ means expectation and $\delta_{ijkl} = 1$ if $i = j = k = l$, otherwise it is zero. Under stationary noise conditions, the matrix $\mathrm{COV}_{\mathrm{noise}}$ is a

block Toeplitz matrix. The idea of GTLS is to postmultiply the measurement data matrix H_m by a matrix T related to the noise covariance matrix COV_{noise} by

$$TT^t = COV_{noise}(1 + i(g-1), 1 + i(g-1))^{-1} \quad \forall i. \tag{15}$$

In other words, the matrix TT^t is equal to the block matrix that is repeated on the diagonal of the matrix COV_{noise}. This matrix is the expectation of the outer product of each row with itself. The formula (15) gives the well-known Choleski factorization (Golub and Van Loan, 1989). The matrix COV_{noise} is known up to a constant factor. Then COV_{noise} can be obtained by measuring the signals that are sent to the estimation algorithm with zero input (no voltage, current and speed signal). Model mismatches, which also contribute to the noise signal, are assumed to be zero (or very small). It is too difficult to incorporate these phenomena into the noise model because such effects are speed-dependent, saturation-dependent and so on.

Now we can apply the standard TLS procedure. Afterwards, we must transform the results back by multiplying again with T.

Multiplying the data matrix with the matrix T is not enough to meet all the requirements of TLS, because correlations of the noise in the columns still exist after multiplication with T. The reason is that not all the information we have about the COV_{noise} matrix is stored in T. Nevertheless, this is the only thing that can be done without increasing the computational complexity too much, and this method gives satisfactory results.

3.4. *Implementation aspects*

After having programmed the algorithm in the C language, we concluded that a sampling rate of 2500 Hz is possible when fast widespread DSP boards would be used. With our formulation of the differential equations of the motor, only parameter changes due to saturation, heating etc. are now possible. The global identification scheme is given in Fig. 1. Note the filtering block above the TLS block in the scheme, which can be used to attenuate high frequencies or as a noise 'prewhitening' filter. These noise 'prewhitening' filters can help to fulfill the requirements imposed by (14), by removing some correlations between the channels. A method to calculate these filters can be found in Van Huffel and Vandewalle (1991) (described as equilibrated GTLS) and Van Hamme (1992) (convergence of the BTLS method). But these two methods require too many computations. The high-frequency attenuations filters are not necessary for the simulation experiments, but can be useful for the practical experiments. This will be tested in the future.

3.5. *Results with the GTLS method*

Before we started with measurements on real induction motors, we generated the input and output signals, to test the method, with $MATRIX_X$ (Integrated Systems, 1991). First we modeled a motor without saturation and then we added the saturation. The modeling of the saturation was based on Krause (1987). Although not all the effects of saturation (space-dependence) can be described with this technique, it suffices because saturation is seen as a disturbance in the signal together with noise, unbalances, etc.

The test consists in reducing the rotor speed continuously and smoothly from maximum speed (300 rad s^{-1}) to stall in 4 s. The simulated motor was a 3 kW induction motor with the parameters as shown in Table 1.

Extra noise was added to the simulated signals such that the signal-to-noise ratio was 40 dB. Figure 4 shows some of the estimated coefficients as a function of time. Figure 5 shows the estimated motor parameters calculated via (5). Even under variable-speed conditions, the coefficients are estimated very well.

Table 1. Physical parameters

Parameter:	R_s	R_r	L_r	L_s	L_m
Exact value	0.94 Ω	0.61 Ω	0.0668 H	0.0668 H	0.065 H

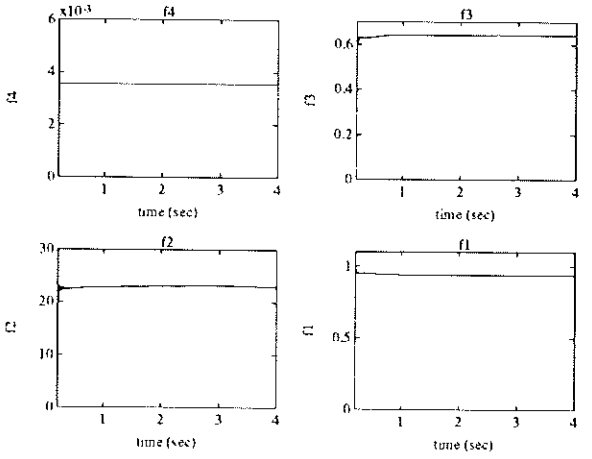

Fig. 4. Estimated transfer function coefficients. All estimates converge to the exact parameters.

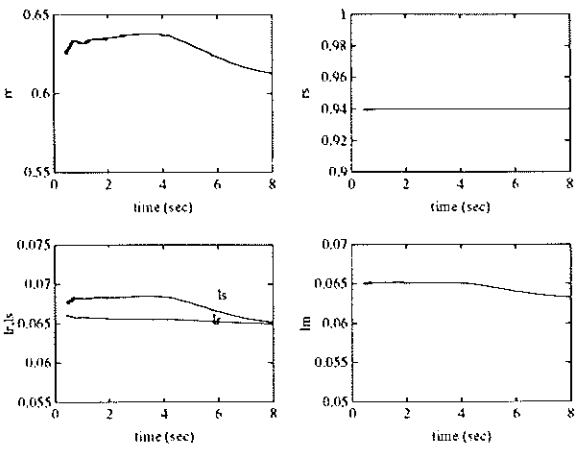

Fig. 5. Estimated induction motor parameters with the GTLS method; see (25) for exact values.

4. PARAMETER ESTIMATION WITH CONSTRAINED OPTIMIZATION

Although the previous method performs well under low-noise conditions (SNR > 40 dB), there are still problems under high-noise conditions (SNR < 25 dB). In Section 4.1 the cause of these problems is analyzed. It is then shown that some extra knowledge of the physical parameters can be incorporated to reduce the noise sensitivity of these parameters. From a mathematical point of view, this means that constraints on the coefficients must be imposed during the optimization procedure. Results are shown at the end of the Section 4.1. Section 4.2 covers the optimization method that is used.

4.1. *Principle and results*

The quality of the estimates is dramatically reduced under very low signal-to-noise conditions (SNR < 25 dB). The result of an estimation with the GTLS method under high-noise conditions (SNR = 20 dB) is shown in Table 2. The corresponding estimated physical parameters are given in Table 3.

Although the physical parameters are badly estimated under these circumstances, the Bode plot of the estimated transfer function error is much less dramatic than might be expected from the coefficient errors. The estimated transfer function fits the exact transfer function rather well at higher frequencies and high rotor speeds. Figure 6 shows the Bode plots for different rotor speeds. The two curves are the exact Bode plot

Table 3. Exact and estimated physical parameters, under low signal-to-noise conditions

Parameter:	R_s	R_r	L_r	L_s	L_m
Exact	0.94 Ω	0.61 Ω	0.0668 H	0.0668 H	0.065 H
Estimate	0.935 Ω	0.68 Ω	0.060 H	0.0547 H	0.055 H

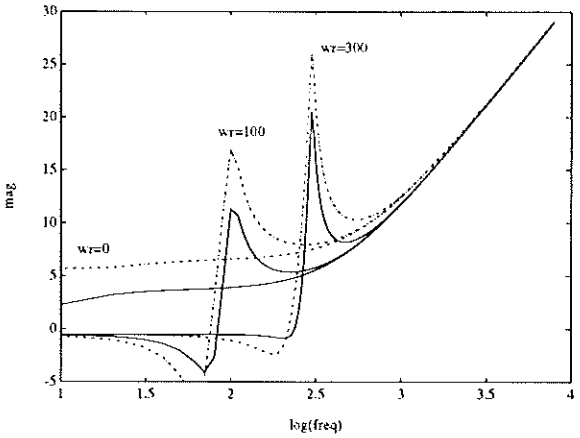

Fig. 6. Comparison of the Bode plots (magnitude) of the transfer function $H(s) = V_s^q/I_s^q$ (– · – · –, exact; ——, high-noise estimate) for different rotor speeds (0, 100 and 300).

and the estimated transfer function under high-noise conditions. The low-frequency errors are due to the lack of excitation in this frequency band. Only the base frequency and some harmonic frequencies are generated by the invertor. So only transients and low-speed conditions will generate low-frequency signals. A second problem arises under low-speed conditions. In this case there is almost a pole–zero cancellation in the transfer function. If we plot these poles and zeros, we get Fig. 7. Poles and zeros are listed for some rotor speeds in Table 4.

This sensitivity can be dramatically reduced by putting constraints on the coefficients. The idea is to connect the very sensitive parameters to the other parameters by simple nonlinear constraints, which exist because of the redundancy in the number of coefficients with respect to the unknown physical parameters:

$$f_0 = \frac{d_0 f_1}{d_2}, \quad d_0 = \frac{(\frac{1}{2}d_1)^2}{d_2}, \quad d_1 = \frac{f_3 - 2e}{f_4}. \quad (16)$$

We propose to optimize the equation error $e = H_m\theta$ (see (11) for a definition of θ), subject

Table 2. Exact and estimated parameters obtained with the GTLS method, under low signal-to-noise conditions (SNR = 20 dB)

Parameter:	d_2	d_3	d_1	e	f_4	f_3	f_2	f_1	f_0
Exact	1.00	18.26	83.39	0.578	0.0036	1.22	22.74	0.94	78.38
Estimate	1.00	22.67	1046	0.577	0.0035	1.24	28.9	0.935	750

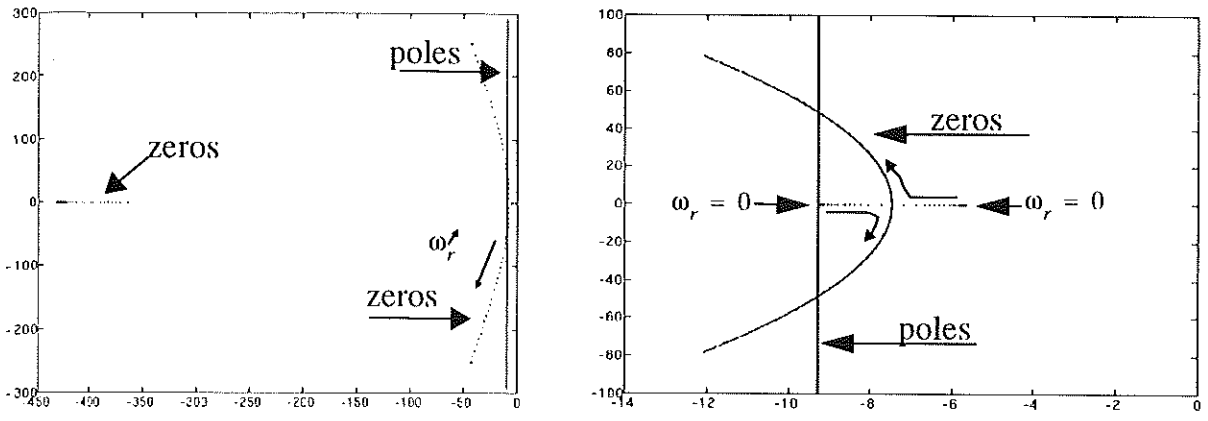

Fig. 7. Left: pole–zero plot (root locus) of the transfer function for different speeds. Right: zoom in on the pole–zero plot. The arrows near the ω_r symbol point towards larger rotor speeds. Note the pole–zero cancellation near the points $-9.28 + 0j$ and $-9.28 \pm 50j$. The real part of the poles is equal to the inverse of the rotor time constant.

Table 4. Poles and zeros of the transfer function at low speeds

ω_r	Pole 1	Pole 2	Zero 1	Zero 2	Zero 3
0	−9.28	−9.28	−9.2814	−5.6657	−433.3359
1	−9.28 + 1j	−9.28 − 1j	−9.1064	−5.8417	−433.3358
2	−9.28 + 2j	−9.28 − 2j	−8.3995	−6.5514	−433.3358
3	−9.28 + 3j	−9.28 − 3j	−7.4778 + 1.4711j	−7.4778 + 1.4711j	−433.3358
4	+9.28 + 4j	−9.28 − 4j	−7.4810 − 2.5378j	−7.4810 + 2.5278j	−433.3357
5	−9.28 + 5j	−9.28 − 5j	−7.4852 − 3.4384j	−7.4852 + 3.4384j	−433.3356

to the constraints (16) and a normalizing constrained by which the trivial solution $\theta = 0$ is avoided. Without the nonlinear constraints, this optimization problem coincides with the TLS optimization problem:

$$f(x) = \frac{\theta^t H_m^t H_m \theta}{\theta^t \theta}. \quad (17)$$

The constraints can be directly included in the optimization function (17). This reduces the number of variables (and the dimension of the search space for the optimization). This is almost impossible, however, if T (see (15)), used by the GTLS algorithm, is not a diagonal matrix. The constraints (16) are written for the unscaled parameters. Transforming the constraints for the scaled parameters will become very complex unless the matrix T is diagonal. This is never the case, because the noises are correlated. So we have to add the constraints separately to the optimization algorithm. In addition, the constraint (16) is not very robust, because two numbers, f_3 and e, of the same magnitude, are subtracted, and therefore the relative error of d_1 becomes very large. Rewriting the constraint in another form does not really solve this problem. While, in principle, the constraint (16) is valid, in practice, owing to the high relative error on d_1, it is useless. A possible solution is to add a penalty term to the optimization function. The choice of $[d_1 - (f_3 - 2e)/f_4]^2$ as a penalty term is obvious. The optimization function is then changed to

$$f(\theta) = \frac{\theta^t H_m^t H_m \theta}{\theta^t \theta} + \alpha(2e - f_3 + d_1 f_4)^2. \quad (18)$$

The importance of the penalty factor can be controlled with the factor α. It must be chosen on a trial-and-error basis. It is not possible to find a constraint that does not contain subtractions. Another approximate constraint is.

$$d_1 \approx \frac{2f_2 d_2}{2f_1 + e}. \quad (19)$$

The idea is that the mutual inductance and the rotor inductance are almost equal, which means that

$$\frac{e}{d_2} \approx R_r. \quad (20)$$

The problem of finding the coefficients now reduces to a constrained optimization problem. The solution method for this optimization problem is commented on below. Table 5 shows the results obtained with this method. The physical parameters are given in Table 6. The error covariance of this estimate can be approximated with a first-order perturbation technique. The optimization function is

$$f(\theta, \lambda_1, \lambda_2, \lambda_3) = h(\theta, H_m) + \lambda_1 c_1(\theta) + \lambda_2 c_2(\theta) + \lambda_3 c_3(\theta), \quad (21)$$

Table 5. Exact and estimated parameters (constrained optimization). The results of the unconstrained method (Table 2) are repeated also for comparison

Parameter:	d_2	d_1	d_0	e	f_4	f_3	f_2	f_1	f_0
Exact	1.00	18.26	83.39	0.578	0.0036	1.22	22.74	0.94	78.38
Unconstrained method	1.00	22.67	1046	0.577	0.0035	1.24	28.9	0.935	750
Estimate	1.00	18.67	87.0	0.572	0.0036	1.21	23.5	0.948	82.67

Table 6. Exact and estimated physical parameters; the results of the unconstrained GTLS method are given for comparison

Parameter:	R_s	R_r	L_r	L_s	L_m
Exact	0.94 Ω	0.61 Ω	0.0668 H	0.0668 H	0.065 H
Estimate (constrained)	0.948 Ω	0.62 Ω	0.067 H	0.0668 H	0.0652 H
Estimate (unconstrained)	0.935 Ω	0.68 Ω	0.060 H	0.0547 H	0.055 H

with

$$\Theta_e = [d_2 \quad d_1 \quad d_0 \quad e \quad f_4 \quad f_3 \quad f_2 \quad f_1 \quad f_0]^t$$

$$h(\theta, H_m) = \frac{\theta^t H_m^t H_m \theta}{\theta^t \theta},$$

$$c_1(\theta) = \frac{\theta(5)}{\theta(1)} - \frac{\theta(6) - 2\theta(4)}{\theta(2)}, \tag{22}$$

$$c_2(\theta) = \frac{\theta(3)}{\theta(1)} - \frac{\theta(2)^2}{4\theta(1)^2},$$

$$c_3(\theta) = \frac{\theta(9)}{\theta(1)} - \frac{\theta(8)\theta(3)}{\theta(1)^2}.$$

Calculating the derivatives with respect to θ, λ_1, λ_2 and λ_3 and setting them equal to zero gives

$$0 = H(\theta, H_m) + \lambda_1 C_1(\theta) + \lambda_2 C_2(\theta) + \lambda_1 C_3(\theta),$$
$$0 = c_1(\theta) = c_2(\theta) = c_3(\theta), \tag{23}$$

where $H(\theta, H_m)$, $C_1(\theta)$, $C_2(\theta)$ and $C_3(\theta)$ are the derivatives of the TLS function and the constraints. suppose that θ_e is the solution of the set of equations (23) under the assumption that the data is noise-free. This means that we may replace θ by θ_e in (23). Now we assume that the solution changes by small amounts $\delta\theta$, $\delta\lambda_1$, $\delta\lambda_2$ and $\delta\lambda_3$ if noise is added to the data. Equations (23) become

$$\begin{aligned} 0 = {} & H(\theta + \delta\theta, H_m + \delta H_m) + (\lambda_1 + \delta\lambda_1) C_1(\theta + \delta\theta) \\ & + (\lambda_2 + \delta\lambda_2) C_2(\theta + \delta\theta) \\ & + (\lambda_3 + \delta\lambda_3) C_3(\theta + \delta\theta), \end{aligned} \tag{24}$$

$$0 = c_1(\theta + \delta\theta) = c_2(\theta + \delta\theta) = c_3(\theta + \delta\theta).$$

After linearizing and rewirting these expressions, and taking (23) into account, we get

$$\begin{bmatrix} \delta\theta \\ \delta\lambda_1 \\ \delta\lambda_2 \\ \delta\lambda_3 \end{bmatrix} = \begin{bmatrix} \dfrac{dH}{d\theta} + \lambda_1 \dfrac{dC_1}{d\theta} + \lambda_2 \dfrac{dC_2}{d\theta} + \lambda_3 \dfrac{dC_3}{d\theta} \\ \hline C_1^t \\ C_2^t \\ C_3^t \end{bmatrix}$$

$$\begin{bmatrix} C_1 & C_2 & C_3 \\ \hline 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}^{-1} \begin{bmatrix} -\dfrac{dH}{dH_m} \delta\bar{H}_m \\ \hline -c_1 \\ -c_2 \\ -c_3 \end{bmatrix}. \tag{25}$$

Remark. The variable $\delta\bar{H}_m$ is generated from δH_m by putting all the elements of δA (data matrix) in one column. An approximation of the error covariance matrix can then be obtained from

$$\begin{aligned} \mathrm{COV} = E([\delta\theta \quad \delta\lambda_1 \quad \delta\lambda_2 \quad \delta\lambda_3]^t \\ \times [\delta\theta \quad \delta\lambda_1 \quad \delta\lambda_2 \quad \delta\lambda_3]). \end{aligned} \tag{26}$$

The covariance of this estimation method is given in Table 7 and compared with the covariance of the unconstrained GTLS solution. We start with the covariance matrix of the norm-1 solution. Then we scale everything by putting the first estimate equal to 1. This means that we divide all the parameters by the first parameter. The error covariance of this scaled solution, called $\mathrm{COV}_{sc}(\theta)$, is related to the

Table 7. Exact and estimated coefficients for the two presented methods

Parameter:	d_2	d_1	d_1	e	f_4	f_3	f_2	f_1	f_0
Value	1.00	18.26	83.39	0.578	0.0036	1.22	22.74	0.94	78.38
Covariance (unconstrained)	0	1.62	4.6×10^4	3.9×10^{-5}	5.4×10^{-11}	1.16×10^{-4}	2.5	2.61×10^{-5}	1.0×10^5
Covariance (constrained)	0	1.321	100	9.0×10^{-6}	4.0×10^{-11}	7.2×10^{-5}	1.59	1.37×10^{-5}	89

covariance of the norm-1 solution, called COV_1, by

$$\begin{aligned} COV_{sc}(\theta)(i,j) = &-\frac{\theta(i)}{\theta(1)^3} COV_1(1,j) \\ &+\frac{\theta(j)\theta(i)}{\theta(1)^4} COV_1(1,1) \\ &+\frac{COV_1(i,j)}{\theta(1)^2} \\ &-\frac{\theta(j)}{\theta(1)^3} COV_1(1,i). \end{aligned} \quad (27)$$

This expression can easily be obtained from first-order approximations.

The error covariances of the estimates of the physical parameters can easily be computed with the first-order perturbation method. The formulas to compute the physical parameters can easily be found from (5). After introducing small perturbations of all the variables, we linearize the equations. After taking the expected value of the square of the perturbations on the physical parameters, we get the error covarinces shown in Table 8. It can be seen from Table 8 that the difference between the constrained and the unconstrained method is not so dramatic as can be expected from Table 7. This is because we have omitted the badly estimated coefficients to calculate the physical parameters with the unconstrained method. Owing to the redundancy in (5), it is possible to find the physical parameters in different ways from the estimated coefficients. We have used the way that uses the fewest of poor quality estimates to have the best possible solution. Nevertheless, the quality of the estimate of the rotor parameters and the mutual inductance is considerably better.

4.2. *Algorithm*

The first version of the algorithm was implemented in Matlab. The quasi-Newton method, used in the Matlab Optimization toolbox (Grace, 1992) gives good results with good initial guesses. Several initial guesses were used to test the algorithm. All the calculations ended in the same solution. Sometimes we needed more than one run of the optimization algorithm to converge to a feasible solution because the algorithm terminates 'successfully' after a number of iterations in an infeasible point. Resetting the Hessian to the identity matrix suffices to start iterating again to the exact and feasible solution. This phenomenon has to be investigated such that a guaranteed feasible and optimum solution is found.

A second version of the algorithm was implemented in C, utilizing the optimization routines of the NAG library (NAG, 1991).

In principle (and contrary to what might be expected), the computational load does not increase very much compared with the method described in Section 3, because the algorithm can be implemented in a recursive way. The cost function (17) is updated after every sample. If a new measurement row vector h_m is calculated, it is added to the matrix H_m. The update of $H_m^t H_m$ is as follows:

$$(H_m^t H_m)_{new} = (H_m^t H_m)_{old} + h_m^t h_m.$$

This is a simple update, which can be performed very rapidly. After this updating, one new step is taken towards the optimum (one step in the quasi-Newton algorithm) and followed by adding a new row h_m to the data matrix H_m. The quasi-Newton method also uses a rank-1 update of the inverse Hessian (for an explanation of the inverse Hessian update mechanism, see Bazaraa *et al.*, 1993, p. 318). This also points towards an elegant on-line implementation, which will be the subject of forthcoming work.

5. CONCLUSIONS

In this paper two methods have been proposed to estimate the parameters of induction motor drives, both of which are capable of estimating all the physical parameters of such motors. The first was found to be too sensitive to noise. A variant of this method, which uses the TLS algorithm with extra constraints has been described in Section 4. This variant performs very well under high-noise conditions. A system based on a Siemens SIMOVERT SE3560 and an induction motor is at present being developed for on-line testing of all the described techniques.

Acknowledgement—This paper presents research results obtained within the framework of the Belgian Programme on Interuniversity Attraction Poles (IUAP-nr 17 and 50)

Table 8. Error covariances of the estimates of the physical parameters

Parameter:	R_s	L_s	R_r	L_r	L_m
Value	0.94 Ω	0.0668 Ω	0.61 H	0.0668 H	0.065 H
Covariance (unconstrained)	2.6×10^{-5}	3.48×10^{-7}	0.060	7.4×10^{-4}	1.8×10^{-4}
Covariance (constrained)	1.37×10^{-5}	2.1×10^{-7}	0.003	5.36×10^{-5}	1.7×10^{-5}

initiated by the Belgian State, Prime Minister's Office, Science Policy Programming. The scientific responsibility is assumed by its authors.

REFERENCES

Bazaraa, M., H. Sherali and C. Shetty (1993). *Nonlinear Programming.* Wiley, New York.

Capolino, G. A. and M. Boussak (1990). Induction machine parameters identification: comparison of different algorithms. In *Proc. Int. Conf. on Electrical Machinery*, 1990, p. 940–945.

Golub, G. H. and C. F. Van Loan (1989). *Matrix Computations.* The John Hopkins University Press, Baltimore.

Grace, A. (1992). *Optimization Toolbox, for use with Matlab.* Mathworks, Natick, MA.

Integrated Systems (1991). *SystemBuild V2.4 Users' Guide.* Integrated Systems, Inc., Santa Clara, CA.

Krause, P. C. (1987). *Analysis of Electric Machinery.* McGraw-Hill, New York.

Matsuo, T. and T. Lipo (1985). A rotor parameter identification scheme for vector controlled induction motor drives. *IEEE Trans. Ind. Applic.*, **IA-21** 624–632.

NAG (1991). *NAG Fortran Library Manual,* Mark 15, Vol. 4: *Minimizing or Maximizing a Function.* Nag Ltd, Oxford.

Parks, T. W. and C. S. Burrus (1987). *Digital Filter Design.* Wiley, New York.

Scharf, L. L. (1992). *Statistical Signal Processing, Detection, Estimation and Time Series Analysis.* Addison-Wesley, Reading, MA.

Schoukens, J. (1990). *System Identification, A frequency Domain Modeling Approach.* Vrije Universiteit Brussel.

Sugimoto, H. and S. Tamai (1987). Secondary resistance identification of an induction motor applied model reference adaptive system and its characteristics. *IEEE Trans. Ind. Applic.*, **IA–23** 296–303.

Van hamme, H. (1992). Identification of linear systems from time- or frequency-domain measurements. PhD dissertation, VU Brussels.

Van Huffel, S. and J. Vandewalle (1991). *The Total Least Squares Problem, Computational Aspects and Analysis.* SIAM, New York.

Zai, L.-C., C. DeMarco and T. A. Lipo (1992). An extended Kalman filter approach to rotor time constant measurements in PWM induction motor drives. *IEEE Trans. Ind. Applic.*, **IA-28**, 96–104.